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TANGLE HYPER GRAPH

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ABSTRACT

We will study a new graph, this graph called tangle hyper graph. We will study the matrices which represent this graph. We will discuss how to change it into simple graph .We will explain the effect of folding on hyper tangle graph.

Keywords: *Tangle graph, hyper graph, Tangle hyper graph.*

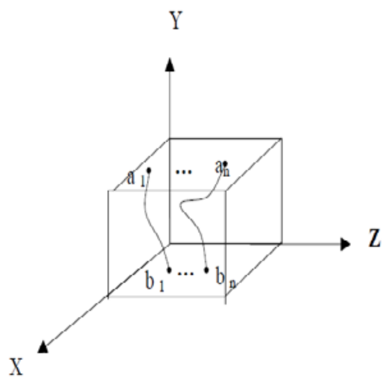
Mathematics Subject Classification: *51H10, 57H10.*

INTRODUCTION

Conway developed tangle theory and invented a system of notation for tabulating knots, nowadays known as Conway notation. Tangle theory can be considered analogous to knot theory except, instead of closed loop we use string whose end are nailed down. Tangles have been shown to be useful in studying DNA topology. Braid groups were introduced explicitly by Emil Artin in 1925, although (as Wilhelm Magnus pointed out in 1974) they were already implicit in Adolf Hurwitz's work on monodromy (1891). In fact, as Magnus says, Hurwitz gave the interpretation of a braid group as the fundamental group of a configuration space (cf. braid theory), an interpretation that was lost from view until it was rediscovered by Ralph Fox and Lee Neuwirth in 1962. Like in most fruitful mathematical theories, the theory of hyper graph has many applications. Hyper graphs model many practical problems in many different science. Hyper graphs have shown their power as a tool to understand problems in awide variety of scientific field. Moreover it well known now that hyper graph theory is a very useful tool to resolve optimization problem such as scheduling problems, location problems and so on.

Definition 1:

Tangle graph: Let D be a unit cube, so D={(x, y, z):0<x, y, z<1} on the top face of cube place n points $a_1, a_2, \dots, a_n$ similarly place on bottom face $b_1, b_2, \dots, b_n$, now join the points $a_1, a_2, \dots, a_n$ with $b_1, b_2, \dots, b_n$ by arcs $d_1, d_2, \dots, d_n$ these arcs are disjoint and each d_i connects some a_j to b_k not connect a_j to a_k or b_j to b_k this called tangle [1].



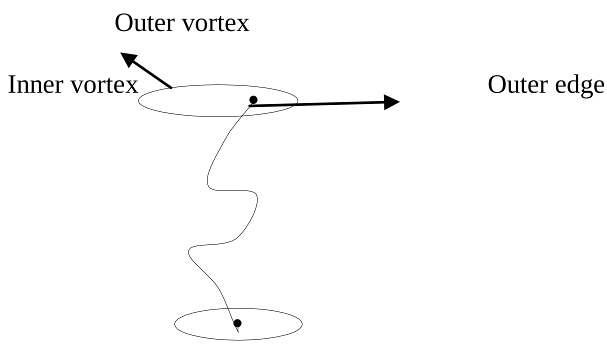


Figure – 1: Simple tangle graph

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Definition 2:

Hyper graph: A hyper graph is a graph which an edge can connect any number of vertices. Formally, a hyper graph ***H*** is a pair ***H*** =(X, E) Where X is a set of elements called nodes or vertices, and E is a set of non-empty sub set of X called hyper edge or edges [2].

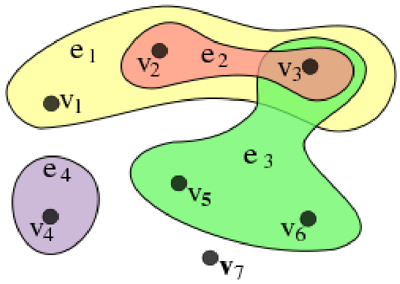


Figure - 1: An example of a hyper graph, with

$$X=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\} \text{ and } E=(e_1,e_2,e_3,e_4)=\{(v_1,v_2,v_3),(v_2,v_3)(v_3,v_5,v_6),(v_4)\}.$$

In this example the edges connect between more than two points.

