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# ON PRE GENERALIZED PRE REGULAR WEAKLY INTERIOR AND PRE GENERALIZED PRE REGULAR WEAKLY CLOSURE IN TOPOLOGICAL SPACES

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***ABSTRACT***

*In this paper, the notation of pgprw-interior and pgprw-closure are introduced and studied in topological spaces. It is proved that the complment of pgprw-interior of A is the closure of the complement of A and some properties of the new concepts have been studied.*

***Keywords:*** *Pgprw closed sets, Pgprw open sets, Pgprw-closure, Pgprw-interior.*

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## 1. INTRODUCTION

N.Levine[1] introduced the concept of generalized sets of a topological space in 1970. Dunham[2] defined generalized closure operator in 1982.Mashhour, Abd El-Monsef and Deeb[3] introduced the concept of pre-closed sets in 1982. Gnanambal[4] introduced and studied the concept of gpr closed sets in topological space. Gnanambal and Balachandran[5], introduced and studied the concept of gpr- interior and gpr-closure operator in topological space in 1999. Recently in the year 2015 Wali and Vivekananda Dembre [6] introduced and studied Pre generalized pre regular weakly closed sets (briefly-pgprw)and Pre generalized pre regular weakly open [7] sets in topological spaces.

## 2. PRELİMİNARİES

A subset A of a topological space $(X, \tau)$ is called

(i) Generalized closed set(briefly g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(ii) Pre-open set if A $\subseteq$ int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.
(iii) Generalized pre regular closed set(briefly gpr-closed) if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.
(iv) A subset A of topological space $(X,\tau)$ is called a pre generalized pre regular ωeakly closed set(briefly pgprω-closed set) if pCl(A) $\subseteq$ U whenever A $\subseteq$ U and U is rgα open in $(X,\tau)$.
(v) A subset A in $(X,\tau)$ is called Pre generalized pre regular weakly open set in X if $A^c$ is Pre generalized pre regular weakly closed set in X.

Throughout this paper space $(X, \tau)$ and $(Y, \sigma)$ (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), $A^c$, P-Cl(A) and P-int(A) denote the Closure of A, Interior of A, Compliment of A, pre closure of A and pre-interior of A in X respectively.

## 3. PGPRW-CLOSURE AND PGPRW- INTERIOR IN TOPOLOGICAL SPACES

In this section the notation of pgprw-closure and pgprw-Interior is defined and some of its basic properties are studied.

**Definition 3.1:** For a subset A of $(X, \tau)$, pgprw-closure of A is denoted by pgprw-cl(A) and defined as pgprw-cl(A)= $\cap$\{G: A $\subseteq$ G, G is pgprw–closed in $(X,\tau)$\} or $\cap$\{G:A $\subseteq$ G, G$\epsilon$ pgprw-C(X)\}

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**Theorem 3.2:** If A and B are subsets of space (X, $\tau$) then

(i) pgprw-cl (X) = X, pgprw-cl(ϕ) = ϕ
(ii) A ⊆ pgprw-cl(A).
(iii) If B is any pgprw-closed set containing A, then pgprw-cl(A) ⊆ B.
(iv) If A⊆B then pgprw-cl(A) ⊆ pgprw-cl(B)
(v) pgprw-cl(A) = pgprw-cl(pgprw-cl(A))
(vi) pgprw-cl(A∪B) = Pgprw-cl(A)∪pgprw-cl(B).

