THE MAGNETIC FIELD AND THERMAL EFFECTS ON STOKE'S SECOND PROBLEM FOR COUPLE STRESS FLUID THROUGH A POROUS MEDIUM

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ABSTRACT

*We have investigated the magnetic field and thermal effects on Stoke's second problem for couple stress fluid through a porous medium. The expressions for the velocity field and the temperature field are obtained analytically. It is observed that, the u and $|u|$ decreases with increasing M, Gr **and** Re, while they increases with increasing Da, p **and** λ. The θ **and** $|u|$ with increasing p.*

1.1. INTRODUCTION

The flow induced by a suddenly accelerating plate on the fluid above it, usually referred to as Stoke's first problem (Stoke's[92]), and the flow due to an oscillating flat plate, usually referred to as Stokes' second problem (see Raleigh [70]) are amongst a handful of unsteady flows of a Navier-Stokes fluid for which one can obtain an exact solution. Such exact solutions serve a dual purpose, that of providing an explicit solution to a problem that has physical relevance and as a means for testing the efficiency of complex numerical schemes for flows in complicated flow domains. The Stokes' second problem describes the oscillatory flat plate in a semi-infinite flow domain with a specific frequency. Tanner [96] has investigated the exact solution to this Stokes' first problem for a Maxwell fluid. The impulsive motion of a flat plate in a viscoelastic fluid was analyzed by Taipei [94]. Exact solution for unsteady flow of non-Newtonian fluid due to an oscillating wall was presented by Rajagopal [67]. Prezoisi and Joseph [63] have discussed the Stoke's first problem for viscoelastic fluids. Erdogan [32] has investigated the unsteady flow of viscous fluid due to an oscillating plane wall by using Laplace transform technique. Puri and Ky the [65] have discussed an unsteady flow problem which deals with non-classical heat conduction effects and the structure of waves in Stokes' second problem. Much work has been published on the flow of fluid over an oscillating plate for different constitutive models (Zeng and Weinbaum [102]; Asghar *et al.* [9]; Ibrahem *et al.*, [49]). The theory of couple stresses in fluids, developed by Stokes [93] represents the simplest generalization of the classical theory and allows for polar effects such as the presence of couple stresses and body couples. Stokes' first and second problems for an incompressible couple stress fluid under isothermal conditions were studied by Devakar and Iyengar [28].

There has been an increase in interest in the effect of porous media, because of their extensive practical applications in geophysics, thermal insulation in buildings, petroleum resources, packed-bed reactors and sensible heat-storage beds. Many studies related to non-Newtonian fluids saturated in a porous medium have been carried out. Dharmadhikari and Kale [27] studied experimentally the effect of non-Newtonian fluids in a porous medium. Chen and Chen [23] investigated the free convection flow along a vertical plate embedded in a porous medium. Rees [79] analyzed the effect of inertia on free convection over a horizontal surface embedded in a porous medium. Nakayama [60] investigated the effect of buoyancy-induced flow over a non-isothermal body of arbitrary shape in a fluid-saturated porous medium. A ray-tracing method for evaluating the radioactive heat transfer in a porous medium was examined by Argento [5].

Past few decades, the study of magneto hydrodynamics flow of electrically conducting fluids in electric and magnetic fields are of considerable interest in modern metallurgical and metal working process. The Hartmann flow is a classical problem that has important applications in MHD power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, the petroleum industry, purification of crude oil and design of various heat exchangers. Ramachandra Rao and Deshikachar [72] have investigated the MHD oscillatory flow of blood through channels of variable cross section. The effect of transverse magnetic field in physiological type of flow, through a uniform circular pipe was studied by Ramachandra Rao and Deshikachar [73]. It has been established that the biological systems in general are greatly affected by the application of external magnetic field. Vajravelu and Rivera [98] have analyzed the hydro magnetic flow at an oscillating plate. The pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field was investigated by Rathod and Tanveer [77]. Reddappa *et al.* [66] have investigated the non-classical heat conduction effects in Stokes' second problem of a micro polar fluid under the influence of a magnetic field.

In view of these, we investigated the magnetic field and thermal effects on Stoke's second problem for couple stress fluid through a porous medium. The expressions for the velocity field and the temperature field are obtained analytically. The effects of various emerging parameters on the velocity field and temperature field are studied in detail with the help of graphs.

4.2. Formulation of the problem:

The equations of motion that characterize couple stress fluid flow are similar to the Navier-Stokes equations and are given by:

$$\frac{d\rho}{dt} + \rho\, div(q) = 0 \tag{1.2.1}$$

$$\rho\frac{dp}{dt} + \rho f + \left[\frac{1}{2}curl(\rho c) + div(\tau^s) + \frac{1}{2}curl(div(M))\right] + J \times B - \frac{\mu}{k_1}q - \rho\left[1 - \alpha(\theta - \theta_\infty)\right]g\delta_{i1} \tag{1.2.2}$$

where ρ is the density of the fluid, α_1 − the co-efficient of thermal expansion, g – the acceleration due to gravity, $B = (B_0 + B_1)$- total magnetic field, B_1 is the induced magnetic field assumed negligible, $\tau^{(s)}$ is the symmetric part of the force stress diad, μ is the viscosity of the fluid, k is the permeability of the porous medium, M_1 is the couple stress diad and f, c are the body force per unit mass and body couple per unit mass respectively.

The constitutive equations concerning the force stress t_{ij}, and the rate of deformation tensor d_{ij} are given by:

$$t_{ij} = -p\delta_{ij} + \lambda div(q)\delta_{ij} + 2\mu d_{ij} - \frac{1}{2}\varepsilon_{ijk}\left[m_{jk} + 4\eta\omega_{k,rr} + \rho c_k\right] \tag{1.2.3}$$

The couple stress tensor m_{ij} that arises in the theory has the linear constitutive relation

$$m_{ij} = \frac{1}{3}m\delta_{ij} + 4\eta\omega_{j,i} + 4\eta\omega_{i,j} \tag{1.2.4}$$

In the above $\omega = \frac{1}{2}$ curl q is the spin vector, $\omega_{i,j}$ is the spin tensor, m is the trace of couple stress tensor m_{ij}, p is the fluid pressure and ρc_k is the body couple vector. Comma in the suffixes denotes covenant differentiation and $\omega_{k,rr}$ stands for $\omega_{k,11} + \omega_{k,22} + \omega_{k,33}$. The quantities λ and μ, are the viscosity coefficients and η, η' are the couple stress viscosity coefficients. These material constants are constrained by the inequalities

$$\mu \geq 0, \qquad 3\lambda + 2\mu \geq 0, \qquad \eta \geq 0, \qquad |\eta'| \leq \eta \tag{1.2.5}$$

There is a length parameter $l = \sqrt{\eta/\mu}$, which is a characteristic measure of the polarity of the fluid model and this parameter is identically zero in the case of non-polar fluids.

