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TRAVERSABILITY OF BLOCK LINE GRAPHS

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ABSTRACT

The black line graph $B_l(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G, with two points adjacent if one corresponds to a point of G and other to a line incident with it or one corresponds to a block B of G and other to a point v of G and v is in B. In this paper, we establish a necessary and sufficient condition for the block line graph of a connected graph to be eulerian. Also we obtain a characterization of graphs whose block line graphs are hamiltonian.

Keywords: *block line graph, eulerian graph, hamiltonian graph.*

Mathematics Subject Classification: *05C.*

1. INTRODUCTION

The graphs considered in this paper are finite, undirected without loops and multiple lines. Any undefined term here may be found in Kulli [1].

If $B = \{u_1, u_2,\ldots,u_r, r\geq 2\}$ is a block of a graph *G*, then we say that point u_1 and block *B* are incident with each other, as are u_2 and *B* and so on. If two distinct blocks B_1 and B_2 of *G* are incident with a common cut point, then they are adjacent blocks. This idea was introduced by Kulli in [2]. The points, lines and blocks are called its members.

The block line graph $B_l(G)$ of a graph *G* is the graph whose point set is the union of the set of points, lines and blocks of *G*, with two points adjacent if one corresponds to a point and other to a line incident with it or one corresponds to a block *B* of *G* and other to a point *v* of *G* and *v* is in *B*. This concept was introduced by Kulli in [3]. Many other graph valued functions in graph theory were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

In this paper, we establish a characterization of graphs whose block line graphs are eulerian. Also some properties of hamiltonian block line graphs are obtained. Traversability of some graph valued functions were studied, for example, in [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

We consider graphs without isolated points.

The following result will be useful to prove our results.

Theorem A [1, *p*76]: A connected graph *G* is eulerian if and only if every point of *G* is the even degree.

2. EULERIAN BLOCK LINE GRAPHS.

Remark 1: If *v* is point of a graph *G* and v_1 is the corresponding point of *v* in $B_l(G)$, then $\deg_{B_l(G)} v_1 = \deg_G v + m$, whose *m* is the number of blocks containing *v*.

Remark 2: If *e* is a line of a graph *G* and e_1 is the corresponding point of *e* in $B_l(G)$, then $\deg_{B_l(G)} e_1 = 2$.

Remark 3: If *B* is a block of a graph *G* and B_1 is the corresponding point of *B* in $B_l(G)$, then $\deg_{B_l(G)} B_1 = n$ where *n* is the number of points in *B*.

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Theorem 4: Let *G* be a nontrivial graph. If *G* is eulerian then $B_l(G)$ is not eulerian.

Proof: Suppose *G* is a nontrivial eulerian graph. Then *G* is connected and by Theorem A, every point of *G* is of even degree. Since every nontrivial connected graph has at least two noncutpoints, it implies that *G* has a noncutpoint *v* of even degree. By Remark 1, we see that $\deg_{B_l(G)} v_1$ is odd, where v_1 is the corresponding point of *v* in $B_l(G)$. Thus by Theorem A, $B_l(G)$ is not eulerian.

A necessary and sufficient condition for a graph whose block line graph is eulerian is presented in the following theorem.

Theorem 5: Let *G* be a nontrivial connected graph. The block line graph $B_l(G)$ is eulerian if and only if *G* satisfies the following conditions.

1. each noncutpoint of *G* is incident with odd number of lines,
2. each cutpoint of *G* is incident with either even number of lines and even number of blocks or odd number of lines and odd number of blocks, and
3. each block of *G* is incident with even number of points.

Proof: Suppose $B_l(G)$ is eulerian. Let *v* be a point of $B_l(G)$. Then *v* is a point or a line or a block of *G*. We have the following 3 cases.

Case-1: Suppose *v* is a point of *G*. Then by Remark 1,

$$\deg_{B_l(G)} v = \deg_G v + m$$

where *m* is the number of blocks containing *v*.

We consider the following two subcases.

