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ON *K* HYPER-BANHATTI INDICES AND COINDICES OF GRAPHS

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ABSTRACT

The vertices and edges of a graph G are called the elements of G. If e=uv is an edge of G, then the vertex u and edge e are incident as are v and e. We introduce the first and second K hyper-Banhatti indices to take account of the contributions of pairs of incident elements. Also we introduce the first and second K hyper-Banhatti coindices to take account the contributions of pairs of nonincident elements. In this paper, we obtain the exact values of the first and second K hyper-Banhatti indices for some standard graphs.

***Keywords*: incident, *K* hyper-Banhatti indices, *K* hyper-Banhatti coindices.**

***Mathematics Subject Classification*: 05*C*.**

1. INTRODUCTION

The graphs considered here are finite, undirected without isolated vertices, loops and multiple edges. Any undefined term in this paper may be found in Kulli [1].

Let $G=(V, E)$ be a graph with $|V| = n$ vertices and $|E| = m$ edges. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v is denoted by uv. If $e=uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) – 2$ with $e=uv$. The vertices and edges of a graph are called its elements.

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences.

In Chemical Science, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [2].

In [3], Kulli introduced the first and second *K* Banhatti indices to take account of the contributions of pairs of incident elements.

The first *K* Banhatti index $B_1(G)$ and the second *K* Banhatti index $B_2(G)$ of a graph G are defined as

$$B_1(G)=\sum_{ue}\left[d_G(u)+d_G(e)\right]$$

$$B_2(G)=\sum_{ue}d_G(u)d_G(v)$$

where ue means that the vertex u and edge e are incident in G.

In [3], Kulli introduced the first and second *K* Banhatti coindices to take account of the contributions of pairs of nonincident elements.

The first *K* Banhatti coindex $\overline{B}_1(G)$ and the second *K* Banhatti coindex $\overline{B}_2(G)$ of a graph G are defined as

$$\overline{B}_1(G)=\sum_{u*e}\left[d_G(u)+d_G(e)\right]$$

$$\overline{B}_2(G)=\sum_{u*e}d_G(u)d_G(e)$$

where $u * e$ means that the vertex u and edge e are nonincident in G.

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Recently many other indices and coindices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

In this paper, we introduce *K* hyper-Banhatti indices and coindices of graphs. Recently many other hyper indices and coindices were studied, for example, in [17, 18, 19, 20, 21].

2. FIRST *K* HYPER-BANHATTI INDEX

We introduce the first *K* hyper-Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 1: The first *K* hyper-Banhatti index of a graph *G* is defined as

$$HB_1(G) = \sum_{ue}\left[d_G(u) + d_G(e)\right]^2$$

where *ue* means that the vertex *u* and edge *e* are incident in *G*.

The following are the first *K* hyper-Banhatti index for cycles, complete graphs, complete bipartite graphs.

Proposition 2: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HB_1(C_n) = 32n.$$

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Every vertex of C_n is incident with exactly two edges. Consider

$$\begin{aligned} HB_1(C_n) &= \sum_{ue}\left[d_{C_n}(u) + d_{C_n}(e)\right]^2 \\ &= \sum_{u_i}\sum_{e_j}\left[d_{C_n}(u_i) + d_{C_n}(e_j)\right]^2 \\ &= \sum_{u_i}^{n}\sum_{e_j}^{2}(2+2)^2 \\ &= \sum_{u_i}^{n}2(4)^2 \\ &= 32n. \end{aligned}$$

Proposition 3: Let K_n be a complete graph with *n* vertices. Then

$$HB_1(K_n) = n(n-1)(3n-5)^2.$$

Proof: Let K_n be a complete graph with *n* vertices. Every vertex of K_n is incident with $n - 1$ edges. Consider

$$\begin{aligned} HB_1(K_n) &= \sum_{ue}\left[d_{K_n}(u) + d_{K_n}(e)\right]^2 \\ &= \sum_{u_i}\sum_{e_j}\left[d_{K_n}(u_i) + d_{K_n}(e_j)\right]^2 \\ &= \sum_{u_i}^{n}\sum_{e_j}^{n-1}\left[(n-1) + (2n-4)\right]^2 \\ &= \sum_{u_i}^{n}(n-1)(3n-5)^2 \\ &= n(n-1)(3n-5)^2. \end{aligned}$$