After neglecting body forces and body couples, the equations governing the couple stress fluid dynamics as given by Stokes [93] are

$$div\ q = 0 \tag{1.2.6}$$

$$\rho\left[\frac{\partial q}{\partial t} + (q.\nabla q)\right] = -\nabla p - \left[\begin{matrix}\mu curl(q)\\ curl(curl(curl(curl(q))))\end{matrix}\right] + J \times B - \frac{\mu}{k_1}q + \alpha_1 g(\theta - \theta_\infty) \tag{1.2.7}$$

Neglecting the displacement currents, the Maxwell equations and the Ohm's law are:

$$div\ B = 0,\ curlB = \mu_m J,\ curlE = -\frac{\partial B}{\partial t}, J = \sigma(E + q \times B) \tag{1.2.8}$$

where σ is the electrical conductivity, μ_m is the magnetic permeability and E is the electric field. The imposed and induced electrical fields are assumed to be negligible. Under the assumption of low magnetic Reynolds number, $J \times B$ reduces to

$$J \times B = -\sigma\mu_e^2 B_0^2 q \tag{1.2.9}$$

We consider the unsteady flow of an incompressible, couple stress fluid through a porous medium which fills the half space $y > 0$ above a flat (solid) plate occupying xz - plane. Initially, we assume that both fluid and plate are at rest. A uniform magnetic field B_0 is applied transverse direction to the flow. It is assumed that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible as in Cowling [13]. At time $t = 0+$, whether we allow the plate to start with a constant velocity U along x-axis or oscillate with velocity $U \cos \omega t$ the flow occurs only in x- direction. Therefore, the velocity is expected to be in the form $q = \left(u(y,t),0,0\right)$ and it automatically satisfies the continuity Eq. (4.2.6).

Under these assumptions the Eq. (4.2.7) becomes

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} - \sigma \mu_e^2 B_0^2 u - \frac{u}{k} u + \alpha g \left(\theta - \theta_\infty\right)$$ (1.2.10)

The energy equation (MCF model) is given by (Ibrahem *et al.*, [50])

$$\tau \theta_{tt} + \theta_t = \frac{\chi}{\rho c_p} \theta_{yy}$$ (1.2.11)

where $\omega_{i,j}$ is the vorticity, χ *is* the thermal conductivity, θ the temperature. and τ the thermal relaxation time.

Introducing the non-dimensional variables

$$\bar{u} = \frac{u}{U}, \bar{y} = \frac{y}{l}, \bar{t} = \frac{U}{l}t, \bar{\theta} = \frac{\theta - \theta_0}{\theta_w - \theta_0}, l^2 = \frac{\eta}{\mu}, R = \frac{\rho U l}{\mu}$$ (1.2.12)

into Equations (4.2.10) and (4.2.11), we get (after dropping bars)

$$R\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{\partial^4 u}{\partial y^4} - \left(H^2 + \frac{1}{Da}\right)u + G\theta$$ (1.2.13)

$$\lambda p\theta_n + p\theta_1 = \theta$$ (1.2.14)

Here $\quad p = \frac{v\rho c_p}{\chi}, \lambda = \frac{\tau U_0^2}{v} = C_p.$

The non-dimensional boundary conditions are

$$\theta(y,t) = e^{i\omega t} \qquad \text{at} \qquad y = 0$$

$$\theta(y,t) \to 0 \qquad \text{at} \qquad y \to \infty$$

$$u(y,t) = e^{i\omega t} \qquad \text{at} \qquad y = 0$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \qquad \text{at} \qquad y = 0$$

$$u(y,t) = 0 \qquad \text{at} \qquad y \to \infty$$ (1.2.15)

4.3. Solution:

To solve the nonlinear system (4.2.13) and (4.2.14) with the boundary conditions (4.2.15), we assume that

$$u(y,t) = U(y)e^{i\omega t}, \theta(y,t) = \Theta(y)e^{i\omega t}$$ (1.3.1)

It we substitute Eq. (4.3.1) in Equations (4.2.13) and (4.2.14) and the boundary conditions (4.2.15), we get

$$R\frac{\partial^4 U}{\partial y^4} - \frac{\partial^2 U}{\partial y^2} + \left(iR\omega + \frac{1}{Da} + H^2\right)U = G\Theta$$ (1.3.2)

$$\Theta'' + \left(\lambda p\omega^2 - i\omega p\right)\Theta = 0$$ (1.3.3)

The corresponding boundary conditions are

$$\Theta(0) = 1, \Theta(\infty) = 0, U(0) = 1, U''(0) = 0, U(\infty) = 0$$ (1.3.4)

Solving the Equations (4.3.2) and (4.3.3) using the boundary conditions Eq. (4.3.4), we get

$$\Theta(y) = e^{-my} \tag{1.3.5}$$

$$U(y) = c_1 e^{-m_1 y} + c_2 e^{-m_2 y} \tag{1.3.6}$$

where $m = \sqrt{-\lambda p \omega^2 + i\omega p}$, $c_1 = \dfrac{m_2^2}{m_2^2 - m_1^2}$, $c_2 = -\dfrac{m_1^2}{m_2^2 - m_1^2}$, $m_1 = \sqrt{\dfrac{1-r}{2}}$, $m_2 = \sqrt{\dfrac{1+r}{2}}$ and

$$r = \sqrt{1 - 4\left(H^2 + \frac{1}{Da}\right)^2 - 4iR\omega}$$

The solution of Equations (4.2.13) and (4.2.14) are given by

$$\theta(y,t) = e^{-(my - i\omega t)} \tag{1.3.7}$$

$$u(y,t) = c_1 e^{-(m_2 y - i\omega t)} \tag{1.3.8}$$

The rate of heat transfer coefficient in terms of Nusselt number Nu at the wall of the plate is given by

