

PRIME RING WITH SYMMETRIC SKEW 4- REVERSE DERIVATIONS

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ABSTRACT

In this paper we introduce the notion of symmetric skew 4-reverse derivation of prime ring and we consider R be a non commutative 2,3- torsion free ring, I be a non zero two sided ideal of R . Suppose α is an anti automorphism of R , and $D: R \times R \times R \times R \rightarrow R$ be a symmetric skew 4-reverse derivation associated with the anti automorphism α . If f is trace of D such that $[f(x), \alpha(x)] = 0$, for all $x \in I$, then $D = 0$.

Key words: prime ring, Reverse Derivation, Symmetric Skew 3- derivation, Symmetric Skew 4- reverse derivation and Anti automorphism.

INTRODUCTION

Bresar and Vukman [3] have introduced the notion of a reverse derivations and Samman and Alyamani [9] have studied some properties of semi prime rings with reverse derivations. AjdaFosner [1] have introduced the notation of symmetric skew 3- derivations of prime or semi prime rings and proved that under certain conditions a prime ring with a non zero symmetric skew 3-derivation has to be commutative. The study of centralizing and commuting mappings on prime rings was initiated by the result of Posner [8] which states that the existence of a non zero centralizing derivation on a prime ring implies that the ring has to be commutative. Vukman [10, 11] investigated symmetric bi derivation on prime and semi prime rings in connection with centralizing mappings. C.JayaSubba Reddy *et al.* [5] has studied prime ring with symmetric skew 3- reverse derivations. Faiza Shujat and Abuzaid Ansari [4] has studied symmetric skew 4-derivations on prime rings. In this paper we proved that under certain conditions a prime ring with a non zero symmetric skew 4-reverse derivation has to be commutative.

PRELIMINARIES

Throughout the paper, R will represent a ring with a center Z and α an anti automorphism of R . Let $n \geq 2$ be an integer. A ring R is said to be n -torsion free if for $x \in R, nx = 0$ implies $x = 0$. For all $x, y \in R$ the symbol $[x, y]$ will denote the commutator $xy - yx$. Recall that a ring R is semi prime if $xRx = 0$, implies that $x = 0$. An additive map $d: R \rightarrow R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. An additive map $d: R \rightarrow R$ is called reverse derivation if $d(xy) = d(y)x + yd(x)$, for all $x, y \in R$ and it is called a skew derivation (α -derivation) of R associated with the automorphism or anti automorphism α if $d(xy) = d(x)y + \alpha(x)d(y)$, for all $x, y \in R$, and it is called a skew reverse derivation (α - reverse derivation) of R associated with auto morphism or anti automorphism α if $d(xy) = xd(y) + \alpha(y)d(x)$, for all $x, y \in R$.

Before starting our main theorem, let us gives some basic definations and well known results which we will need in our further investigation.

Let D be a symmetric 3-additive map of R , then obviously

$$D(-x, y, z) = -D(x, y, z), \text{ for all } x, y, z \in R \tag{1}$$

Namely, for all $y, z \in R$, the map $D(\cdot, y, z): R \rightarrow R$ is endo morphissm of the additive group of R .

The map $f: R \rightarrow R$ defined by $f(x) = D(x, x, x, x)$, $x \in R$ is called trace of D

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Note that f is not additive on R . But for all $x, y \in R$, we

$$f(y + x) = f(y) + f(x) + 3D(y, x, x) + 3D(x, y, y)$$

Recall also that by (1), f is odd function

More precisely, for all $x, y, z, u, v, w \in R$, we have

$$D(xu, y, z) = xD(u, y, z) + \alpha(u)D(x, y, z),$$

$$D(x, yv, z) = yD(x, v, z) + \alpha(v)D(x, y, z),$$

$$D(x, y, zw) = zD(x, y, w) + \alpha(w)D(x, y, z).$$

Of course, if D is symmetric, then the above three relations are equivalent to each other.

MAIN RESULT

Theorem: Let R be a 2,3 –torsion free non commutative prime ring and I be a nonzero ideal of R . Suppose α is an anti automorphism of R and $D: R^4 \rightarrow R$ is a symmetric skew 4- reverse derivation associated with α . If f is trace of D such that $[f(x), \alpha(x)] = 0$, for all $x \in I$, then $D = 0$.

