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ON *K* HYPER-BANHATTI INDICES AND COINDICES OF GRAPHS

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ABSTRACT

The vertices and edges of a graph G are called the elements of G. If e=uv is an edge of G, then the vertex u and edge e are incident as are v and e. We introduce the first and second K hyper-Banhatti indices to take account of the contributions of pairs of incident elements. Also we introduce the first and second K hyper-Banhatti coindices to take account the contributions of pairs of nonincident elements. In this paper, we obtain the exact values of the first and second K hyper-Banhatti indices for some standard graphs.

***Keywords*:** incident, *K* hyper-Banhatti indices, *K* hyper-Banhatti coindices.

***Mathematics Subject Classification*:** 05*C.*

1. INTRODUCTION

The graphs considered here are finite, undirected without isolated vertices, loops and multiple edges. Any undefined term in this paper may be found in Kulli [1].

Let $G=(V, E)$ be a graph with $|V| = n$ vertices and $|E| = m$ edges. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v is denoted by uv. If $e=uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) – 2$ with $e=uv$. The vertices and edges of a graph are called its elements.

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences.

In Chemical Science, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [2].

In [3], Kulli introduced the first and second *K* Banhatti indices to take account of the contributions of pairs of incident elements.

The first *K* Banhatti index $B_1(G)$ and the second *K* Banhatti index $B_2(G)$ of a graph G are defined as

$$B_1(G)=\sum_{ue}\left[d_G(u)+d_G(e)\right]$$

$$B_2(G)=\sum_{ue}d_G(u)d_G(v)$$

where ue means that the vertex u and edge e are incident in G.

In [3], Kulli introduced the first and second *K* Banhatti coindices to take account of the contributions of pairs of nonincident elements.

The first *K* Banhatti coindex $\overline{B}_1(G)$ and the second *K* Banhatti coindex $\overline{B}_2(G)$ of a graph G are defined as

$$\overline{B}_1(G)=\sum_{u*e}\left[d_G(u)+d_G(e)\right]$$

$$\overline{B}_2(G)=\sum_{u*e}d_G(u)d_G(e)$$

where $u * e$ means that the vertex u and edge e are nonincident in G.

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Recently many other indices and coindices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

In this paper, we introduce *K* hyper-Banhatti indices and coindices of graphs. Recently many other hyper indices and coindices were studied, for example, in [17, 18, 19, 20, 21].

2. FIRST *K* HYPER-BANHATTI INDEX

We introduce the first *K* hyper-Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 1: The first *K* hyper-Banhatti index of a graph *G* is defined as

$$HB_1(G) = \sum_{ue}\left[d_G(u) + d_G(e)\right]^2$$

where *ue* means that the vertex *u* and edge *e* are incident in *G*.

The following are the first *K* hyper-Banhatti index for cycles, complete graphs, complete bipartite graphs.

Proposition 2: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HB_1(C_n) = 32n.$$

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Every vertex of C_n is incident with exactly two edges. Consider

$$\begin{aligned} HB_1(C_n) &= \sum_{ue}\left[d_{C_n}(u) + d_{C_n}(e)\right]^2 \\ &= \sum_{u_i}\sum_{e_j}\left[d_{C_n}(u_i) + d_{C_n}(e_j)\right]^2 \\ &= \sum_{u_i}^{n}\sum_{e_j}^{2}(2+2)^2 \\ &= \sum_{u_i}^{n} 2(4)^2 \\ &= 32n. \end{aligned}$$

Proposition 3: Let K_n be a complete graph with *n* vertices. Then

$$HB_1(K_n) = n(n-1)(3n-5)^2.$$

Proof: Let K_n be a complete graph with *n* vertices. Every vertex of K_n is incident with $n-1$ edges. Consider

$$\begin{aligned} HB_1(K_n) &= \sum_{ue}\left[d_{K_n}(u) + d_{K_n}(e)\right]^2 \\ &= \sum_{u_i}\sum_{e_j}\left[d_{K_n}(u_i) + d_{K_n}(e_j)\right]^2 \\ &= \sum_{u_i}^{n}\sum_{e_j}^{n-1}\left[(n-1) + (2n-4)\right]^2 \\ &= \sum_{u_i}^{n}(n-1)(3n-5)^2 \\ &= n(n-1)(3n-5)^2. \end{aligned}$$