$$Nu = -\frac{\partial \theta}{\partial y}\bigg|_{y=0} = me^{i\omega t} \tag{1.3.9}$$
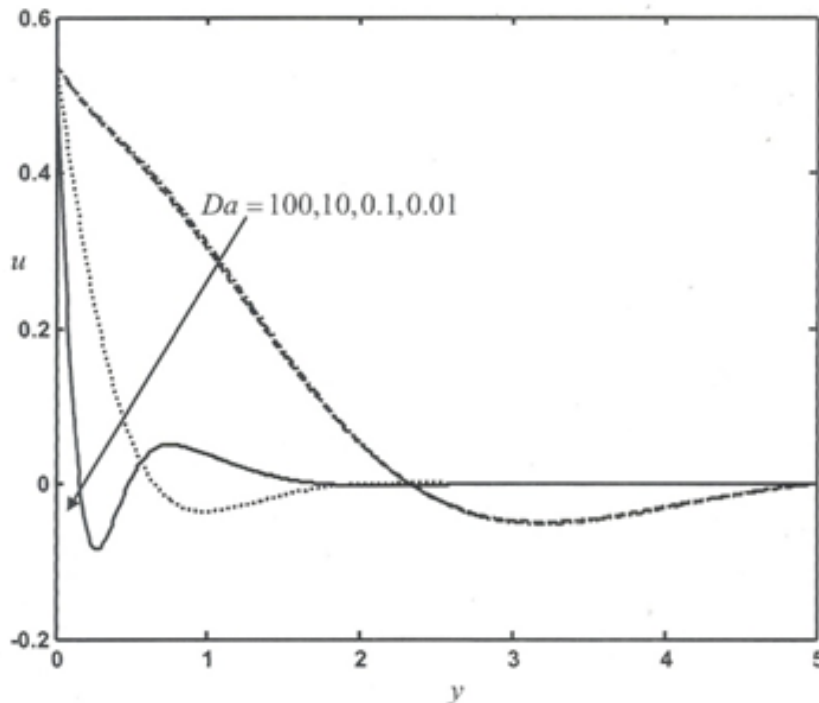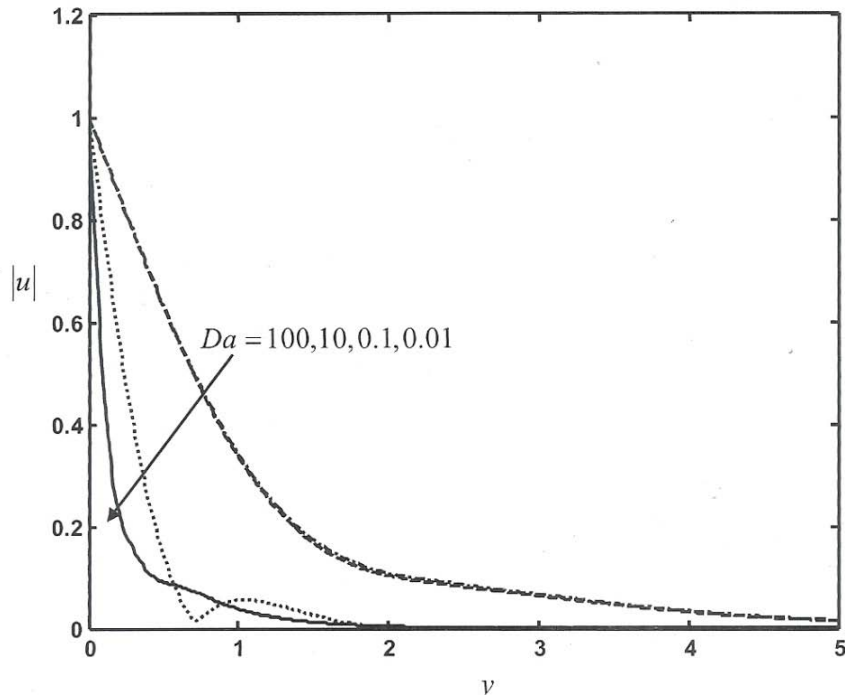
1.4. Discussion of the results

Fig. 1.4.1: variation of velocity u with y for different values of
Darcy number Da with $\lambda = 0.005, p = 1, H = 1, \omega = 10, Re = 0.5, t = 0.1$ *and* $Gr = 5$.

Fig. 1.4.1 depicts variation of velocity u with y for different values of Darcy number Da $with$ $\lambda = 0.005$, $p = 1$, $H = 1, \omega = 10, Re = 0.5, t = 0.1$ and $Gr = 5$. It is found that, the velocity u oscillates with y. Further it is found that, the velocity u initially increases and then decreases with increasing Da.

variation of velocity $|u|$ with y for different values of Darcy number Da with $\lambda = 0.005, p = 1, H = 1, \omega = 10$, $Re = 0.5, t = 0.1$ and $Gr = 5$ is shown in

Fig. 1.4.2: variation of velocity $|u|$ with y for different values of Darcy number Da with $\lambda = 0.005, p = 1, H = 1, \omega = 10, Re = 0.5, t = 0.1$ and $Gr = 5$.

