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KNOT SYMMETRIC ALGEBRAS

M. Kamaraj* and R. Mangayarkarasi **

**Department of Mathematics, Government Arts College Melur - 625106, Madurai district, TamilNadu, India*

E-mail: kamarajm17366@rediffmail.co.in

***Department of Mathematics, EMG Yadava Women's CollegeMadurai - 625 014, Madurai district, TamilNadu, India*

E-mail: amuthandatas@gmail.com

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ABSTRACT

In this paper we introduce a new class of Algebra FT_n called as Knot Symmetric Algebras.We define multiplication in FT_n and prove the associativity. when $x=1$, the algebra FT_n is the symmetric group algebra FS_n.

INTRODUCTION:

Let G be a group of linear transformation on a vector space V and let $\pi^{\otimes n}$ be the representation of G on V^n = $V \otimes V \otimes ...... \otimes V$, the n-th tenser power of V. In [Br], R. Brauer defined algebras motivated by the problem of decomposing $\pi^{\otimes n}$ in to irreducible representations of G. In [PK], M. Parvathi and M. Kamaraj introduced the signed Brauer's algebras which are generalization of Brauer's algebras. In [PS], the signed Brauer's algebras have been realised as centralizer of direct product of two orthogonal groups. The symmetric group algebra FS_n have graphs without horizontal edges as generators, whereas Brauer's algebras and signed Brauer's algebras are generated by the graphs also with horizontal edges. These concepts motivated us to generalize the symmetric group FS_n. In this paper we introduce a class of algebras called as **Knot symmetric algebras** which are the generalization of symmetric group algebras. when $x=1$, the Knot symmetric algebras become as the symmetric group algebras. In this paper, we use set theoretic notions of generators, instead of graphs.

1. PRELIMINARY:

Throughout this paper let F denote a field. Let M be a finite monoid with identity e.

Algebra:

Definition: 1.1 An Algebra A over a field F is a ring A with an identity element which is at the same time a vector space F. More over the scalar multiplication in the vector space and ring multiplication are required to satisfy the axiom $\alpha(ab) = (\alpha a)b = a(\alpha b)$

Formal sums:

Definition 1.2 Let M be a finite monoid of order n. The collection $FM = \{ y : y = \sum_{m \in M} \alpha m_{\alpha}, \alpha \in F \}$ is

**Corresponding author: R. Mangayarkarasi **, *E-mail: amuthandatas@gmail.com*

constructed. This collection is said to be a collection of formal sums. Two elements y and z from this collection are considered to be equal if and only if they have the same coefficients.

The operations on the formal sums and products are defined as follows:

$$\sum_{m\in M}\alpha\, m_\alpha + \sum_{m\in M}\beta\, m_\beta = \sum_{m\in M}(\alpha+\beta)\, m_{\alpha+\beta}$$

$$\Big(\sum_{m\in M}\alpha\, m_\alpha\Big)\Big(\sum_{m\in M}\beta\, m_\beta\Big) = \sum_{m\in M}\alpha\beta\, m_\varepsilon\, m_\beta$$

With these definitions the set of all formal sums FM forms an algebra. FM is called a monoid algebra of M over F.

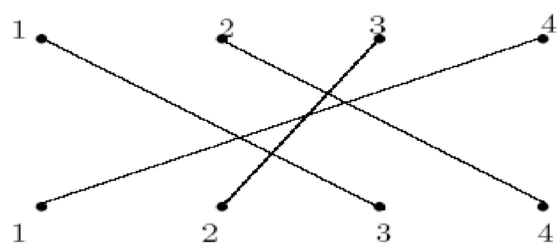
Remark: If M is a group, then FM is called group algebra.