**Proof:**

(i) By definition of pgprw-closure, X is only pgprw-closed set containing X. Therefore pgprw-cl(X) = Intersection of all the pgprw-closed set containing X=∩{X} = X therefore pgprw-cl (X) = Xand again by definition of pgprw-closure. Pgprw-cl(ϕ) = Intersection of all pgprw-closed sets containing ϕ = ϕ∩any pgprw-closed set containing ϕ = ϕ. Therefore pgprw-cl(ϕ)=ϕ.
(ii) By definition of pgprw-closure of A, it is obvious that A ⊆ pgprw-cl(A).
(iii) Let B be any pgprw-closed set containing A. Since pgprw-cl(A) is the intersection of all pgprw-closed set containing A, pgprw-cl(A) is contained in every pgprw-closed set containing A. Hence in particular pgprw-cl(A)⊆B
(iv) Let A and B be subsets of (X,$\tau$) such that A⊆B by definition of pgprw-closure,
Pgprw-cl(B)=∩{F:B ⊆F ϵ pgprw-C(X)}.If B ⊆F ϵ pgprw-C(X), then pgprw-cl(B) ⊆ F. since A ⊆B, A⊆B ⊆ Fϵpgprw-C(X), we have pgprw-cl(A)⊆F, pgprw-cl(A)⊆∩{F : B ⊆F ϵ pgprw-C(X)} = pgprw-cl(B).
Therefore pgprw-cl(A) ⊆ pgprw-cl(B).
(v) Let A be any subset of X by definition of pgprw-closure , pgprw-cl(A) = ∩{F : A ⊆F $\epsilon$ pgprw-C(X)}. If A ⊆F ϵ pgprw-C(X) then pgprw-cl(A) ⊆ F, since F is pgprw-closed set containing pgprw-cl(A) by (iii) pgprw-cl(pgprw-cl(A))⊆F;Hence pgprw-cl(pgprw-cl(A))=∩{F:A⊆F$\epsilon$pgprw-C(X)}=pgprw-cl(A).Therefore; pgprw-cl(pgprw-cl(A))=pgprw-cl(A).
(vi) Let A and B be subsets of X, Clearly A⊆A∪B, B⊆A∪B from (iv) pgprw-cl(A)⊆ pgprw-cl(A∪B),pgprw-cl(B)⊆pgprw-cl(A∪B);hence, pgprw-cl(A)∪ pgprw-cl(B)⊆ pgprw-cl(A∪B)……..(1) Now we have to prove that pgprw-cl(A∪B) ⊆ pgprw-cl(A)∪ pgprw-cl(B).Suppose x∉pgprw-cl(A)∪ pgprw-cl(B) then ∃pgprw-closed set $A_1$ and $B_1$ with A⊆$A_1$ , B⊆$B_1$ and x∉$A_1$∪$B_1$.We have A∪B⊆$A_1$∪$B_1$ and $A_1$∪$B_1$ is the pgprw-closed set(we know that union of two pgprw closed subsets of X is pgprw closed set in X ) such that x∉$A_1$∪$B_1$.Thus x∉pgprw-cl(A∪B).
(vii) Hence pgprw-cl(A∪B)⊆pgprw-cl(A)∪ pgprw-cl(B)------(2). From (1) and (2) we have pgprw-cl(A∪B)= pgprw-cl(A)∪pgprw-cl(B).

**Theorem 3.3:** If A ⊆X is pgprw-closed set then pgprw-cl(A) =A

**Proof:** Let A be pgprw-closed subset of X. we know that A ⊆ pgprw-cl(A) –(1)Also A⊆A and A is pgprw-closed set by theorem3.2 (iii) pgprw-cl(A)⊆A –(2)Hence pgprw-cl(A) =A.

The Converse of the above need not be true as seen from the following example.

**Example 3.4:** Let X={a,b,c,d}, $\tau$={X,ϕ,{a},{c,d},{a,c,d}}.A={a,c} pgprw-cl(A)={a,c}=A then A is not pgprw-closed set.

**Theorem 3.5:** If Aand B are subsets of space X then pgprw-cl(A∩B)⊆pgprw-cl(A)∩pgprw-cl(B).

**Proof:** Let A and B be subsets of X, Clearly A∩B ⊆ A, A∩B ⊆ B by theorem 3.2(iv) Pgprw-cl(A∩B)⊆pgprw-cl(A), Pgprw-cl(A∩B)⊆pgprw-cl(B); hence pgprw-cl(A∩B) ⊆ pgprw-cl(A)∩pgprw-cl(B).