Proof: Let $[f(x), \alpha(x)] = 0$, for all $x \in I$. (2)

Linearization of (2) yields that

$$[f(x + y), \alpha(x + y)] = 0$$

$$[f(x) + 4D(x, x, x, y) + 6D(x, x, y, y) + 4D(x, y, y, y) + f(y), \alpha(x)] = 0$$

$$[f(x), \alpha(x)] + 4[D(x, x, x, y), \alpha(x)] + 6[D(x, x, y, y), \alpha(x)] + 4[D(x, y, y, y), \alpha(x)] + [f(y), \alpha(x)] + [f(x), \alpha(y)] + 4[D(x, x, x, y), \alpha(y)] + 6[D(x, x, y, y), \alpha(y)] + 4[D(x, y, y, y), \alpha(y)] + [f(y), \alpha(y)] = 0, \text{ for all } x \in I. \quad (3)$$

From (2) & (3), we get

$$4[D(x, x, x, y), \alpha(x)] + 6[D(x, x, y, y), \alpha(x)] + 4[D(x, y, y, y), \alpha(x)] + [f(y), \alpha(x)] + [f(x), \alpha(y)] + 4[D(x, x, x, y), \alpha(y)] + 6[D(x, x, y, y), \alpha(y)] + 4[D(x, y, y, y), \alpha(y)] = 0, \text{ for all } x \in I. \quad (4)$$

Replacing y by $-y$ in (4) we find

$$-4[D(x, x, x, y), \alpha(x)] + 6[D(x, x, y, y), \alpha(x)] - 4[D(x, y, y, y), \alpha(x)] + [f(y), \alpha(x)] - [f(x), \alpha(y)] + 4[D(x, x, x, y), \alpha(y)] - 6[D(x, x, y, y), \alpha(y)] + 4[D(x, y, y, y), \alpha(y)] = 0, \text{ for all } x \in I. \quad (5)$$

Comparing (4) and (5) and using 2-torsion freeness of R we get

$$4[D(x, x, x, y), \alpha(x)] + 4[D(x, y, y, y), \alpha(x)] + [f(x), \alpha(y)] + 6[D(x, x, y, y), \alpha(y)] = 0, \text{ for all } x \in I. \quad (6)$$

Substitute $y + z$ for y in (6) and use (6) to get

$$4[D(x, x, x, y + z), \alpha(x)] + 4[D(x, y + z, y + z, y + z), \alpha(x)] + [f(x), \alpha(y + z)] + 6[D(x, x, y + z, y + z), \alpha(y + z)] = 0$$

$$4[D(x, x, x, y), \alpha(x)] + 4[D(x, x, x, y), \alpha(x)] + 4[D(x, y, y, y), \alpha(x)] + 4[D(x, y, y, z), \alpha(x)] + 4[D(x, y, z, y), \alpha(x)] + 4[D(x, y, z, z), \alpha(x)] + 4[D(x, z, y, y), \alpha(x)] + 4[D(x, z, y, z), \alpha(x)] + 4[D(x, z, z, y), \alpha(x)] + 4[D(x, z, z, z), \alpha(x)] + [f(x), \alpha(y)] + [f(x), \alpha(z)] + 6[D(x, x, y, y), \alpha(y)] + 6[D(x, x, y, z), \alpha(y)] + 6[D(x, x, z, y), \alpha(y)] + 6[D(x, x, z, z), \alpha(y)] + 6[D(x, x, y, y), \alpha(z)] + 6[D(x, x, y, z), \alpha(z)] + 6[D(x, x, z, y), \alpha(z)] + 6[D(x, x, z, z), \alpha(z)] = 0$$

$$12[D(x, x, x, y), \alpha(x)] + 12[D(x, z, y, y), \alpha(x)] + [D(x, x, y, z), \alpha(y)] + 6[D(x, x, z, z), \alpha(y)] + 6[D(x, x, y, y), \alpha(z)] + 12[D(x, x, y, z), \alpha(z)] = 0, \text{ for all } x, y, z \in I \quad (7)$$

Replacing z in $-z$ in (7) and compare with (7) we obtain

$$-12[D(x, x, x, y), \alpha(x)] + 12[D(x, z, y, y), \alpha(x)] + 12[D(x, x, y, z), \alpha(y)] - 6[D(x, x, z, z), \alpha(y)] + 6[D(x, x, y, y), \alpha(z)] - 12[D(x, x, y, z), \alpha(z)] = 0$$

$$2(12[D(x, z, y, y), \alpha(x)] + 12[D(x, x, y, z), \alpha(y)] + 6[D(x, x, y, y), \alpha(z)]) = 0$$

Using of two torsion free ring we have

$$12[D(x, z, y, y), \alpha(x)] + 12[D(x, x, y, z), \alpha(y)] + 6[D(x, x, y, y), \alpha(z)] = 0, \text{ for all } x, y, z \in I. \quad (8)$$