2. KNOT SYMMETRIC ALGEBRA:

We introduce a new class of algebras which we call them as Knot Symmetric Algebras. Let S_n denote the set of all Brauer diagrams (graphs), with $2n$ vertices and without horizontal edges. For Brauer diagrams see $[w]$. Let $\pi\in S_n$ the vertices of π are represented in two rows such that each row contains n vertices. The vertices of each row are indexed with $1,2\ldots\ldots,n$ from left to right in order. Let $E(\pi)$ denote the set all edges of π.
(i.e) $E(\pi)=\{e_i=(i,\pi(i));1\le i\le n\}$

Define $S(\pi)$ as a subset of $E(\pi)\times E(\pi)$ such that $S(\pi)=\{(e_i,e_j),i\prec j\}$. It is obvious that $|S(\pi)|=\dfrac{n(n-1)}{2}$

Example:



Here $\pi=\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 2 & 1\end{pmatrix}$

$E(\pi)=\{\,e_1=(1,3),\,e_2=(2,4),e_3=(3,2),e_4=(4,1)\,\}$

$S(\pi)=\{\,(e_1,e_2),(e_1,e_3),(e_1,e_4),(e_2,e_3),(e_2,e_4),(e_3,e_4)\,\}$

Let f_π be a mapping from $S(\pi)$ to $\{-1,0,1\}$ such that

$$f_\pi\,(e_i,e_j)=\begin{cases}0 & if\ \pi(i)\prec\pi(j)\\ 1\,(or)-1 & if\ \pi(i)\succ\pi(j)\end{cases}$$

Knot mapping:

Definition: 2.1 A mapping f_π defined above is called a Knot mapping.

knot Number: Define $K(\pi)=\{(e_i\, e_j)\in S(\pi)\,;\pi(i)\succ\pi(j)\}$

Definition: 2.2 $|k\,(\pi)|$ is called Knot number of π.

Result 1: The number of Knot mappings of π is $2^{|K(\pi)|}$.

Result 2: If $(e_i,e_j)\in K\,(\pi)$ then $f_\pi(e_i,e_j)\neq 0$ for any arbitary knot mapping f_π.

Result 3: If $(e_i,e_j)\in K(\pi)$, then $f_\pi\,(e_i,e_j)+g_\pi(e_i,e_j)=0$ if and only if $f_\pi(e_i,e_j)\neq g_\pi(e_i,e_j)$.

Let x be indeterminate. Define $N(\pi)=$ $\{x^m\, f_\pi\,;m\in z, f_\pi$ is a Knot Mapping}. For any two Knot Mapping f_π and g_π, Define $E(f_\pi,g_\pi)=\{(e_i,e_j)\in K(\pi); f_\pi(e_i,e_j)+g_\pi(e_i,e_j)=0\}$

Knot Relation:

Define a relation ~ in $N(\pi)$ such that $x^m f_\pi \sim x^l\, g_\pi$ if either $m=l$ and $f_\pi=g_\pi$, or $l-m=2\sum_{(e_i,e_j)\in E(f_\pi,g_\pi)} f_\pi(e_i,e_j)$

The relation defined above is called Knot Relation. Our aim is to prove that Knot Relation is an equivalence relation. First we will prove the following Lemma.

Lemma: 2.3 Let f_π, g_π and $h_\pi\in N(\pi)$, Then $(A-B)\cup(B-A)=C$, where $A=E(f_\pi,g_\pi)$, $B=E(g_\pi,h_\pi)$, $C=E(f_\pi,h_\pi)$.

Note: In this proof, f_π, g_π and h_π are denoted by f, g and h respectively.

Proof of the Lemma: Claim $A-B\subseteq C$

Let $(e_i,e_j)\in A-B$

(i.e) $f(e_i,e_j)+g(e_i,e_j)=0$ and $g(e_i,e_j)+h(e_i,e_j)\neq 0$

(i.e) $f(e_i,e_j)+g(e_i,e_j)=0$ and $g(e_i,e_j)=h(e_i,e_j)$.