Proposition 4: Let $K_{m,n}$ be a complete bipartite graph. Then

$$HB_1(K_{m,n}) = mn[(m + 2n - 2)^2 + (2m + n - 2)^2].$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Every vertex of V_1 is incident with *n* edges and every vertex of V_2 is incident with *m* edges. Let $V_1 = \{v_1, v_2, ..., v_m\}$ and $V_2 = \{w_1, w_2, ..., w_n\}$. Consider

$$\begin{aligned} HB_1(K_{m,n}) &= \sum_{ue}\left[d_{K_{m,n}}(u) + d_{K_{m,n}}(e)\right]^2 \\ &= \sum_{v_i}^{m}\sum_{e_j}^{n}\left[d_{K_{m,n}}(v_i) + d_{K_{m,n}}(e_j)\right]^2 + \sum_{w_j}^{n}\sum_{e_i}^{m}\left[d_{K_{m,n}}(w_j) + d_{K_{m,n}}(e_i)\right]^2 \\ &= \sum_{v_i}^{m}\sum_{e_j}^{n}\left[n + (m+n-2)\right]^2 + \sum_{w_j}^{n}\sum_{e_i}^{m}\left[m + (m+n-2)\right]^2 \end{aligned}$$

$$= \sum_{v_i}^{m}\left[n\left(m+2n-2\right)^2\right] + \sum_{w_j}^{n}\left[m\left(2m+n-2\right)^2\right]$$
$$= mn(m + 2n - 2)^2 + nm(2m + n - 2)^2$$
$$= mn[(m + 2n - 2)^2 + (2m + n - 2)^2].$$

The following results are immediate from Proposition 4.

Corollary 5: Let $K_{n,\,n}$ be a complete bipartite graph. Then

$$HB_1\left(K_{n,\,n}\right) = 2n^2\left(3n-2\right)^2.$$

Corollary 6: Let $K_{1,\,n}$ be a star. Then

$$HB_1\left(K_{1,n}\right) = n\left(5n^2-4n+1\right).$$

Theorem 7: Let G be an r -regular graph with n vertices. Then

$$HB_1\left(G\right) = nr\left(3r-2\right)^2.$$

Proof: Let G be an r -regular graph with n vertices. Every vertex of G is incident with r edges. Consider

$$HB_1\left(G\right) = \sum_{ue}\left[d_G\left(u\right)+d_G\left(e\right)\right]^2$$
$$= \sum_{u_i}^{n}\sum_{e_j}^{r}\left[r+\left(2r-2\right)\right]^2$$
$$= \sum_{u_i}^{n} r\left(3r-2\right)^2$$
$$= nr(3r-2)^2.$$

3. SECOND *K* HYPER-BANHATTI INDEX

We introduce the second K hyper-Banhatti index of a graph in terms of incident vertex-edge degrees.

Definition 8: The second K hyper-Banhatti index of a graph G is defined as

$$HB_2\left(G\right) = \sum_{ue}\left(d_G\left(u\right)d_G\left(e\right)\right)^2$$

where ue means that the vertex u and edge e are incident in G.

The following are the second K hyper-Banhatti index for cycles, complete graphs, complete bipartite graphs.