Substitute $y + u$ for y in (8) and use (8) we get

$$12[D(x, z, y + u, y + u), \alpha(x)] + 12[D(x, x, y + u, z), \alpha(y + u)] + 6[D(x, x, y + u, y + u), \alpha(z)] = 0.$$

$$12[D(x, z, y, y), \alpha(x)] + 12[D(x, z, y, u), \alpha(x)] + 12[D(x, z, u, y), \alpha(x)] + 12[D(x, z, u, u), \alpha(x)] \\ + 12[D(x, x, y, z), \alpha(y)] + 12[D(x, x, u, z), \alpha(y)] + 12[D(x, x, y, z), \alpha(u)] \\ + 12[D(x, x, u, z), \alpha(u)] + 6[D(x, x, y, y), \alpha(z)] + 6[D(x, x, y, u), \alpha(z)] + 6[D(x, x, u, y), \alpha(z)] \\ + 6[D(x, x, u, u), \alpha(z)] = 0.$$

$$24[D(x, z, y, u), \alpha(x)] + 12[D(x, x, y, z), \alpha(u)] + 12[D(x, x, u, z), \alpha(y)] + 12[D(x, x, y, u), \alpha(z)] = 0, \\ \text{for all } x, y, z \in I. \tag{9}$$

Since R is 2 and 3-torsion free and replacing y, u by x in (9), we have

$$24[D(x, z, x, x), \alpha(x)] + 12[D(x, x, x, z), \alpha(x)] + 12[D(x, x, x, z), \alpha(x)] + 12[D(x, x, x, x), \alpha(z)] = 0. \\ 48[D(x, x, x, z), \alpha(x)] + 12[D(x, x, x, x), \alpha(z)] = 0. \\ 4[D(x, x, x, z), \alpha(x)] + [f(x), \alpha(z)] = 0, \text{ for all } x, z \in I. \tag{10}$$

Again replaced z by yz in (10) and using (10) we obtain

$$4[D(x, x, x, yz), \alpha(x)] + [f(x), \alpha(yz)] = 0 \\ 4[yD(x, x, x, z) + \alpha(z)D(x, x, x, z), \alpha(x)] + [f(x), \alpha(z)\alpha(y)] = 0 \\ 4y[D(x, x, x, z), \alpha(x)] + 4[y, \alpha(x)]D(x, x, x, z) + 4\alpha(z)[D(x, x, x, z), \alpha(x)] + 4[\alpha(z), \alpha(x)]D(x, x, x, z) \\ + [f(x), \alpha(z)]\alpha(y) + \alpha(z)[f(x), \alpha(y)] = 0 \\ \alpha(z)(4[D(x, x, x, z), \alpha(x)] + [f(x), \alpha(y)] + 4y[D(x, x, x, z), \alpha(x)] \\ + 4[y, \alpha(x)]D(x, x, x, z) + 4[\alpha(z), \alpha(x)]D(x, x, x, z) + [f(x), \alpha(z)]\alpha(y) = 0 \\ 4y[D(x, x, x, z), \alpha(x)] + 4[y, \alpha(x)]D(x, x, x, z) + 4[\alpha(z), \alpha(x)]D(x, x, x, z) + [f(x), \alpha(z)]\alpha(y) = 0, \\ \text{for all } x, y, z \in I. \tag{11}$$

Substitute x for z in (11) and view of (2) we find

$$4[y, \alpha(x)]f(x) = 0, \text{ for all } x, y \in I. \tag{12}$$

Using 2- torsion freeness of R we obtain

$$[y, \alpha(x)]f(x) = 0, \text{ for all } x, y \in I. \tag{13}$$

Substitute zy for y to get

$$[zy, \alpha(x)]f(x) = 0$$

$$[z, \alpha(x)]yf(x) + z[y, \alpha(x)]f(x) = 0$$

$$[z, \alpha(x)]yf(x) = 0, \text{ for all } x, y \in I. \tag{14}$$

Primeness of R yields that $[z, \alpha(x)] = 0$ or $f(x) = 0$, for all $x \in I \cap Z(R), z \in I$.

Next we will show that $f(x) = 0$, for all $x \in I$.

Let $z \in I \cap Z(R)$ and $x \in I \setminus Z(R)$.

Then $x + z, x - z \in I / Z(R)$ and we have

$$f(x + z) = f(z) + 4D(x, x, x, z) + 4D(x, z, z, z) + 6D(x, x, z, z) \tag{15}$$

and

$$f(x + z) = f(z) - 4D(x, x, x, z) - 4D(x, z, z, z) + 6D(x, x, z, z) \tag{16}$$

Comparing (15) & (16) and using 2- torsion free condition, we get

$$f(z) + 6D(x, x, z, z) = 0. \tag{17}$$

On suitable linearization and using (17) we arrive at $f(x) = 0$, for all $x \in I$. Hence we have $D(x, y, z, w) = 0$, for all $x, y, z, w \in I$.

Substitute xr for x for all $x \in I, r \in R$ to get

$$D(xr, y, z, w) = 0$$

$$xD(r, y, z, w) + \alpha(r)D(x, y, z, w) = 0$$

$$xD(r, y, z, w) = 0 \tag{18}$$

This implies that $ID(r, y, z, w) = 0$, for all $y, z, w \in I, r \in R$.

Since R is prime we obtain $D(r, y, z, w) = 0$, for all $y, z, w \in I, r \in R$.

Repeating this process until we get $D(r, s, t, p) \in R$.

Hence $D = 0$.

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