Thus $f(e_i,e_j)+h(e_i,e_j)=0$

Hence $(e_i,e_j)\in C$

Therefore $A-B\subseteq C$

similarly we can prove that $B-A\subseteq C$

Therefore $(A-B)\cup(B-A)\subseteq C$,

Conversely to prove that $C\subseteq(A-B)\cup(B-A)$

Suppose $(e_i,e_j)\in C$

(i.e) $f(e_i,e_j)+h(e_i,e_j)=0$ (1)

To prove that $(e_i,e_j)\in$ $(A-B)\cup(B-A)$

Suppose $(e_i,e_j)\notin A-B$

$(e_i,e_j)\notin A \ \ or \ \ (e_i,e_j)\in B$

suppose $(e_i,e_j)\notin A$

Then $f(e_i,e_j)+g(e_i,e_j)\neq 0$

(i.e) $f(e_i,e_j)=g(e_i,e_j)$

Using (1) we get

$g(e_i,e_j)+h(e_i,e_j)=0$ at $(e_i,e_j)\in B$

Therefore $(e_i,e_j)\in B-A$

If $(e_i,e_j)\in B$

then $g(e_i,e_j)+h(e_i,e_j)=0$

suppose $(e_i,e_j)\in A$

(i.e) $f(e_i,e_j)+g(e_i,e_j)=0$

$f(e_i,e_j)+g(e_i,e_j)+g(e_i,e_j)+h(e_i,e_j)=0$

$f(e_i,e_j)+h(e_i,e_j)=-2\,g(e_i,e_j)\neq 0$

This implies that $(e_i,e_j)\notin C$

Hence $(e_i,e_j)\notin A$

Therefore $(e_i,e_j)\in B-A$

Therefore if $(e_i,e_j)\notin A-B$ then $(e_i,e_j)\in B-A$

Therefore $C\subseteq (A-B)\cup(B-A)$

Hence $(A-B)\cup(B-A)=C$

Theorem: 2.4 The Knot relation defined above is an equivalence relation.

Proof: To prove the Equivalence relation, we need to prove the following three properties namely reflexive, symmetric, and transitive properties.

Reflexive Property: Reflexive property is obvious from the definition. If there is no confusion we will denote f , g and h instead of f_π, g_π and h_π respectively.

Symmetric Property:

Let $x^m f \sim x^l g$

If $m=l$ and $f=g$ then clearly $x^l g \sim x^m f$

Otherwise $l-m=2\sum_{(e_i,e_j)\in A} f(e_i,e_j)$

Now $m-l=-2\sum_{(e_i,e_j)\in A} f(e_i,e_j)$

$=2\sum_{(e_i,e_j)\in A} -f(e_i,e_j)$

$=2\sum_{(e_i,e_j)\in A} g(e_i,e_j)$ since $f(e_i,e_j)+g(e_i,e_j)=0$

Hence $x^l g \sim x^m f$

Transitive Property:

Let $x^m f \sim x^l g$ and $x^l g \sim x^p h$ If $m=l$ and , $f=g$, $l=p$ and $g=h$

Then clearly $m=p$ and $f=h$

(i.e) $x^m f \sim x^p h$

If $m=l$, $f=g$ and $p-l= 2\sum_{(e_i,e_j)\in B} g(e_i,e_j)$

Therefore $p-m= 2\sum_{(e_i,e_j)\in C} f(e_i,e_j)$

Hence $x^m f \sim x^p h$

Similarly we can prove that,

if $l-m=2\sum_{(e_i,e_j)\in A} f(e_i,e_j)$ and $p=l,\ g=h$

If $l-m=2\sum_{(e_i,e_j)\in A} f(e_i,e_j)$ and $p-l= 2\sum_{(e_i,e_j)\in B} g(e_i,e_j)$

$p-m=l-m+p-l$

$=2\sum_{(e_i,e_j)\in A} f(e_i,e_j)+2\sum_{(e_i,e_j)\in B} g(e_i,e_j)$

$=2\{\sum_{(e_i,e_j)\in (A-B)} f(e_i,e_j)+\sum_{(e_i,e_j)\in (A\cap B)} f(e_i,e_j)+\sum_{(e_i,e_j)\in (B-A)} g(e_i,e_j)+\sum_{(e_i,e_j)\in (B\cap A)} g(e_i,e_j)\}$