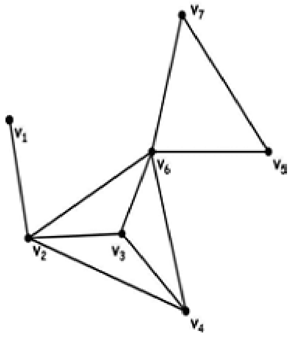
Incident matrices of hyper graph:

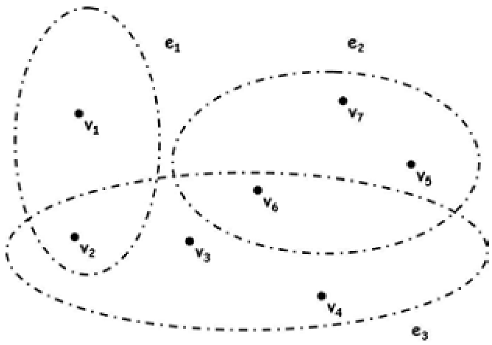
$$\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\0&0&0&1\\0&0&1&0\\0&0&1&0\\0&0&0&0\end{bmatrix}$$

Where the vortex ($\mathbf{v_7}$) in Fig. called isolated point.

Example:

	e_1	e_2	e_3
v_1	1	0	0
v_2	1	0	1
v_3	0	0	1
v_4	0	0	1
v_5	0	1	0
v_6	0	1	1
v_7	0	1	0
v_2	1	0	1





Definition 3:

Directed hyper graph:
A *directed hyper graph* H = (V, A) is a pair, where V is a finite set of vertices and A is a set of hyper arcs. A *hyper arc* $a \in A$ is an ordered pair (X, Y) where X and Y are disjoint not empty subsets of V ($X \cap Y = \emptyset, X, Y \neq \emptyset$). Set X (Y) is called

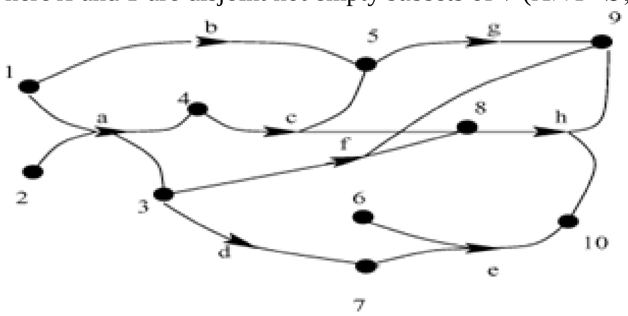
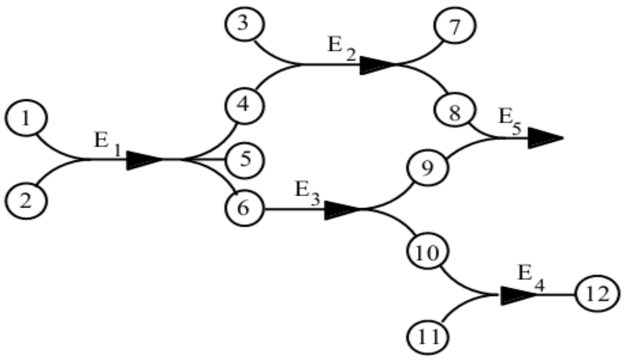


Figure 1 – Directed hypergraph $H = (V, A)$.

the *origin* (*destination*) of a and denoted $Org(a)$ ($Dest(a)$). A directed hyper graph $H = (V, A)$ with no isolated vertices has size $|H| = \sum_a = (X, Y) \in A^{|X| + |Y|}$ [3].

Incidence matrix of directed hyper graph:



	E_1	E_2	E_3	E_4	E_5
1	-1	0	0	0	0
2	-1	0	0	0	0
3	0	-1	0	0	0
4	1	-1	0	0	0
5	1	0	0	0	0
6	1	0	-1	0	0
7	0	1	0	0	0
8	0	1	0	0	-1
9	0	0	1	0	-1
10	0	0	1	-1	0
11	0	0	0	-1	0
12	0	0	0	1	0

Figure - 3

Definition 4:

Definition 4-1: Empty hyper graph is the hyper graph such that:
$V = \emptyset$, $E = \emptyset$

Definition4-2: A trivial hyper graph is a hyper graph such that:
$V \neq \emptyset$, $E = \emptyset$

Loop in hyper graph is a hyper edge e ϵ E such that IeI=1 [4].

Definition 5: Let $H = (V, A)$ be a directed hyper graph.