From Fig. 1.4.2. It is noted that the velocity $|u|$ increases with an increase in Da.
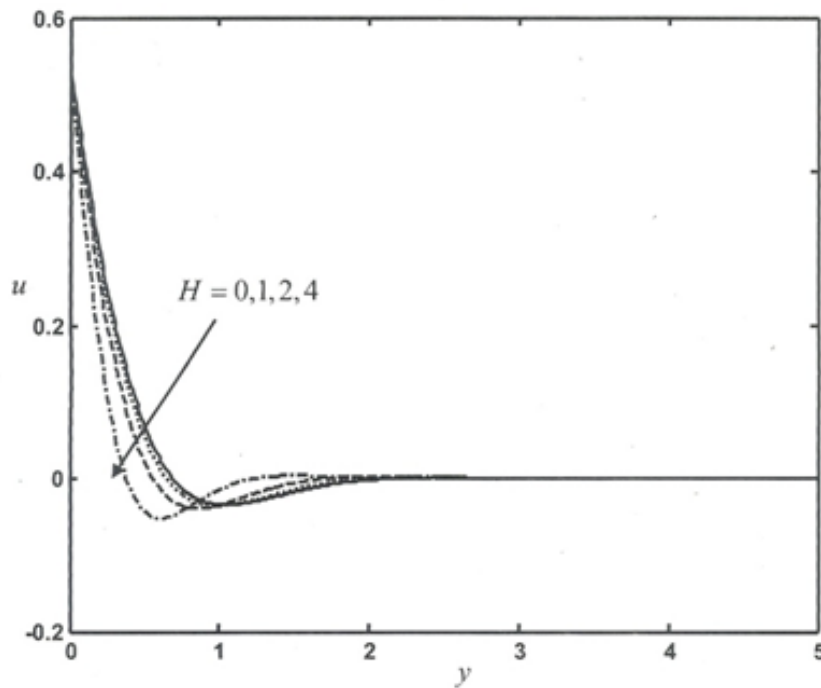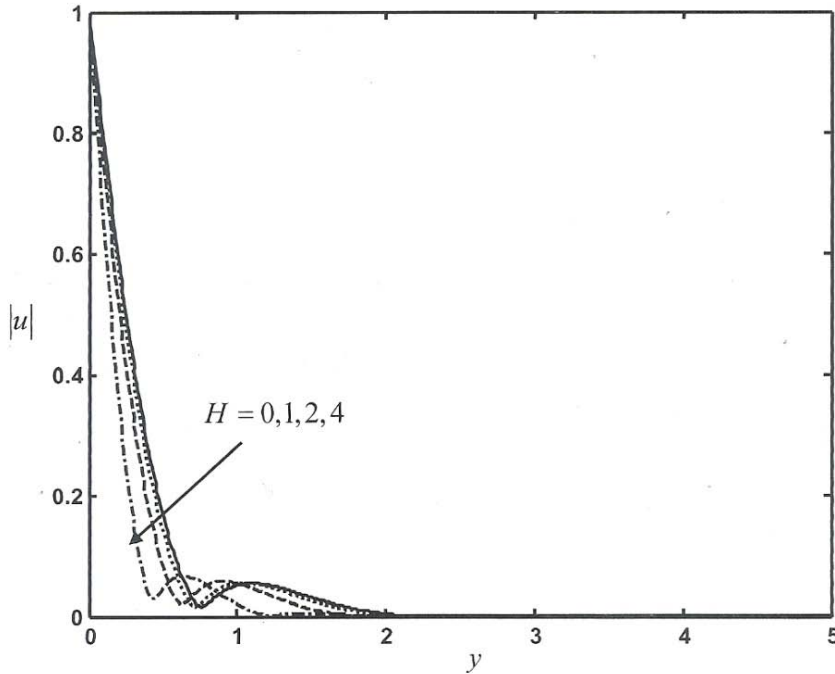

Fig. 1.4.3: variation of velocity u with y for different values of Hartman number H with $\lambda = 0.005, p = 1, Da = 0.1, \omega = 10, Re = 0.5, t = 0.1$ and $Gr = 5$.

Fig. 1.4.3 shows that variation of velocity u with y for different values of Hartmann number H with $\lambda = 0.005$, $p = 1, \omega = 10, Re = 0.5, Da = 0.1, t = 0.1$ a n d $Gr = 5$. It is observed that the velocity u first decreases and then increases with increasing H.

variation of velocity $|u|$ with y for different values of Hartmann number H with $\lambda = 0.005, p = 1, \omega = 10$, $Re = 0.5$, $Da = 0.1, t = 0.1$ and $Gr = 5$ is shown in

Fig. 1.4.4: variation of velocity $|u|$ with y for different values of Hartman number H with $\lambda = 0.005, p = 1, Da = 0.1, \omega = 10, Re = 0.5, t = 0.1$ and $Gr = 5$.

From Fig. 1.4.4. It is found that, the absolute velocity $|u|$ decreases with an increase in H. Further, it is found that velocity is more for non-conducting (magnetic) (i.e., $H \rightarrow 0$) couple stress fluid than that of conducting couple stress fluid.
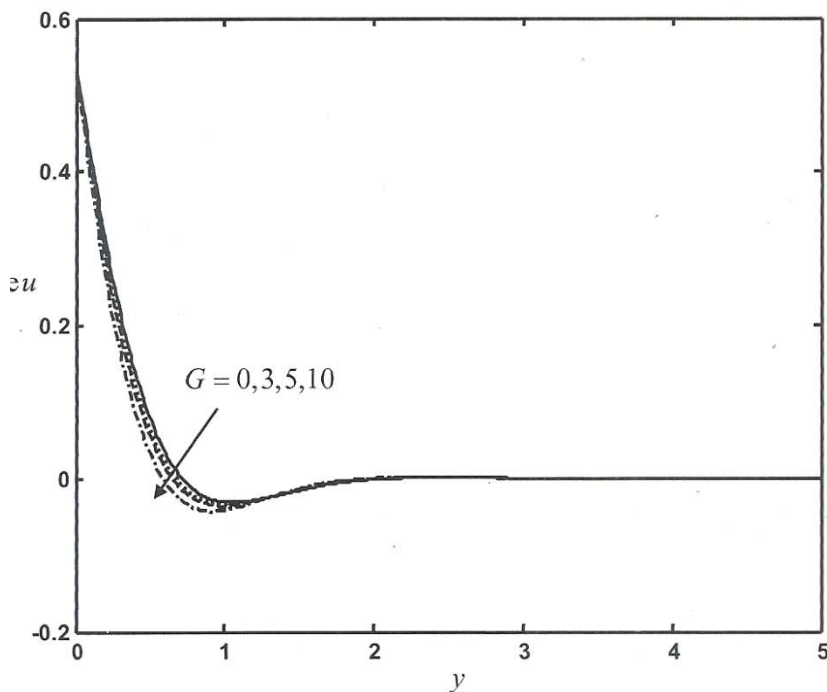

Fig. 1.4.5: variation of velocity u with y for different values of Grsh of number Gr with $\lambda = 0.005, p = 1, \omega = 10, Re = 0.5, Da = 0.1, t = 0.1$ and $H = 5$.

Fig.1.4.5 depicts variation of velocity u with y for different values of Grashof number Gr with $\lambda = 0.005, p = 1, \omega = 10, Re = 0.5, Da = 0.1, t = 0.1$ and $H = 1$. It is noted that, the u initially decreases and then increases with increasing Gr.