$=2\{\sum_{(e_i,e_j)\in (A-B)} f(e_i,e_j)+\sum_{(e_i,e_j)\in (B-A)} g(e_i,e_j)+\sum_{(e_i,e_j)\in (A\cap B)} f(e_i,e_j)+\sum_{(e_i,e_j)\in (B\cap A)} g(e_i,e_j)\}$

$=2\{\sum_{(e_i,e_j)\in (A-B)} f(e_i,e_j)+\sum_{(e_i,e_j)\in (B-A)} g(e_i,e_j)+\sum_{(e_i,e_j)\in (A\cap B)} (f(e_i,e_j)+g(e_i,e_j))\}$

Since if $(e_i,e_j)\in A$, we have $f(e_i,e_j)+g(e_i,e_j)=0$

$=2\{\sum_{(e_i,e_j)\in (A-B)} f(e_i,e_j)+\sum_{(e_i,e_j)\in (B-A)} g(e_i,e_j)\}=2\sum_{(e_i,e_j)\in (A-B)\cup(B-A)} f(e_i,e_j)$

$= 2\sum_{(e_i,e_j)\in C} f(e_i,e_j)$ since $(A-B)\cup(B-A)=C$

Hence $x^m f \sim x^p h$. Hence the Knot relation is an Equivalence relation.

Let $\overline{N(\pi)}=N(\pi)/\sim$. That is $\overline{N(\pi)}$ is the collection of disjoint equivalence classes with respect to the Knot relation.

Define $T_n=$ { $(\pi, x^m f_\pi):\pi\in S_n, f_\pi\in\overline{N(\pi)}$ and m is an integer}

we define multiplication in T_n as follows.

Let $a, b \in T_n$ and $a = (\pi, x^m f_\pi), b = (\sigma, x^l g_\pi)$

Define $ab = (\sigma o \pi, x^{m+l+\alpha} h_{\sigma o \pi})$, where α and $h_{\sigma o \pi}$ are defined as follows.

Let $(e_i^{`}, e_j^{`}) \in S(\sigma o \pi), (u_i, u_j) \in S(\pi), (v_p, v_q) \in S(\sigma), p, q \in \{\pi(i), \pi(j)\}$

$f_\pi (u_i, u_j) = u$ and $g_\sigma (v_p, v_q) = v$

Now $\alpha = \sum_{(e_i^{`}, e_j^{`}) \in s(\sigma o \pi)^{`}} \alpha(e_i^{`}, e_j^{`})$ where $\alpha(e_i^{`}, e_j^{`}) = (u+v)|uv|$ and $h_{\sigma o \pi}(e_i^{`}, e_j^{`}) = (u+v)(1-\delta_{u,v})$,

where $\delta(u,v) = \begin{cases} 0 & if\ u \neq v \\ 1 & if\ u = v \end{cases}$

KNOT MULTIPLICATION:

Definition: 2.5 The multiplication defined above is called Knot multiplication. Let $(e_i, e_j) \in S(\gamma o \sigma o \pi)$, then there exists $(u_i, u_j) \in S(\pi)$, $(v_p, v_q) \in S(\sigma), (w_r, w_s) \in S(\gamma)$ such that $\{p, q\} = \{\pi(i), \pi(j)\}$ and $\{r, s\} = \{\sigma o \pi(i), \sigma o \pi(j)\}$

Lemma 2.6 If $u, v \in \{0, 1, -1\}$ and $u \neq v$ then $(u+v)|uv| = 0$

Proof: suppose $u = 0$ *or* $v = 0$ then the result is obvious, otherwise , if $u = 1$ then $v = -1$, hence $(u+v)|uv| = 0$

similarly, if $u = -1$, then $v = 1$, , therefore $(u+v)|uv| = 0$

Result 2.7 If $u, v \in \{0, 1, -1\}, u \neq v, v \neq 0$, then $u(u+v) = 0$ and $|u \quad |(u+v) = 0$

Proof: If $u = 0$, the result is obvious, otherwise $u + v = 0$ that is $u(u+v) = 0$

Theorem 2.8 The Knot multiplication is associative in T_n.