**Remark 3.6:** In-general; pgprw-cl(A)∩pgprw-cl(B)⊈pgprw-cl(A∩B) as seen from the following example.

**Example 3.7:** Consider X={a, b, c, d}, $\tau$={X, ϕ,{a},{b},{a, b},{a, b, c}}, A={b, c}, B={c, d}, A∩B={c}, pgprw-cl(A) = {b, c, d}, pgprw-cl(B) ={c, d}, pgprw-cl(A∩B)={c} and pgprw-cl(A)∩pgprw-cl(B)={c,d} therefore pgprw-cl(A)∩pgprw-cl(B) ⊈pgprw-cl(A∩B).

**Theorem 3.8:** For an x $\epsilon$X, x$\epsilon$ pgprw-cl(A) if and if A∩V ≠ ϕ for every pgprw-open set V containing x.

**Proof:** Let x$\epsilon$ pgprw-cl(A). To prove A∩V ≠ ϕ for every pgprw-open set V containing x by contradiction.

 

Suppose ∃pgprw-open set V containing x s.t A∩V = ϕ. Then A ⊆ X-V, X-V is pgprw-closed set, pgprw-cl(A) ⊆ X-V. This shows that x∉pgprw-cl(A) which is contradiction. Hence A∩V ≠ ϕ for every pgprw-open set V containing x. Conversely: Let A∩V ≠ ϕ for every pgprw-open set V containing x. To prove x$\epsilon$ pgprw-cl(A). we prove the result by contradiction. Suppose x∉pgprw-cl(A) then there exist a pgprw-closed subset Fcontaining A s.t x ∉ F. Then x$\epsilon$X-F is pgprw-open.Also (X-F)∩A=ϕ which is contradiction. Hence x$\epsilon$ pgprwcl(A).

**Theorem 3.9:** If Ais subset of space X then

(i) Pgprw-cl(A) ⊆ cl(A).
(ii) Pgprw-cl(A) ⊆ pcl(A).

**Proof:**

(i) Let A be subset of space X by definition of Closure Cl(A) = ⊆∩{F: A ⊆F ϵ C(X)}.If A⊆ F ϵ C(X) then A⊆F ϵ pgprw-C(X) because every closed set is pgprw-closed that is pgprw-cl(A)⊆F therefore pgprw-cl(A) ⊆∩{F: A⊆F $\epsilon$ C(X)} = cl(A). Hence pgprw-cl(A) ⊆ cl(A).
(ii) Let A be subset of space X by definition of p-closure pcl(A)= ⊆∩{F: A ⊆F ϵ p-C(X)}.If A⊆ F ϵ p-C(X) then A⊆F ϵ pgprw-C(X) because every p-closed set is pgprw-closed that is pgprw-cl(A) ⊆ F therefore pgprw-cl(A) ⊆∩{F: A⊆F $\epsilon$ p-C(X)} = pcl(A). Hence pgprw-cl(A) ⊆ p-cl(A)

**Remark 3.10:** Containment relation in the above theorem3.9 may be proper as seen from following example**.**

**Example 3.11:** Let X={a, b, c, d}, $\tau$={X, ϕ,{a},{b},{a, b}, {a, b, c}}, A={a} cl(A)={a,c,d} pgprw-cl(A)={a,d}, pcl(A) = {a, c, d} It follows that pgprw-cl(A)⊂cl(A) and pgprw-cl(A) ⊂ pcl(A)

**Theorem 3.12:** If A is subset of space X then gpr-cl(A) ⊆ pgprw-cl(A) where gpr-cl(A) = ∩{F: A ⊆ F ϵ GPR-C(X)}

**Proof:** Let A be a subset of X by definition of pgprw-closure, pgprw-cl(A) = ∩{F: A ⊆ F ϵ pgprw-C(X)} If A ⊆ F ϵ pgprw-C(X) then A ⊆ F ϵ GPR-C(X), because every pgprw-closed is gpr-closed i.e gpr-cl(A) ⊆ F therefore gpr-cl(A) ⊆ ∩{F : A ⊆ F ϵ pgprw-C(X)}= pgprw-cl(A). Hence gpr-cl(A) ⊆ pgprw-cl(A).