Proposition 4: Let $K_{m,n}$ be a complete bipartite graph. Then

$$HB_1(K_{m,n}) = mn[(m+2n-2)^2 + (2m+n-2)^2].$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Every vertex of V_1 is incident with *n* edges and every vertex of V_2 is incident with *m* edges. Let $V_1 = \{v_1, v_2, ..., v_m\}$ and $V_2 = \{w_1, w_2, ..., w_n\}$. Consider

$$\begin{aligned} HB_1(K_{m,n}) &= \sum_{ue}\left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e)\right]^2 \\ &= \sum_{v_i}^{m}\sum_{e_j}^{n}\left[d_{K_{m,n}}(v_i) + d_{K_{m,n}}(e_j)\right]^2 + \sum_{w_j}^{n}\sum_{e_i}^{m}\left[d_{K_{m,n}}(w_j) + d_{K_{m,n}}(e_i)\right]^2 \\ &= \sum_{v_i}^{m}\sum_{e_j}^{n}\left[n + (m+n-2)\right]^2 + \sum_{w_j}^{n}\sum_{e_i}^{m}\left[m + (m+n-2)\right]^2 \end{aligned}$$

$$= \sum_{v_i}^{m} \left[n(m + 2n - 2)^2 \right] + \sum_{w_j}^{n} \left[m(2m + n - 2)^2 \right]$$
$$= mn(m + 2n - 2)^2 + nm(2m + n - 2)^2$$
$$= mn[(m + 2n - 2)^2 + (2m + n - 2)^2].$$

The following results are immediate from Proposition 4.

Corollary 5: Let $K_{n,\,n}$ be a complete bipartite graph. Then

$$HB_1(K_{n,\,n}) = 2n^2(3n-2)^2.$$

Corollary 6: Let $K_{1,\,n}$ be a star. Then

$$HB_1(K_{1,n}) = n(5n^2 - 4n + 1).$$

Theorem 7: Let G be an r -regular graph with n vertices. Then

$$HB_1(G) = nr(3r-2)^2.$$

Proof: Let G be an r -regular graph with n vertices. Every vertex of G is incident with r edges. Consider

$$HB_1(G) = \sum_{ue} \left[d_G(u) + d_G(e) \right]^2$$
$$= \sum_{u_i}^{n} \sum_{e_j}^{r} \left[r + (2r-2) \right]^2$$
$$= \sum_{u_i}^{n} r(3r-2)^2$$
$$= nr(3r-2)^2.$$

3. SECOND *K* HYPER-BANHATTI INDEX

We introduce the second K hyper-Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 8: The second K hyper-Banhatti index of a graph G is defined as

$$HB_2(G) = \sum_{ue} \left(d_G(u) d_G(e) \right)^2$$

where ue means that the vertex u and edge e are incident in G.

The following are the second K hyper-Banhatti index for cycles, complete graphs, complete bipartite graphs.

Proposition 9: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HB_2(C_n) = 32n.$$

Proof: Let C_n be a cycle with n vertices. Every vertex of C_n is incident with exactly two edges. Consider

$$HB_2(C_n) = \sum_{ue} \left(d_{C_n}(u) d_{C_n}(e) \right)^2$$
$$= \sum_{u_i} \sum_{u_j} \left(d_{C_n}(u_i) d_{C_n}(e_j) \right)^2$$
$$= \sum_{u_i}^{n} \sum_{e_j}^{2} (2 \times 2)^2$$
$$= \sum_{u_i}^{n} (2)(4)^2$$
$$= 32n.$$

Proposition 10: Let K_n be a complete graph with n vertices. Then

$$HB_2(K_n) = 4n(n-1)^3 (n-2)^2.$$

Proof: Let K_n be a complete graph with n vertices. Every vertex of K_n is incident with n–1 edges. Consider

$$HB_2(K_n) = \sum_{ue}\left(d_{K_n}(u)d_{K_n}(e)\right)^2$$
$$= \sum_{u_i}\sum_{e_j}\left(d_{K_n}(u_i)d_{K_n}(e_j)\right)^2$$
$$= \sum_{u_i}^{n}\sum_{e_j}^{n-1}\left((n-1)(2n-4)\right)^2$$
$$= \sum_{u_i}^{n}(n-1)(n-1)^2(2n-4)^2$$
$$= 4n(n-1)^3(n-2)^2.$$

Proposition 11: Let $K_{m,n}$ be a complete bipartite graph. Then

$$HB_2(K_{m,n}) = mn(m^2+n^2)(m+n-2)^2.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and $|V_1|=m$, $|V_2|=n$, $V(K_{m,n}) = V_1 \cup V_2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Let $V_1 = \{v_1, v_2, ..., v_m\}$ and $V_2 = \{w_1, w_2, ..., w_n\}$. Consider

$$HB_2(K_{m,n}) = \sum_{ue}\left(d_{K_{m,n}}(u)d_{K_{m,n}}(e)\right)^2$$
$$= \sum_{v_i}^{m}\sum_{e_j}^{n}\left(d_{K_{m,n}}(v_i)d_{K_{m,n}}(e_j)\right)^2 + \sum_{w_j}^{n}\sum_{e_i}^{m}\left(d_{K_{m,n}}(w_j)d_{K_{m,n}}(e_i)\right)^2$$
$$= \sum_{v_i}^{m}\sum_{e_j}^{n} n^2(m+n-2)^2 + \sum_{w_j}^{n}\sum_{e_i}^{m} m^2(m+n-2)^2$$
$$= \sum_{v_i}^{m} n^3(m+n-2)^2 + \sum_{w_j}^{n} m^3(m+n-2)^2$$
$$= mn^3(m+n-2)^2 + nm^3(m+n-2)^2$$
$$= mn(n^2+m^2)(m+n-2)^2.$$

The following results are immediate from Proposition 11.