Subcase-1: Suppose *v* is a noncutpoint of *G*. Then $m = 1$. By Theorem A, $\deg_{B_l(G)} v$ is even. Hence $\deg_G v$ is odd.

Thus (1) holds.

Subcase-2: Suppose *v* is a cutpoint of *G*. By Theorem A $\deg_{B_l(G)} v$ is even. Hence both $\deg_G v$ and *m* are either even or odd. Thus (2) holds.

Case-2: Suppose *v* is a line *e* of *G*. Then by Remark 2, $\deg_{B_l(G)} v = 2.$

Case-3: Suppose *v* is a block *B* of *G*. Then by Remark 3, deg $\deg_{B_l(G)} B = n,$ where *n* is the number of points in *B*. By Theorem A, $\deg_{B_l(G)} B$ is even. Thus *n* is even. Thus (3) holds.

Conversely suppose (1), (2) and (3) hold. Suppose *v* is a point of $B_l(G)$. Then *v* is a point or a line or a block of *G*. If *v* is a point of *G*, then *v* is either a noncutpoint or a cutpoint of *G*. If *v* is a noncutpoint of *G*, then by Condition (1) and Remark 1, $\deg_{B_l(G)} v$ is even. If *v* is a cutpoint of *G*, then by Condition (2) and Remark 1, $\deg_{B_l(G)} v$ is even. If *v* is a line *e* of *G*, then by Remark 2, $\deg_{B_l(G)} e$ is even. If *v* is a block *B* of *G*, then by Condition (3) and Remark 3, $\deg_{B_l(G)} B$ is even. Thus every point of $B_l(G)$ is of even degree. By Theorem A, $B_l(G)$ is eulerian.

Corollary 6: If *G* is a nontrivial path, then $B_l(G)$ is eulerian.

Proof: This follows from Theorem 5.

Corollary 7: If *G* is a cycle, then $B_l(G)$ is not eulerian.

Proof: This follows from Theorem 4.

3. HAMILTONIAN BLOCK LINE GRAPHS

Remark 8[3]: If *v* is a cut point in *G*, then the corresponding point v_1 of *v* in $B_l(G)$ is also a cutpoint.

Proposition 9: If a connected graph *G* has a cut point, then $B_l(G)$ is not hamiltonian.

Proof: This follows from Remark 8.

We obtain a characterization of graphs whose block line graphs are hamiltonian.

Theorem 10: The block line graph $B_l(G)$ of G is hamiltonian if and only if G is P_2.

Proof: Suppose G is P_2. Then $B_l(G) = C_4$ and hence $B_l(G)$ is hamiltonian.

Conversely suppose $B_l(G)$ is hamiltonian. We now prove that $G = P_2$. On the contrary, assume $G \neq P_2$. We now consider the following cases.

Case-1: Suppose G is disconnected. Then $B_l(G)$ is disconnected Hence $B_l(G)$ is not hamiltonian.

Case-2: Suppose G is a connected graph with a cutpoint. By Proposition 9, $B_l(G)$ has a cutpoint and hence $B_l(G)$ is not hamiltonian.

Case-3: Suppose G is a block B with $p \geq 3$ points. Then G has a cycle $C_n = v_1\, v_2\, v_3 \ldots\, v_n\, v_1$, $n \geq 3$. In $B_l(G)$,

$C_{2n} = v_1\, e_1\, v_2\, e_2 \ldots e_{n-1}\, v_n\, e_n\, v_1$ is a cycle. Let u be a point in $B_l(G)$ corresponding to the block B. Then u is adjacent with all the points v_i, $1 \leq i \leq n$, in $B_l(G)$ Since every pair of points v_i and v_j are not adjacent in $B_l(G)$, it implies that $B_l(G)$ has a subgraph homeomorphic to $K_{2,3}$. Thus $B_l(G)$ is not Hamiltonian.