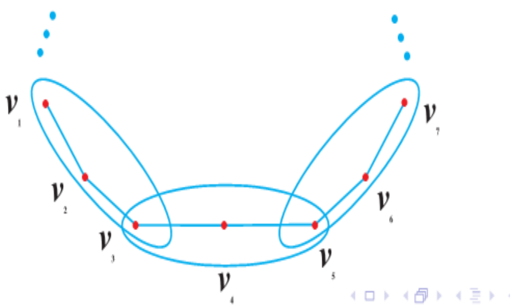
1. If every hyper arc $a \in A$ is such that $|Dest(a)| = 1$ than H is called a *B-graph*;
2. If every hyper arc $a \in A$ is such that $|Org(a)| = 1$ than H is called a *F-graph*;
3. If every hyper arc $a \in A$ is such that $|Dest(a)| = 1$ or $|Org(a)| = 1$ than H is called a*BF-graph*. A digraph is a particular case of BF-graphs, with $|Org(a)| = 1$ and $|Dest(a)| = 1$ for all arcs.

Cycle:

Loose cycle:

C_m is a loose cycle in $K_n^{(3)}$, if it has vertices $\{v_1, \ldots, v_m\}$ and edges

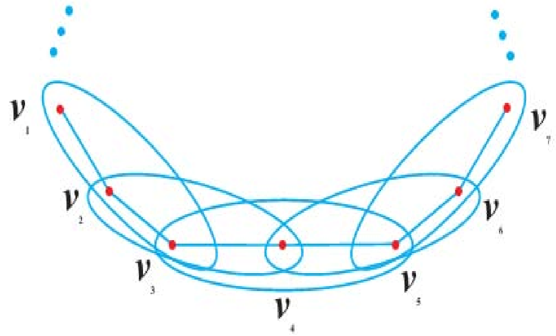
$$\{(v_1, v_2, v_3), (v_3, v_4, v_5)(v_5, v_6, v_7), \ldots, (v_{m-1}, v_m, v_1)\}.$$ (So in particular m is even).



Tight cycle:

C_m is a tight cycle in $K_n^{(3)}$, if it has vertices $\{v_1, \ldots, v_m\}$ and edges

$$\{(v_1, v_2, v_3), (v_2, v_3, v_4)(v_3, v_4, v_5), \ldots, (v_m, v_1, v_2)\}.$$



Thus every set of 3 consecutive vertices along the cycle forms an edge.

Berge cycle:

$C_m = (v_1, e_1, v_2, e_2, \ldots, v_m, e_m, v_1)$ is a berge cycle in $K_n^{(r)}$,, if

- $v_1, \ldots v_m$ are all distinct vertices.
- $e_1, \ldots e_m$ are all distinct edges.
- $v_k, v_{k+1} \in e_k$ for $k = 1, \ldots m$, .where $v_{m+1} = v_1$

T –tight- Berge- cycle:

$C_m = (v_1, v_2, \ldots, v_m)$ is a t- tight Berge-cycle in $K_n^{(r)}$, if for each set $\{v_i, v_{i+1}, \ldots, v_{i+t-1}\}$ of t consecutive vertices along the cycle (mod m)., there is an edge e_i containing it and these edges are all distinct.

Special cases: For t=2 we get ordinary Berge- cycle and for wq=r we get the tight cycle.

Conversation hyper graph into simple graph:

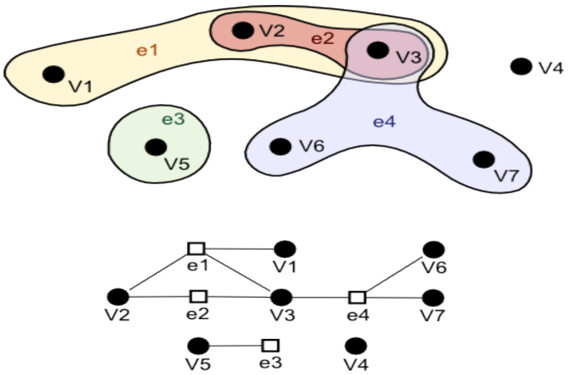


Figure - 6

Tangle hyper graph:

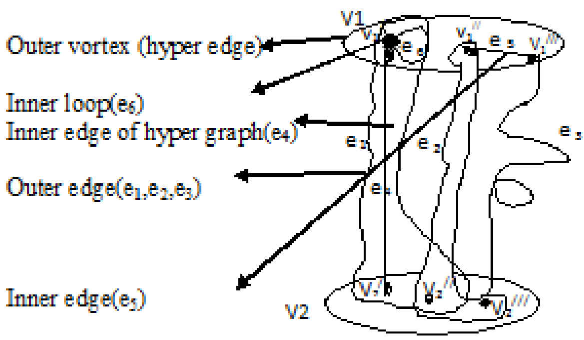


Figure - 7

In this figure outer edge (e_1, e_2, e_3) of tangle hyper graph, where this edge connected between more than two vertices. The inner edge of tangle graph is the edge between two inner vertices of outer vertices and inner edge of hyper graph is the edge between inner vertices of hyper edge of hyper graph.