The variation of velocity $|u|$ with y for different values of Grashof number Gr with $\lambda = 0.005, p = 1, \omega = 10, Re = 0.5, Da = 0.1, t = 0.1$ and $H = 1$ is depicted in
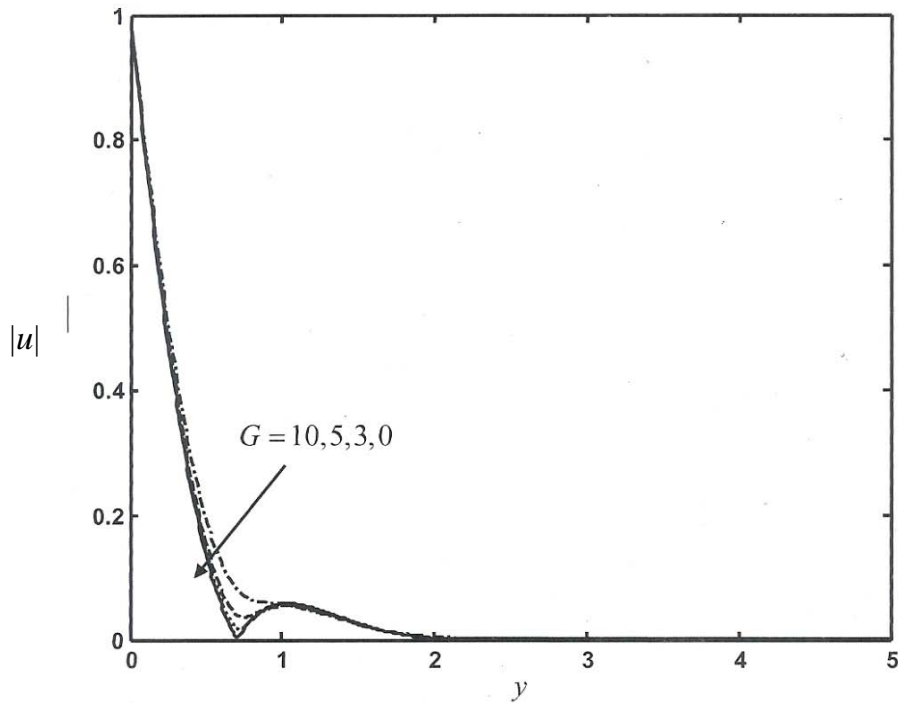
Fig. 1.4.6: The variation of velocity $|u|$ with y for different values of Grashof number Gr with $\lambda = 0.005, p = 1, \omega = 10, Re = 0.5, Da = 0.1, t = 0.1 \; and \; H = 1$.

From Fig. 1.4.6it observed that the absolute velocity $|\boldsymbol{u}|$ first increases and then decreases with an increase inGr.
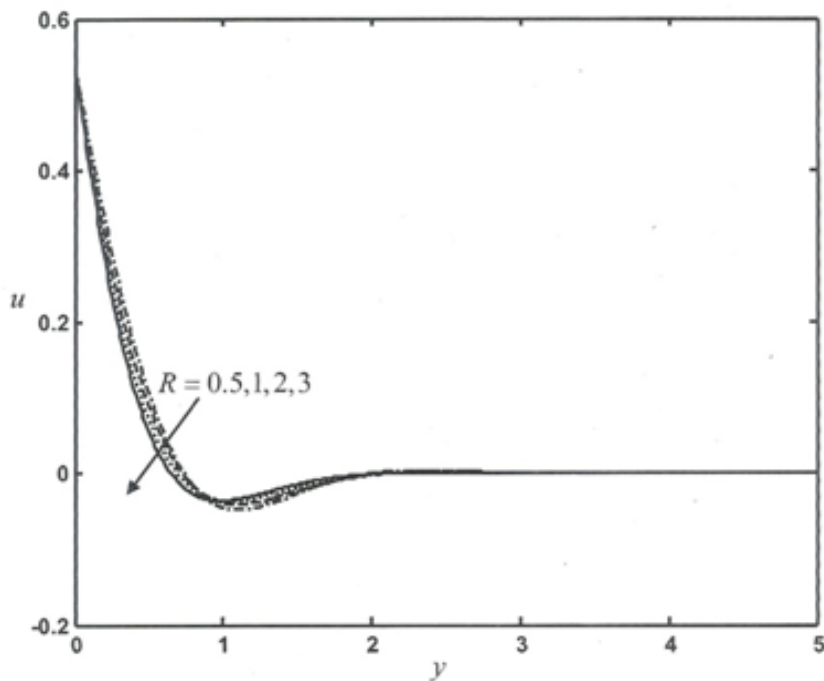

Fig. 1.4.7: variation of velocity u with y for different values of Reynolds number Re with $\lambda=0.005, p=1, \omega=10, Gr=5, Da=0.1, t=0.1$ and $H=1$.

Fig.1.4.7 illustrates the variation of velocity $\boldsymbol{u}$ with y for different values of couple stress Reynolds number$Re$with $\lambda = \boldsymbol{0.005}, \; p = \boldsymbol{1}, \omega = \boldsymbol{10}, \boldsymbol{G} = \boldsymbol{5}, \boldsymbol{t} = \boldsymbol{0.1} \; \boldsymbol{and} \; \boldsymbol{H} = \boldsymbol{1}$. As $\boldsymbol{R}$ increases, it is seen that the velocity $\boldsymbol{u}$ first decreases and then increases.

variation of velocity $|\boldsymbol{u}|$ with$y$ for different values of couple stress Reynolds number $\boldsymbol{Re}$ with $\lambda = \boldsymbol{0.005}, \; p = 1, \; \omega = 10, \; Da = 0.1, Gr = 5, t = 0.1 \; \boldsymbol{and} \; H = \boldsymbol{1}$. is illustrated.
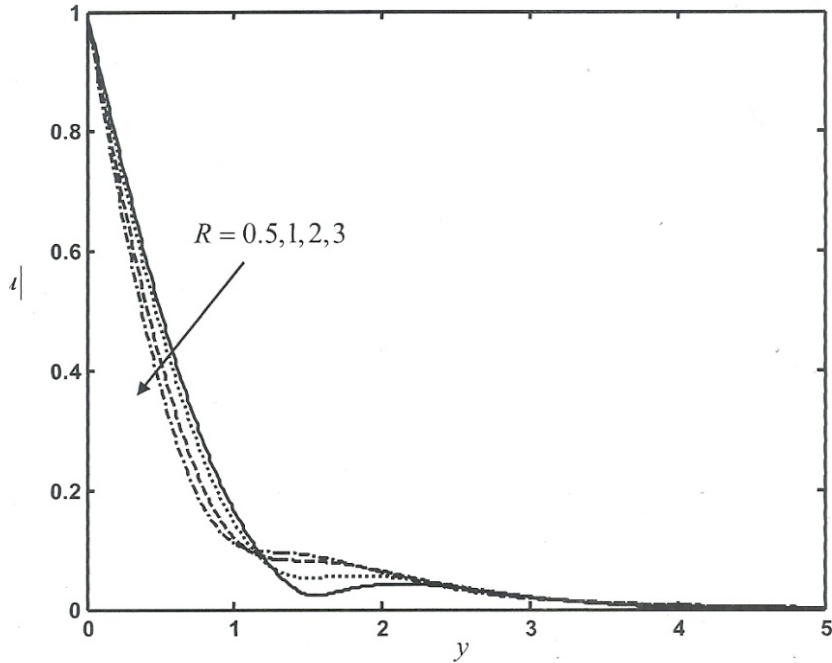
Fig. 1.4.8: variation of velocity $|u|$ with y for different values of Reynolds number Re with $\lambda = 0.005, p = 1, \omega = 10, G = 5, Da = 0.1, t = 0.1 \ and \ H = 1$.