**Theorem 3.13:** Pgprw-closure is a Kuratowski closure operator on a space X.

**Proof:** Let A and B be the subsets space X.

(i) pgprw-cl(X) = X, pgprw-cl(ϕ) = ϕ
(ii) A ⊆ pgprw-cl(A)
(iii) pgprw-cl(A) = pgprw-cl (pgprw-cl(A))
(iv) pgprw-cl(A∪B) = Pgprw-cl(A)∪pgprw-cl(B) by theorem 3.2
Hence pgprw-closure is a Kuratowski closure operator on a space X.

**Definition 3.14:** For a subset A of (X, $\tau$), pgprw-interior of A is denoted by pgprw-int(A) and defined as pgprw-int(A)= ∪{G: G ⊆ A and G is pgprw-open in X} or ∪{G: G ⊆ A and G ϵpgprw-O(X)}
i.e pgprw-int(A) is the union of all pgprw-open set contained in A.

**Theorem 3.15:** Let A and B be subset of space X then

(i) pgprw-int(X) = X, pgprw-int(ϕ)=ϕ
(ii) pgprw-int(A) ⊆ A
(iii) If B is any pgprw-open set contained in A, then B⊆ pgprw-int(A)
(iv) IfA⊆B then pgprw-int(A)⊆pgprw-int(B)
(v) pgprw-int(A)= pgprw-int(pgprw-int(A))
(vi) pgprw-int(A∩B)=pgprw-int(A)∩ pgprw-int(B)

**Proof :** (i) and (ii) by definition of pgprw-interior of A, it is obvious.

(iii) Let B be any pgprw-open set s.t B ⊆ A. Let x ϵB , B is an pgprw-open set contained in A, x is an pgprw-interior of A i.e. x ϵpgprw-int(A). Hence B ⊆ pgprw-int(A) &(iv), (v), (vi) similar proof as theorem 3.2 and definition of pgprw-interior.

**Theorem 3.16:** If a subset A of X is pgprw-open then pgprw-int(A) = A

**Proof**: Let A be pgprw-open subset of X. We know that pgprw-int(A) ⊆ A –(1) Also A is pgprw-open set contained in Afrom Theorem 3.15 (iii) A ⊆ pgprw-int(A) --(2). Hence From(1) and (2) pgprw-int(A) = A

 

**Theorem 3.17:** If A and B are subsets of space X then pgprw-int(A)∪ pgprw-int(B)⊆ pgprw-int(A∪B)

**Proof:** We know that A ⊆ A∪B and B ⊆ A∪B, We have Theorem 3.15(iv) pgprw-int(A) ⊆ pgprw-int(A∪B) and pgprw-int(B )⊆ pgprw-int(A∪B). This implies that pgprw-int(A)∪ pgprw-int(B)⊆ A∪B

**Remark 3.18:** The converse of the above theorem need not be true as seen from the following example.

**Example 3.19:** Let X={a, b, c, d} and $\tau$={X, ϕ, {a},{b},{a, b}, {a, b, c}}, A={b, c}, B ={a, d}, A∪B={a, b, c, d}, pgprw-int(A)={b, c}, pgprw-int(B)={a}, pgprw-int(A∪B)=X, pgprw-int(A) ∪ pgprw-int(B) = {a, b, c}; therefore pgprw-int(A∪ B) ⊈ pgprw-int(A)∪ pgprw-int(B).