Proof: Let $a, b, c \in T_n, a = (\pi, x^m f_\pi), b = (\sigma, x^l g_\sigma), c = (\gamma, x^t p_\gamma)$.
Claim $(ab)c = a(bc)$. we know that $.\gamma o(\sigma o \pi) = (\gamma o \sigma) o \pi$. Let $ab = (\sigma o \pi, x^{m+l+\alpha} h_{\sigma o \pi})$. where α and $h_{\sigma o \pi}$ are as follows:

Let $(e_i^{`}, e_j^{`}) \in S(\sigma o \pi), (u_i, u_j) \in S(\pi), (v_p, v_q) \in S(\sigma)$. Where $\{p, q\} = \{\pi(i), \pi(j)\}$.

Let $f_\pi (u_i, u_j) = u, g_\sigma(v_p v_q) = v$

Now $\alpha = \sum_{(e_i^{`}, e_j^{`}) \in s(\sigma o \pi)} \alpha(e_i^{`}, e_j^{`})$, where $\alpha(e_i^{`}, e_j^{`}) = (u+v)|uv|$

$h_{\sigma o \pi}(e_i^{`}, e_j^{`}) = (u+v)(1-\delta_{u,v})$

Let $(ab)c = (\gamma o(\sigma o \pi), x^{m+l+t+\alpha+\beta} h_{\gamma o(\sigma o \pi)})$. Where β and $h_{\gamma o(\sigma o \pi)}$ are as follows:

Let $(e_i, e_j) \in S(\gamma o(\sigma o \pi)), (w_r, w_s) \in S(\gamma)$. where $\{r, s\} = \{(\sigma o \pi)(i), (\sigma o \pi)(j)\}$.

Let $h_{\sigma o \pi}(e_i^{`}, e_j^{`}) = e$, $p_{\gamma}(w_r, w_s) = w$

Now $\beta = \sum_{(e_i, e_j) \in S(\gamma o(\sigma o \pi))} \beta(e_i, e_j)$

$\beta(e_i, e_j) = (e+w)\left|e\, w\right|$

$h_{\gamma o(\sigma o \pi)}(e_i, e_j) = (e+w)(1-\delta_{e,w})$

Let $bc = (\gamma o \sigma, x^{l+t+\eta}\, h^{`}_{\gamma o \sigma})$, where $\eta$ and $h^{`}_{\gamma o \sigma}$ are defined as follows:

Let $(v_p^{`}, v_q^{`}) \in S(\gamma o \sigma)$,

Now $\eta = \sum \eta(v_p^{`}, v_q^{`})$

$\eta(v_p^{`}, v_q^{`}) = (v+w)\left|v\, w\right|$

$h^{`}_{\gamma o \sigma}(v_p^{`}, v_q^{`}) = (v+w)(1-\delta_{v,w})$

Let $a(bc) = ((\gamma o \sigma) o \pi, x^{m+l+t+\eta+\xi}\, h^{`}_{(\gamma o \sigma) o \pi})$, where $\xi$ and $h^{`}_{(\gamma o \sigma) o \pi}$ are defined as follows:

Let $\{p, q\} = (\pi(i), \pi(j))$,

$h^{`}_{\gamma o \sigma}(v_p^{`}, v_q^{`}) = y$

Now $\xi = \sum_{(e_i, e_j) \in S((\gamma o \sigma) o \pi)} \xi(e_i, e_j)$

$\xi(e_i, e_j) = (u+y)\left|u\, y\right|$

$h^{`}_{(\gamma o \sigma) o \pi}(e_i, e_j) = (u+y)(1-\delta_{u,y})$

Claim: $(ab)c = a(bc)$

We know that $(\gamma o \sigma) o \pi = \gamma o(\sigma o \pi)$

Let $(e_i, e_j) \in S(\gamma o \sigma o \pi)$

By the definition of $\overline{N}(\gamma o \sigma o \pi)$, we should prove either