From Fig. 1.4.8 it is noted that the absolute velocity $|w|$ decreases with increasing Re.
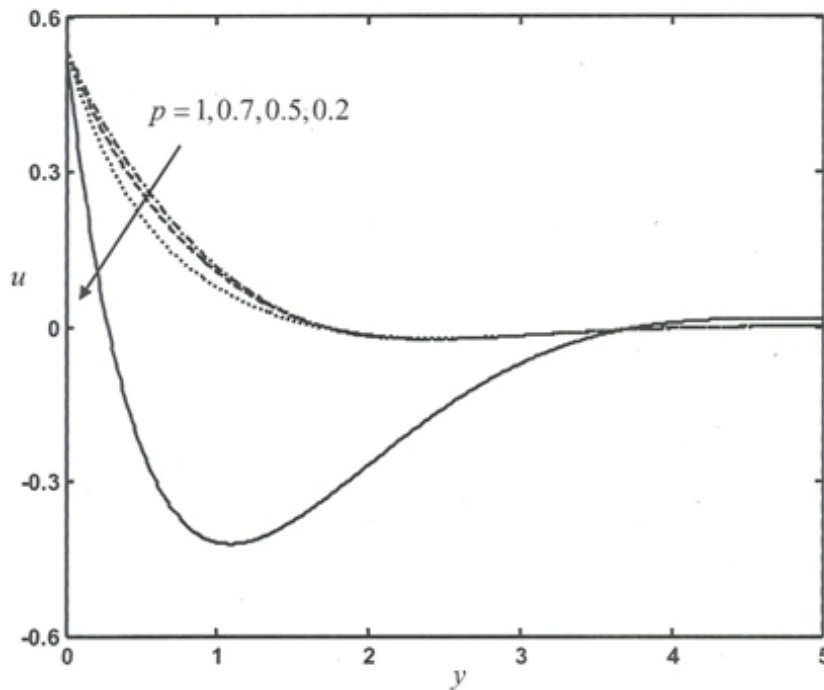

Fig. 1.4.9: variation of velocity u with y for different values of p with $\lambda = 0.005, Re = 0.5, \omega = 10, Da = 0.1, Gr = 5, t = 0.1 \ and \ H = 1$.

Fig. 1.4.9 shows that variation of velocity u with y for different values of p with $\lambda = \mathbf{0.005}, \mathbf{Re} = \mathbf{0.5}, \omega = \mathbf{10}, Gr = 5$, $Da = 0.1, \mathbf{t} = \mathbf{0.1 \ and \ H} = \mathbf{1}$. It is observed that, velocity u first increases and then decreases with an increase in p.

variation of velocity $|u|$ with y for different values of $\mathbf{p}$ with $\lambda = \mathbf{0.005}, Re = 0.5, \omega = 10, Da = 0.1, Gr = 5$, $\mathbf{t} = \mathbf{0.1}$ and $H = \mathbf{1}$ is shown in
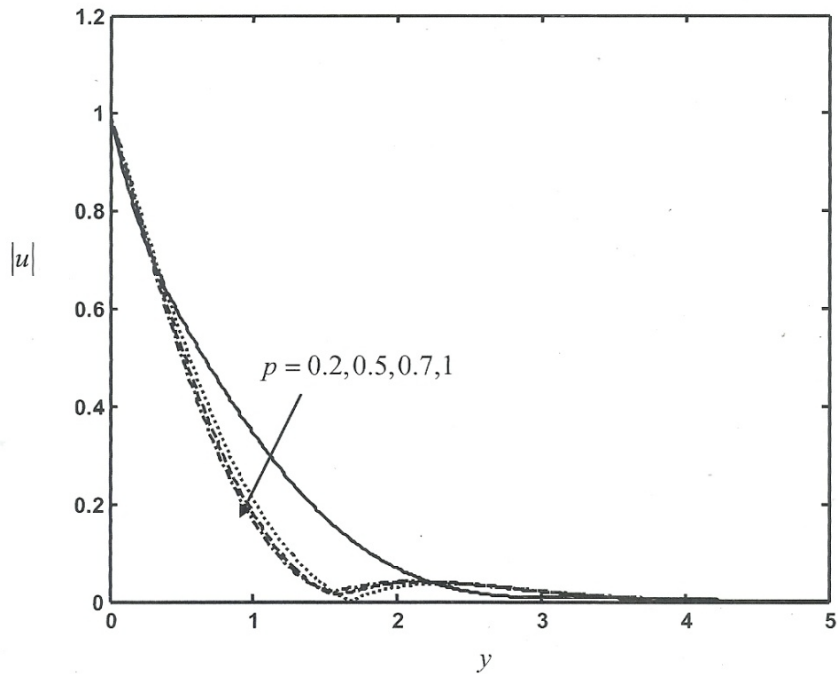
Fig. 1.4.10: The variation of velocity $|u|$ with y for different values of p with
$$\lambda = 0.005, Re = 0.5, \omega = 10, Da = 0.1, Gr = 5, t = 0.1 \text{ and } H = 1.$$

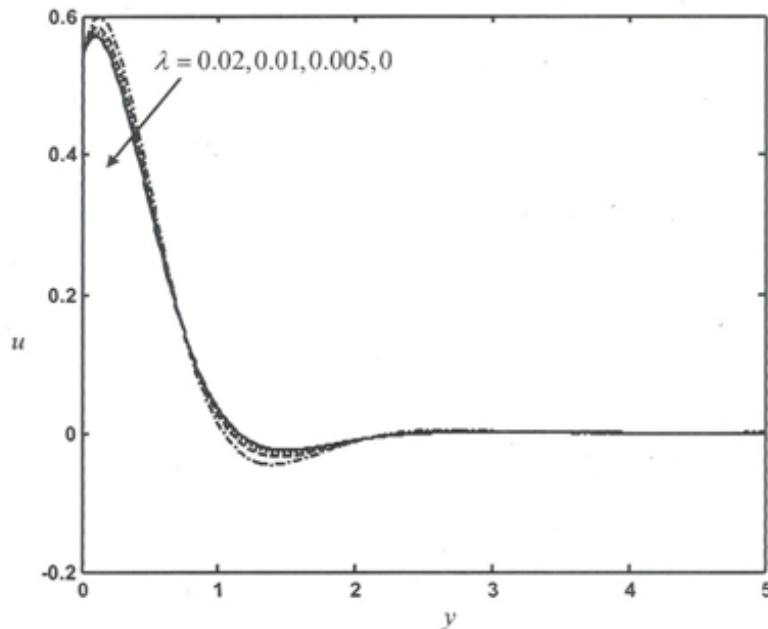Fig. 1.4.10 it is found that, absolute velocity $|u|$ initially increases and then decreases with increasing p.