Thus from the above 3 cases, we conclude that $G = P_2$.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R. Kulli, The semitotal block graph and the total-block graph of a graph, *Indian J. Pure Appl. Math.*, 7, 625-630 (1976).
3. V.R. Kulli, On block line graphs, middle line graphs and middle block graphs, *International Journal of Mathematical Archive*, 6(5), 80-86 (2015).
4. V.R. Kulli, On the plick graph and the qlick graph of a graph, *Research Journal*, 1, 48-52 (1988).
5. V.R.Kulli, On line block graphs, *International Research Journal of Pure Algebra*, 5(4), 40-44 (2015).
6. V.R.Kulli, The block line forest of a graph, *Journal of Computer and Mathematical Sciences*, 6(4), 200-205 (2015).
7. V.R. Kulli, On full graphs, *Journal of Computer and Mathematical Sciences*, 6(5), 261-267 (2015).
8. V.R. Kulli, The semifull graph of a graph, *Annals of Pure and Applied Mathematics*, 10(1), 99-104 (2015).
9. V.R. Kulli, On qlick transformation graphs, *International Journal of Fuzzy Mathematical Archive*, 8(1), 29-35 (2015).
10. V.R. Kulli, On semifull line graphs and semifull block graphs, *Journal of Computer and Mathematical Sciences*, 6(7) (2015) 388-394.
11. V.R. Kulli and D.G.Akka, On semientire graphs, *J. Math. and Phy. Sci*, 15, 585-589 (1981).
12. V.R. Kulli and N.S.Annigeri, The ctree and total ctree of a graph, *Vijnana Ganga*, 2, 10-24 (1981).
13. V.R. Kulli and *B.*Basavanagoud, On the quasivertex total graph of a graph, *J. Karnatak University Sci.*, 42, 1-7 (1998).
14. V.R. Kulli and M.S. Biradar, The line splitting graph of a graph, *Acta Ciencia Indica*, 28, 57-64 (2001).
15. V.R. Kulli and M.S. Biradar, The point block graph of a graph, *Journal of Computer and Mathematical Sciences*, 5(5), 476-481 (2014).
16. V.R.Kulli and M.H.Muddebihal, Lict and litact graph of a graph, *Journal of Analysis and Computation*, 2, 33-43 (2006).
17. V.R. Kulli and K.M.Niranjan, The semi-splitting block graph of a graph, *Journal of Scientific Research*, 2(3) (2010) 485-488.
18. V.R. Kulli and N.S. Warad, On the total closed neighbourhood graph of a graph, *J. Discrete Mathematical Sciences and Cryptography*, 4, 109-114 (2001).
19. V.R. Kulli, On middle neighborhood graphs, *International Journal of Mathematics and its Applications*, 3 (4-D), 79-83 (2015).
20. V. R. Kulli, The neighborhood graph of a graph, International Journal of Fuzzy Mathematical Archive, 8(2), 93-99 (2015).
21. V.R.Kulli and D.G.Akka, Traversability and planarity of total block graphs. *J. Mathematical and Physical Sciences*, 11, 365-375 (1977).
22. V.R. Kulli and D.G.Akka, Traversability and planarity of semitotal block graphs, *J Math. and Phy. Sci.*, 12, 177-178(1978).
23. V.R. Kulli and M.S. Biradar, The middle blict graph of a graph, *International Research Journal of Pure Algebra* 5(7), 111-117 (2015).
24. V.R. Kulli and M.S. Biradar, On eulerian blict graphs and blitact graphs, submitted.

25. V.R. Kulli and M.S. Biradar, On eulerian line splitting graphs, submitted.
26. V.R.Kulli, B Janakiram and K.M. Niranjan, The vertex minimal dominating graph *Acta Ciencia Indica*, 28, 435-440 (2002).
27. V.R.Kulli, B. Janakiram and K.M. Niranjan, The dominating graph, *Graph Theory Notes of New York*, *New York Academy of Sciences*, 46, 5-8 (2004).
28. B.Basavanagoud and V.R.Kulli, Traversability and planarity of quasi-total graphs, *Bull. Cal. Math. Soc.*, 94(1), 1-6 (2002).
29. B.Basavanagoud and V.R.Kulli, Hamiltonian and eulerian properties of plick graphs, *The Mathematics Student*, 74(1-4), 175-181 (2004).

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