$h_{\gamma o(\sigma o \pi)}(e_i, e_j) = h^{`}_{(\gamma o \sigma) o \pi}(e_i, e_j)$ and $\eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j) = \alpha(e_i^{`}, e_j^{`}) + \beta(e_i, e_j)$

and (or) $h_{\gamma o(\sigma o \pi)}(e_i, e_j) + h^{`}_{(\gamma o \sigma) o \pi}(e_i^{`}, e_j^{`}) = 0$

and $\eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j) - \alpha(e_i^{`}, e_j^{`}) - \beta(e_i, e_j) = 2h_{\gamma o(\sigma o \pi)}(e_i, e_j)$

case 1: $u = v = w$

$h_{\sigma o \pi}(e_i^{`}, e_j^{`}) = (u+v)(1-\delta_{u,v}) = 0 = e$

$\alpha(e_i^{`}, e_j^{`}) = (u+v)\left|u\, v\right| = 2u\left|u\ \ \right| = 2u \qquad \sin ce \quad \left|u^2\ \ \right| = \left|u\ \ \right|$ (1)

$$
\begin{aligned}
&h_{\gamma o(\sigma o \pi)}(e_i,e_j)=(e+w)(1-\delta_{e,w})\\
&=w(1-\delta_{o,w})\\
&=u(1-\delta_{0,u}) \qquad (2)\\
&\beta(e_i,e_j)=(e+w)\left|e\,w\right|\\
&=0
\end{aligned}
$$

$$
\begin{aligned}
&h_{\gamma o\sigma}(v_p`,v_q`)=(v+w)(1-\delta_{v,w})\\
&=0\\
&=y \quad (\text{since } \ v=w;\delta_{v,w}=1)\\
&\eta(v_p`,v_q`)=(v+w)\left|v\,w\right|\\
&=2u\left|u\quad\right| \qquad (3)\\
&h`_{(\gamma o\,\sigma)o\pi}(e_i,e_j)=(u+y)(1-\delta_{u,y})\\
&=u(1-\delta_{u,o}) \quad \to (4)\\
&\xi(e_i,e_j)=(u+y)\left|u\,y\right|\\
&=0
\end{aligned}
$$

Hence $h_{\gamma o(\sigma o \pi)}(e_i,e_j)=h`_{(\gamma o \sigma)o\pi}(e_i,e_j)$ and $\eta(v_p`,v_q`)+\xi(e_i,e_j)=\alpha(e_i`,e_j`)+\beta(e_i,e_j)$

case 2: $u=v\neq w$

$$
\begin{aligned}
&h_{\sigma o\pi}(e_i`,e_j`)=(u+v)(1-\delta_{u,v})=0=e\\
&\alpha(e_i`,e_j`)=2u\left|u\quad\right|=2u\\
&h_{\gamma o(\sigma o \pi)}(e_i,e_j)=(e+w)(1-\delta_{e,w})=(w)(1-\delta_{0,w})\\
&\beta(e_i,e_j)=(e+w)\left|e\,w\right|=0\\
&h_{\gamma o\sigma}(v_p`,v_q`)=(v+w)(1-\delta_{v,w})=(v+w)=y\\
&\eta(v_p`,v_q`)=(v+w)\left|v\,w\right|\\
&=0 \quad by \quad the\ lemma\ (2.6)\\
&h`_{(\gamma o\sigma)o\pi}(e_i,e_j)=(u+y)(1-\delta_{u,y})=(u+v+w)(1-\delta_{u,v+w})=(2u+w)(1-\delta_{u,v+w})\\
&\xi(e_i,e_j)=(u+y)\left|u\,y\right|=(u+v+w)\left|u\,(v+w)\right|=(2u+w)\left|u\,(u+w)\right|
\end{aligned}
$$