Fig. 1.4.11: variation of velocity u with y for different values of λ with
$$p = 1, Re = 0.5, \omega = 10, Da = 0.1, Gr = 5, t = 0.1 \text{ and } M = 1.$$

Fig. $1.4.11$ *depicts variation of velocity* u *with* y *for different* values of λ with $\lambda = \mathbf{1}, Re = \mathbf{0.5}, \omega = \mathbf{10},$ $Da = \mathbf{0.1}, Gr = \mathbf{5}, t = \mathbf{0.1} \text{ and } H = \mathbf{1}$.It is noted that, the velocity u first increases and then decreases with an increase in λ.

variation of velocity $|u|$ with y for different values of λ with $\lambda = 1, Re = 0.5, \omega = 10, Da = 0.1, Gr = 5, t = 0.1$ and $H = 1$ is depicted in
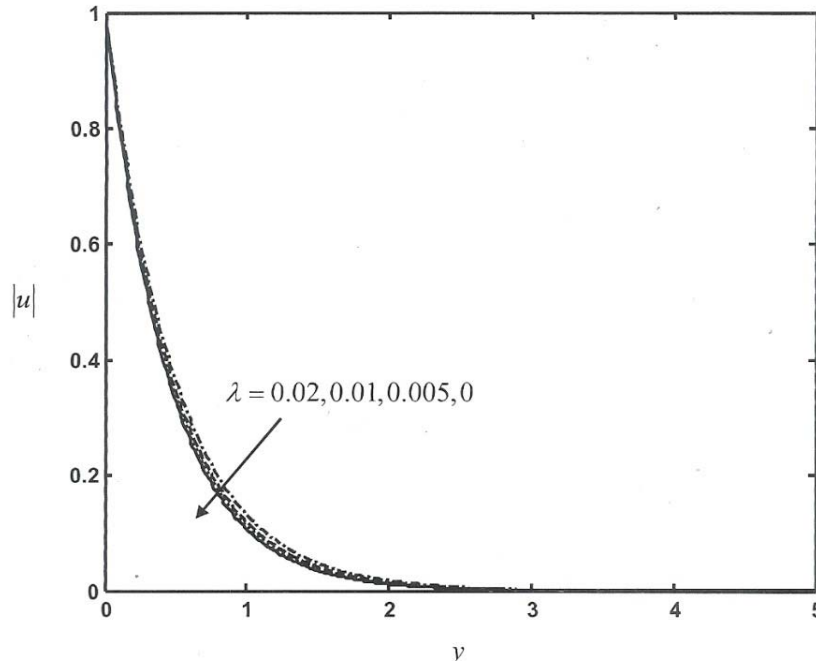
Fig. 1.4.12: variation of velocity $|u|$ with y for different values of λ with
$$p = 1, Re = 0.5, \omega = 10, Da = 0.1, Gr = 5, t = 0.1 \ and \ M = 1.$$

From Fig. 1.4.12 it is observed that the absolute velocity $|u|$ increases with increasing λ.
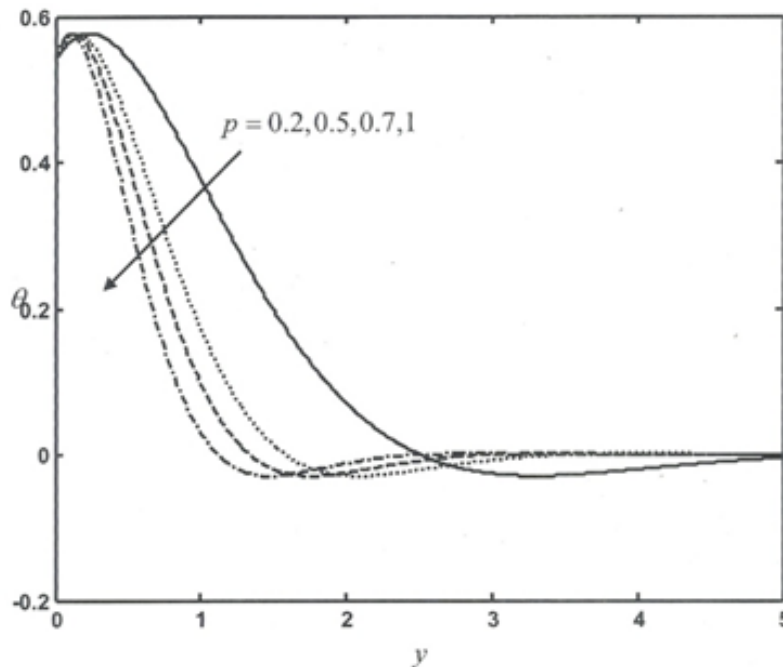

Fig. 1.4.13: variation of temperature θ with y for different values of p with $\lambda = 0.005, and \ \omega = 10.$

Fig. 1.4.13 illustrates variation of temperature θ with y for different values of p with $\lambda = \boldsymbol{0.005 \ and} \ \omega = \boldsymbol{10}$. It is found that, the temperature θ initially increases and then decreases with increase in p.