**Theorem 3.20:** If A is a subset of X then (i) int(A) ⊆ pgprw-int(A)(ii) p-int(A) ⊆ pgprw-int(A).

**Proof:**

(i) Let A be a subset of a space X. Let x ϵint(A) => x ϵ ∪{G: G is open, G ⊆ A}=>∃ an open set G s.t. x ϵ G ⊆ A =>∃ an pgprw-open set G s.t. x ϵ G ⊆ A, as every open set is an pgprw-open set in X=> x ϵ ∪{G : G is pgprw-open set in X}=> x ϵ pgprw-int(A). Thus x ϵ int(A) => x ϵ pgprw-int(A). Hence int(A)⊆ pgprw-int(A).

(ii) Let A be a subset of a space X. Let x ϵ p − int(A) => x ϵ ∪{G : G is p-open, G ⊆ A}=>∃ an p-open set G s.t. x ϵ G ⊆ A =>∃ an pgprw-open set G s.t. x ϵ G ⊆ A, as every p-open set is an pgprw-open set in X=> x ϵ ∪{G: G is pgprw-open set in X}=> x ϵ pgprw-int(A). Thus x ϵ p-int(A) x ϵ pgprw-int(A). Hence p-int(A)⊆ pgprw-int(A).

**Remark 3.21:** Containment relation in the above theorem may be proper as seen from the followingexample

**Example 3.22** Let X={a, b, c, d} $\tau$={X, ϕ, {a},{b},{a, b}, {a, b, c}}; A={b, c}, Int(A)={b}, p-int(A) = {b}, Pgprw-int(A)={b,c}thereforeint(A)⊆pgprw-int(A) and p-int(A)⊆pgprw-int(A)

**Theorem 3.23:** If A issubset of X, then pgprw-int(A)⊆ gpr-int(A), where gpr-int(A) is given by gpr-int(A) = ∪{G ⊆ X: G is gpr-open, G ⊆ A}

**Proof:** Let A be a subset of a space X. Let x ϵpgprw −int(A) => x ϵ ∪{G: G is pgprw-open , G ⊆ A}=>∃ an pgprw-open set G s.t. x ϵ G ⊆ A, as every pgprw-open set is an gpr-open set in X=> x ϵ ∪{G: G is gpr-open, G ⊆ A}=> xϵ gpr-int(A). Thus xϵ pgprw-int(A) => xϵ gpr-int(A) Hence pgprw-int(A) ⊆ gpr-int(A).

**Theorem 3.24:** For any subset A of X

(i) X – pgprw-int(A) = pgprw-cl(X – A)
(ii) pgprw-int(A) = X – pgprw-cl(X – A)
(iii) pgprw-cl(A) = X – pgprw-int(X – A)
(iv) X – pgprw-cl(A) = pgprw-int(X – A)

**Proof:**

(i) xϵX-pgprw-int(A) then x is not in pgprw-int(A) i.e. every pgprw-open set G containing x s.t. G ⊈ A. This implies every pgprw open set G containing x intersects (X – A) i.e. $G \cap (X - A) \neq \phi$. Then by theorem 3.8 xϵ pgprw-cl(X-A) thereforeX-pgprw-int(A) ⊆ pgprw-cl (X-A)–(1) and Let, x ∈pgprw-cl(X-A), then every x∈pgprw open set, G containing x intersects X – A i.e. $G \cap (X - A) \neq \phi$, i.e. every pgprw-open G containing x s.t. $G \subseteq A$. Then by definition 3.14, x is not in pgprw-int(A), i.e. xϵX-pgprw-int(A) and so – pgprw-cl (X-A) ⊆ X- pgprw-int(A)- (2). Thus X – pgprw-int(A) = pgprw-cl(X – A).

(ii) Follows by taking complements in (i).

(iii) Follows by replacing A by X-A in (i)

(iv) Follows by taking complements in (iii).

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