A adjacent matrix:

$$\begin{bmatrix} 0 & 1^{ou}_{ou} & 1^{/} & 1^{/} & 1^{/} & 1^{ou}_{in} & 1^{ou}_{in} & 1^{ou}_{in} \\ 1^{ou}_{ou} & 0 & 1^{ou}_{in} & 1^{ou}_{in} & 1^{ou}_{in} & 1^{/} & 1^{/} & 1^{/} \\ 1^{/} & 1^{ou}_{in} & 1^{0} & 0 & 0 & 1 & 1 & 0 \\ 1^{ou}_{in} & 1^{ou}_{in} & 0 & 0 & 1^{0} & 1 & 1 & 1 \\ 1^{ou}_{in} & 1^{ou}_{in} & 0 & 1^{0} & 0 & 0 & 0 & 1^{1,1}_{2} \\ 1^{ou}_{in} & 1^{/} & 1 & 0 & 0 & 0 & 1 & 0 \\ 1^{ou}_{in} & 1^{/} & 1 & 1 & 0 & 1 & 0 & 1 \\ 1^{ou}_{in} & 1^{/} & 0 & 1 & 1^{1,1}_{2} & 0 & 1 & 0 \end{bmatrix}$$

Where {1^0 is the ordinary edge, $1^/$ is the edge between inner vertex and outer vertex such that inner vertex is found in outer vertex, $1^{i,j}_k$ {I number of fiber, j number of rolls, k number of curves} and 1^{ou}_{in} express the edge between inner and outer vertex such that inner vertex of other outer vertex.}

Lemma: The edge between inner and outer vertex of different vertices is different from edge between outer and inner vertex of the same outer vertex.

Incident matrix:

$$\begin{bmatrix} 1^{//} & 1^{//} & 1^{//} & 1^{0} & 1^{0} & 1^{0} \\ 1^{//} & 1^{//} & 1^{//} & 1^{0} & 0 & 0 \\ 1 & 0 & 0 & 1^{0} & 1^{0} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1^{0} \\ 0 & 0 & 1^{1,1}_{2} & 0 & 0 & 1^{0} \\ 1 & 0 & 0 & 1^{0} & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1^{1,1}_{2} & 0 & 0 & 0 \end{bmatrix}$$

Where {1^0 is the ordinary edge, 1 is the hyper arc between inner vertices and, $1^{i,j}_k$ is the edge of tangle, $1^{//}$ is express the edge between outer vertices and hyper arc t between ordinary edge and outer vertices.}

Note: The outer vertices of tangle graph can be expressed as hyper edge.

Conversation tangle hyper graph to simple tangle graph:

Folding:

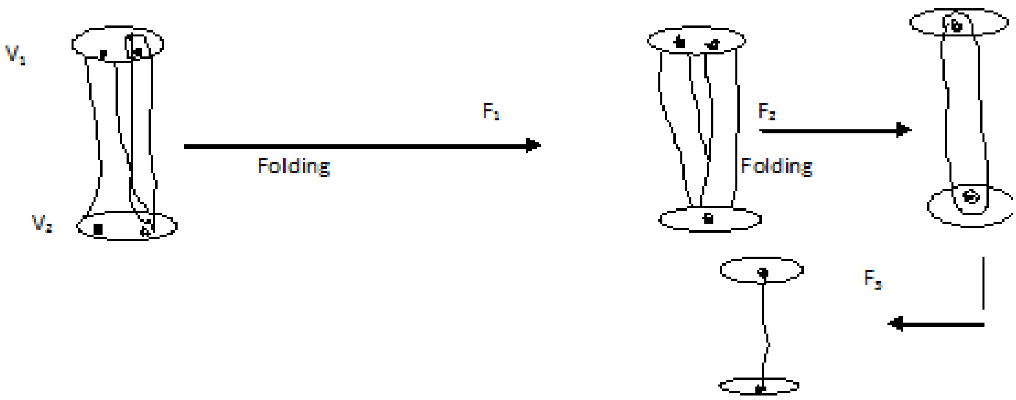


Figure - 8

In this figure: F_1 folding inner vertices of outer vortex (bottom) of hyper tangle graph into one vortex, but hyper edge not folding. F_2 also folding inner vertices of outer vortex (top) into one vortex, these folding will give us simple hyper tangle graph. F_3 folding the hyper edge into simple edge and will give us braid which is the special case of tangle graph.

Lemma: All graphs are just a subset of hyper graph.