variation of temperature $|\theta|$ with y for different values of $\boldsymbol{p}$ with $\lambda = \boldsymbol{0.005 \ and} \ \omega = \boldsymbol{10}$ is shown in
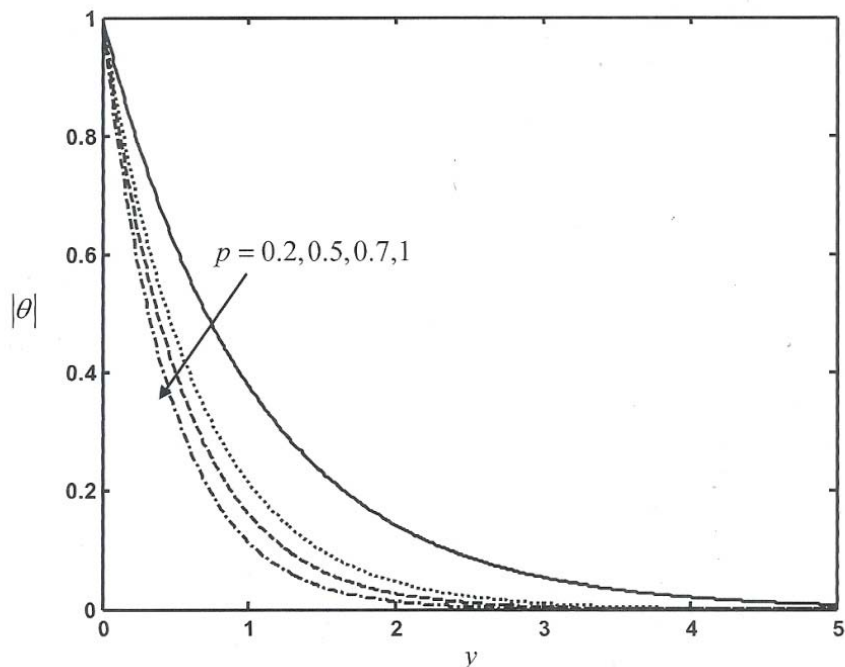
Fig. 1.4.14: variation of temperature $|\theta|$ with y for different values of p with $\lambda = 0.005$ and $\omega = 10$.

From Fig. 1.4.14 it is noted that the absolute temperature $|\theta|$ decreases with increasing p.

Table – 1.4.1: Effect of p on Nusselt number Nu with $t = 0.1, \lambda = 0.005$ and $\omega = 1$.

P	Nu
0.2	0.2822
0.5	0.4462
0.7	0.5280
1	0.6310

Table-1.4.1: shows that effect of p on Nusselt number Nu with $t = 0.1, \lambda = 0.005$ and $\omega = 1$. It is found that the Nu increases with increasing p

Table – 1.4.2: Effect of ω on Nusselt number Nu with $t = 0.1, \lambda = 0.005$ and $p = 1$

P	Nu
0	0
1	0.6310
2	0.7755
3	0.7966

Table-1.4.2 depicts the effect of ω on Nusselt number Nu with $t = 0.1, \lambda = 0.005$ and $p = 1$. It is noted that Nu increases with increasing ω.

REFERENCES

1. C. Argento and D. Bouvard, A Ray Tracing Method for Evaluating the Radiative Heat Transfer in Porous Medium, Int. J. Heat Mass Transfer, 39(1996), 3175-3180.
2. S. Asghar, T. Hayat and A.M. Siddiqui, Moving boundary in a non-Newtonian fluid, Int. J. Nonlinear Mech., 37(2002), 75-80.
3. H. Chen and C. Chen, Free convection flow of non-newtonian fluids along a vertical plate embedded in a porous medium, Journal of Heat Transfer, 110(1988), 257-260.
4. R.V. Dharmadhikari,and D.D.Kale, Flow of non-newtonian fluids through porous media, Chem. Eng. Sci., 40(1985),527-529.
5. M. Devakar, T.K.V. Iyengar, Stokes' problems for an incompressible couple stress fluid, Nonlinear Analysis: Modelling and Control, 1(2) (2008), 181-190.
6. M. E. Erdogan, Plane surface suddenly set in motion in a non-Newtonian fluid, Acta Mech., 108(1995), 179-187.

7. F. S. Ibrahim and F. M. Hady, Mixed convection over a horizontal plate with vectored mass transfer in a transverse magnetic field, Astrophysics and Space Science, Astrophys Space Sci., 114 (1985), 335.

8. A.Nakayama and H. Koyama, Buoyancy-induced flow of non-newtonian fluids over a non-isothermal body of arbitrary shape in a porous medium, Applied Scientific Research, 48(1991), 55-70.

9. M. Pakdemirli, Boundary layer flow of power-law fluids past arbitrary profiles, IMA J. Appl. Math., 50(1993), 133-148.

10. P. Puri and P.K. Kythe, Thermal effects in Stokes' second problem, Acta Mech., 112(1998), 44-50.

11. A.Reddappa, M. V. Subba Reddy and K. R. Krishna Prasad, Thermal effects in Stokes' second problem for unsteady magneto hydrodynamic flow of a Micropolar fluid, Journal of Pure and Applied Physics, Vol. 21,No.3(2009), 365-373.

12. K.R. Rajagopal, A note on unsteady unidirectional flows of a non-Newtonian fluid, Int.J. Non-Linear Mech., 17 (1982), 369-373.

13. L. Raleigh, On the motion of solid bodies through viscous liquid, Phil. Mag 21(6) (1911), 697-711.

14. Ramachandra Rao and K. S. Deshikachar, MHD oscillatory flow of blood through channels of variable cross section, Int. J. Eng. Sci. 24 (1986), no. 10, 1615-1628.

15. Ramachandra Rao and K. S. Deshikachar, Physiological type flows in a circular pipe in the presence of a transverse magnetic field, J. Indian Inst. Sci., 68 (1988), 247-260.

16. V. P. Rathod and S. Tanveer, Pulsatile flow of couple stress fluid through a porous medium with periodic body acceleration and magnetic field, Bull. Malays. Math. Sci. Soc, 32(2) (2009), 245-259.

17. S. Rees and A. P. Bassom, Int. J. Eng. Sci., 34(1996), 113.

18. G. Stokes, on the effect of the internal friction on the motion of pendulums, Trans. Cambridge Philos.Sco. 9(1851), 8 - 106.

19. V. K. Stokes, Couple stresses in fluids, Phys. Fluids, 9(1966), 1709-1715.

20. Taipei, The impulsive motion of a flat plate in a viscoelastic fluid, Acta Mech, 39(1981), 277-279.

21. R. Tanner, Notes on the Rayleigh parallel problem for a viscoelastic fluid, ZAMP, 13(6) (1962), 573-580.

22. K. Vajravelu and J. Rivera, Hydromagnetic flow at an oscillating plate, Int. J. Non-Linear Mech., 38(2003), 305-312.

23. Y. Zeng and S. Weinbaum, Stokes' problem for moving half planes, J. Fluid Mech., 287(1995), 59-74.

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