Corollary 12: Let $K_{n,n}$ be a complete bipartite graph. Then

$$HB_2(K_{n,n}) = 8n^4(n-1)^2.$$

Corollary 13: Let $K_{1,n}$ be a star. Then

$$HB_2(K_{1,n}) = n(n^2+1)(n-1)^2.$$

Theorem 14: Let G be an r-regular graph with n vertices. Then

$$HB_2(G) = 4nr^3(r-1)^2.$$

Proof: Let G be an r-regular graph with n vertices. Then every vertex of G is incident with r edges. Consider

$$HB_2(G) = \sum_{ue}\left(d_G(u)d_G(e)\right)^2$$
$$= \sum_{u_i}^{n}\sum_{e_j}^{r}\left(r(2r-2)\right)^2$$
$$= \sum_{u_i}^{n} r^3(2r-2)^2$$
$$= 4nr^3(r-1)^2.$$

4. *K* HYPER-BANHATTI COINDICES

We define *K* hyper-Banhatti coindices of a graph in terms of nonincident vertex-edge degrees.

Definition 15: The first and second *K* hyper-Banhatti coindices of a graph *G* are defined as

$$\overline{HB}_1(G)=\sum_{u*e}\left[d_G(u)+d_G(e)\right]^2$$

$$\overline{HB}_2(G)=\sum_{u*e}\left(d_G(u)d_G(e)\right)^2$$

where $u * e$ means that the vertex u and edge e are not incident elements in G.

We study these invariants in a separate paper.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. I. Gutman and N. Trinajstić, Graph theory and molecular orbitals. Total π-electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* 17, 535-538 (1972).
3. V.R.Kulli, On *K* Banhatti indices of graphs, *Journal of Computer and Mathematical Sciences*, 7(4), 213-218 (2016).
4. B. Basavanagoud, I. Gutman and V.R. Desai, Zagreb indices of generalized transformation graphs and their complements, *Kragujevac J. Sci*, 37, 99-112 (2015).
5. N. De, Sk. Md. Abu Nayeem and A. Pal, Reformulated first Zagreb index of some graph operations, *Mathematics*, 3, 945-960 (2015).
6. S.M.Hosamani and I. Gutman, Zagreb indices of transformation graphs and total transformation graphs, *Applied Mathematics and Computation* 247, 1156-1160 (2014).
7. H. Hua, A. R. Ashrafi and L. Zhang, More on Zagreb coindices of graphs, *Filomat* 26:6, 1215-1225 (2012), DOI 10.2298/FIL1206215H.
8. A. Ilić and B. Zhou, On reformulated Zagreb indices, *Discrete Appl. Math.* 160, 204-209 (2012).
9. M.H. Khalifeh, H. Yousefi-Azari and A.R. Ashrafi, The first and second Zagreb indices of some graph operations, *Discrete Appl. Math.* 157, 804-811 (2009).
10. V.R.Kulli, On *K* indices of graphs, *International Journal of Fuzzy Mathematical Archive*, 10(2), 105-109 (2016).
11. V.R.Kulli, On *K* coindices of graphs, *Journal of Computer and Mathematical Sciences*, 7(3), 107-112 (2016).
12. V.R. Kulli, On *K* edge index and coindex of graphs, *International Journal of Fuzzy Mathematical Archive*, 10(2), 111-116 (2016).
13. V.R. Kulli, The first and second κ_a indices and coindices of graphs, International Journal of Mathematical Archive, submitted.
14. S. Nikolić, G. Kovaćević, A. Milićević and N. Trinajstić, The Zagreb indices 30 years after, *Croatica Chemica Acta CCACAA* 76(2), 113-124 (2003).
15. P.S.Ranjini, V. Lokesha and I.N. Cangül, On the Zagreb indices of the line graphs of the subdivision graphs. *Applied Mathematics and Computation* 218, 699-702 (2011).
16. K. Xu, K. Tang, H. Liu and J. Wang, The Zagreb indices of bipartite graphs with more edges, *J. Appl. Math. and Informatics*, 33(3-4), 365-377 (2015).
17. B. Basavanagoud and S. Patil, A note on hyper-Zagreb coindex of graph operations, *J. Appl. Math. Comput, DOI* 10.1007/s 12190-016-0986-y.
18. M.R. Farahani, M.R. Rajesh Kanna and R. Pradeep Kumar, On the hyper-Zagreb indices of nano-structures, *Asian Academic Research Journal of Multidisciplinary*, 3(1), (2016).
19. V.R. Kulli, On the second hyper-Zagreb coindex of a graph, submitted.
20. G.H. Shirdel, H. Rezapour and A.M. Sayadi, The hyper-Zagreb index of graph operations, *Iran J. Math. Chem*. 4(2), 213-220 (2013).
21. M. Veylaki, M.J. Nikmehr and H.R. Tavallaee, The third and hyper-Zagreb coindices of some graph operations, *J. Appl. Math. Comput.* (2015) doi: 10.1007/s 12190-015-0872-z.

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