Proposition 9: Let C_n be a cycle with $n \geq 3$ vertices. Then

$$HB_2(C_n) = 32n.$$

Proof: Let C_n be a cycle with n vertices. Every vertex of C_n is incident with exactly two edges. Consider

$$HB_2\left(C_n\right) = \sum_{ue}\left(d_{C_n}\left(u\right)d_{C_n}\left(e\right)\right)^2$$
$$= \sum_{u_i}\sum_{u_j}\left(d_{C_n}\left(u_i\right)d_{C_n}\left(e_j\right)\right)^2$$
$$= \sum_{u_i}^{n}\sum_{e_j}^{2}\left(2\times 2\right)^2$$
$$= \sum_{u_i}^{n}\left(2\right)\left(4\right)^2$$
$$= 32n.$$

Proposition 10: Let K_n be a complete graph with n vertices. Then

$$HB_2(K_n) = 4n(n-1)^3\,(n-2)^2.$$

Proof: Let K_n be a complete graph with n vertices. Every vertex of K_n is incident with n–1 edges. Consider

$$HB_2(K_n) = \sum_{ue}\left(d_{K_n}(u)d_{K_n}(e)\right)^2$$
$$= \sum_{u_i}\sum_{e_j}\left(d_{K_n}(u_i)d_{K_n}(e_j)\right)^2$$
$$= \sum_{u_i}^{n}\sum_{e_j}^{n-1}\left((n-1)(2n-4)\right)^2$$
$$= \sum_{u_i}^{n}(n-1)(n-1)^2(2n-4)^2$$
$$= 4n(n-1)^3(n-2)^2.$$

Proposition 11: Let $K_{m,n}$ be a complete bipartite graph. Then

$$HB_2(K_{m,n}) = mn(m^2+n^2)(m+n-2)^2.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and $|V_1|=m$, $|V_2|=n$, $V(K_{m,n}) = V_1 \cup V_2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Let $V_1 = \{v_1, v_2, ..., v_m\}$ and $V_2 = \{w_1, w_2, ..., w_n\}$. Consider

$$HB_2(K_{m,n}) = \sum_{ue}\left(d_{K_{m,n}}(u)d_{K_{m,n}}(e)\right)^2$$
$$= \sum_{v_i}^{m}\sum_{e_j}^{n}\left(d_{K_{m,n}}(v_i)d_{K_{m,n}}(e_j)\right)^2 + \sum_{w_j}^{n}\sum_{e_i}^{m}\left(d_{K_{m,n}}(w_j)d_{K_{m,n}}(e_i)\right)^2$$
$$= \sum_{v_i}^{m}\sum_{e_j}^{n} n^2(m+n-2)^2 + \sum_{w_j}^{n}\sum_{e_i}^{m} m^2(m+n-2)^2$$
$$= \sum_{v_i}^{m} n^3(m+n-2)^2 + \sum_{w_j}^{n} m^3(m+n-2)^2$$
$$= mn^3(m+n-2)^2 + nm^3(m+n-2)^2$$
$$= mn(n^2+m^2)(m+n-2)^2.$$

The following results are immediate from Proposition 11.

Corollary 12: Let $K_{n,n}$ be a complete bipartite graph. Then

$$HB_2(K_{n,n}) = 8n^4(n-1)^2.$$

Corollary 13: Let $K_{1,n}$ be a star. Then

$$HB_2(K_{1,n}) = n(n^2+1)(n-1)^2.$$

Theorem 14: Let G be an r-regular graph with n vertices. Then

$$HB_2(G) = 4nr^3(r-1)^2.$$

Proof: Let G be an r-regular graph with n vertices. Then every vertex of G is incident with r edges. Consider

$$HB_2(G) = \sum_{ue}\left(d_G(u)d_G(e)\right)^2$$
$$= \sum_{u_i}^{n}\sum_{e_j}^{r}\left(r(2r-2)\right)^2$$
$$= \sum_{u_i}^{n} r^3(2r-2)^2$$
$$= 4nr^3(r-1)^2.$$

4. *K* HYPER-BANHATTI COINDICES

We define *K* hyper-Banhatti coindices of a graph in terms of nonincident vertex-edge degrees.

Definition 15: The first and second *K* hyper-Banhatti coindices of a graph *G* are defined as

$$\overline{HB}_1(G) = \sum_{u*e}\left[d_G(u) + d_G(e)\right]^2$$

$$\overline{HB}_2(G) = \sum_{u*e}\left(d_G(u)\,d_G(e)\right)^2$$

where $u * e$ means that the vertex *u* and edge *e* are not incident elements in *G*.

We study these invariants in a separate paper.

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