Subcase 1: $w=0$

$$
\begin{aligned}
&h_{\gamma o(\sigma o \pi)}(e_i,e_j)=0\\
&h`_{(\gamma o\sigma)o\pi}(e_i,e_j)=0\\
&\alpha(e_i`,e_j`)=2u\\
&\eta(v_p`,v_q`)=0\\
&\xi(e_i,e_j)=2u
\end{aligned}
$$

Hence $h_{\gamma o(\sigma o \pi)}(e_i,e_j)=h`_{(\gamma o\sigma)o\pi}(e_i,e_j)$

and $\alpha(e_i`,e_j`)+\beta(e_i,e_j)=2u$

$$=\eta(v_p`,v_q`)+\xi(e_i,e_j)$$

Subcase 2: $w \neq 0$

$h_{\gamma o(\sigma o \pi)}(e_i, e_j) = w$

$h'_{(\gamma o \sigma) o \pi}(e_i, e_j) = (2u + w)$

$\eta(v_p', v_q') = 0$

$\xi(e_i, e_j) = 0, \quad by\ the \quad result \quad (2.7)$

$\alpha(e_i', e_j') = 2u$

If $u = 0$

$h_{\gamma o(\sigma o \pi)}(e_i, e_j) = w$

$= h'_{(\gamma o \sigma) o \pi}(e_i, e_j)$ and

$\alpha(e_i', e_j') + \beta(e_i, e_j) = 0$

$= \eta(v_p', v_q') + \xi(e_i, e_j)$

If $u \neq 0$ then $u + w = 0$

$h_{\gamma o(\sigma o \pi)}(e_i, e_j) + h'_{(\gamma o \sigma) o \pi}(e_i, e_j) = 2(u + w) = 0$

$\alpha(e_i', e_j') + \beta(e_i, e_j) - \eta(v_p', v_q') - \xi(e_i, e_j) = 2u = 2\, h_{(\gamma o \sigma) o \pi}(e_i, e_j)$

Case 3: $u \neq v = w$

we can prove this case , by similar arguments in case (2)

Case 4: $u \neq v \neq w$

$h_{\sigma o \pi}(e_i', e_j') = (u + v)(1 - \delta_{u,v}) = u + v = e$

$\alpha(e_i', e_j') = (u + v)\left|u\, v\right| = 0$

$h_{\gamma o(\sigma o \pi)}(e_i, e_j) = (e + w)(1 - \delta_{e,w}) = (u + v + w)(1 - \delta_{u+v,w})$

$\beta(e_i, e_j) = (u + v + w)\left|(u + v)\, w\right|$

$h_{\gamma o \sigma}(v_p', v_q') = (v + w)(1 - \delta_{v,w})$

$= (v + w)$

$= y$

$\eta(v_p', v_q') = (v + w)\left|v\, w\right|$

$= 0$

$h'_{(\gamma o \sigma) o \pi}(e_i, e_j) = (u + y)(1 - \delta_{u,y}) = (u + v + w)(1 - \delta_{u,v+w})$

$\xi(e_i, e_j) = (u + v + w)\left|u\,(v + w\right|$

Subcase 1: $u = 0$

since $u = 0, v \neq 0$

$h_{\gamma o(\sigma o\pi)}(e_i, e_j) = (u+w)(1-\delta_{v,w}) = v+w$

$\beta(e_i, e_j) = (v+w)\left|vw\quad\right| = (v+w)\left|w\quad\right| = 0$

$h^{`}_{(\gamma o\sigma)o\pi}(e_i, e_j) = (v+w)(1-\delta_{0,v+w})$

$\xi(e_i, e_j) = 0$

If $w = 0$

$h_{\gamma o(\sigma o\pi)}(e_i, e_j) = v = h^{`}_{(\gamma o\sigma)o\pi}(e_i, e_j)$

$\alpha(e_i^{`}, e_j^{`}) + \beta(e_i, e_j) = 0 = \eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j)$

otherwise $v + w = 0$

$h_{\gamma o(\sigma o\pi)}(e_i, e_j) = 0 = h^{`}_{(\gamma o\sigma)o\pi}(e_i, e_j)$

$\alpha(e_i^{`}, e_j^{`}) + \beta(e_i, e_j) = 0 = \eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j)$

Subcase: 2: $u \neq 0$

Now either $v = 0 \quad (or) \quad u + v = 0$

$\xi(e_i, e_j) = (u+v+w)\left|v+w\right|$

]If $w = 0$ then $u + v = 0$

$h_{\gamma o(\sigma o\pi)}(e_i, e_j) = (u+v)(1-\delta_{u+v,0}) = 0$

$\beta(e_i, e_j) = 0$

$\xi(e_i, e_j) = 0$

$h^{`}_{(\gamma o\sigma)o\pi}(e_i, e_j) = 0$

Hence $h_{\gamma o(\sigma o\pi)}(e_i, e_j) = h^{`}_{(\gamma o\sigma)o\pi}(e_i, e_j)$

and $\alpha(e_i^{`}, e_j^{`}) + \beta(e_i, e_j) = 0$

$= \eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j)$

If $w \neq 0$ then $v = 0$

(or) $v + w = 0$ and $u + v = 0$

If $v = 0$

$h_{\gamma o(\sigma o\pi)}(e_i, e_j) = (u+w)(1-\delta_{u,w})$

$= 0\,(either \quad u = w \quad (or) \quad u + w = 0)$

$h_{(\gamma o\sigma)o\pi}(e_i, e_j) = 0$

$\beta(e_i, e_j) = (u+w)\left|u\,w\right| = u+w$

$\xi(e_i, e_j) = (u+w)\left|u\,w\right| = u+w$

Hence $h_{\gamma o(\sigma o\pi)}(e_i, e_j) = 0$

$= h^{`}_{(\gamma o\sigma)o\pi}(e_i, e_j)$

$\alpha(e_i^{`}, e_j^{`}) + \beta(e_i, e_j) = u + w$

$= \eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j)$

If $v+w=0$ and $u+v=0$

$$h_{\gamma o(\sigma o \pi)}(e_i, e_j) = u$$

$$\beta(e_i, e_j) = 0$$

$$h^{`}_{(\gamma o \sigma) o \pi}(e_i, e_j) = u$$

$$\xi(e_i, e_j) = 0$$

Therefore $h_{\gamma o(\sigma o \pi)}(e_i, e_j) = u$

$$= h^{`}_{(\gamma o \sigma) o \pi}(e_i, e_j)$$

and $\alpha(e_i^{`}, e_j^{`}) + \beta(e_i, e_j) = 0$

$$= \eta(v_p^{`}, v_q^{`}) + \xi(e_i, e_j)$$

Hence the result.

Theorem: 2.9 FT_n is algebra.

Proof: FT_n is the algebra generated by the elements of T_n over the field F.

Corollary: 2.10 When $x=1$ the Knot symmetric algebras FT_n become as the symmetric group algebras FS_n.

Proof: when $x=1, T_n = S_n$.

REFERENCES:

[PK] M. parvathi and M. Kamaraj, signed Brauer's algebras, communications in algebra, 26(3), 839 – 855(1998).

[Br] R. Brauer, algebras which are connected with the semisimple continuous groups, Ann.of math. 38(1937), 854 – 872.

[W] H. wenzl, on the structure of Brauer's centralizer algebras, Ann. of math (128)(1988) 173 – 193.

[PS] M. Parvathi and C. Selvaraj signed Brauer's algebras as centralizer algebras. communication in algebra 27(12), 5985 – 5998(1999).

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