Directed tangle hyper graph:

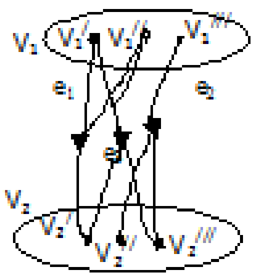


Figure - 9

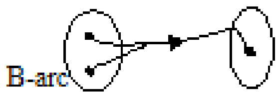
A directed tangle hyper graph $T_h(V, E)$ is a pair , where V is a finite set of vertices these vertices consists of inner and outer vertices, These vertices connects by edges called hyper arcs, when take herozinatal line intersect these arcs at more than one point (tangle).

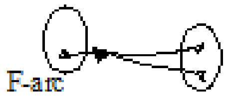
Definition: Let $T_h(V, E)$ be a directed tangle hyper graph

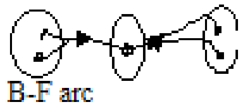
A Backward hyper arc, or simply B-arc, is a hyper arc $E=(T(E),H(E))$ with $|H(E)|=1$

A forward hyper arc, or simply F-arc, is a hyper arc $E=(T(E),H(E))$ with $|T(E)|=1$

A digraph is a particular case of BF-graphs, with $|T(E)| = 1$ and $|H(E)| = 1$ for all arcs.







Matrices:

Incident matrix:

Incident matrix of Fig. (9**)**

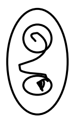
$$\begin{bmatrix} 1^0 & 1^0 & 1^0 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

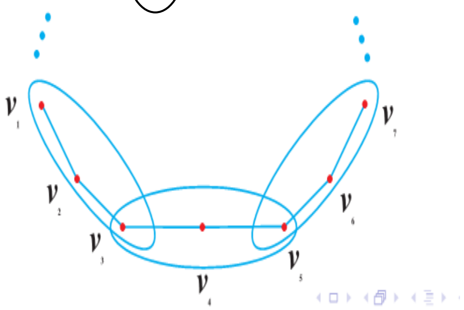
Where $\{1^0$ is the ordinary edge, (1) express if vertex is (tail), (-1) express if vertex (head), and (0) if there no any relation between edge and vertex$\}$.

Cycle of tangle hyper graph:

If we consider the hyper arc of hyper graph as like the outer vortex of tangle graph then we will define loose cycle as we define it in hyper graph. Let $\{v_1,v_2,v_3,\ldots..v_n\}$ set of vertices these vertices are tangle vertices and edges are $\{(v_1,v_2,v_3),(v_3,v_4,v_5),\ldots(v_{n-1},v_n,v_1)\}$

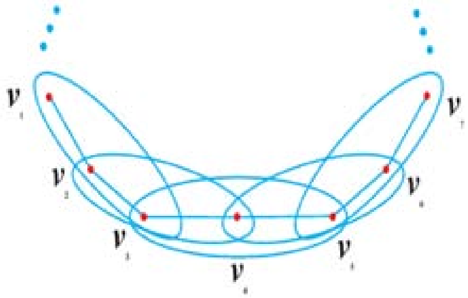
The vertices are tangle take the form

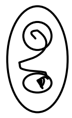




C_m is a tight cycle in $K_n^{(3)}$, if it has vertices $\{v_1,\dots,v_m\}$ and edges

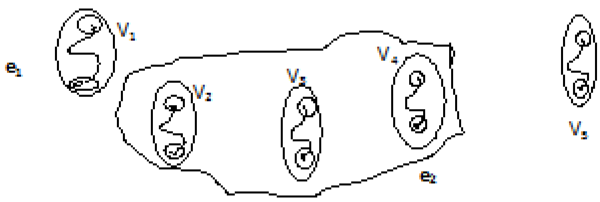
$$\{(v_1,v_2,v_3),(v_2,v_3,v_4)(v_3,v_4,v_5),\dots,(v_m,v_1,v_2)\}.$$



The vertices are tangle take the form 

Hypergraph of tangle vertices:

We will define anew hypergraph whose vertices are tangle. We study the incident and adjacient matrices. We study the fiber of tangle anylatically.



Incident and adjacent matrices:

$$I[T_h]=\begin{bmatrix} 1_2^{1,0} & 0 \\ 1_2^{1,0} & 1_2^{1,0} \\ 1_2^{1,0} & 1_2^{1,0} \\ 0 & 1_2^{1,0} \end{bmatrix} \qquad A[T_h]=\begin{bmatrix} 0 & 1_2^{1,0} & 1_2^{1,0} & 0 & 0 \\ 1_2^{1,0} & 0 & 1_2^{1,0} & 1_2^{1,0} & 0 \\ 1_2^{1,0} & 1_2^{1,0} & 1_2^{1,0} & 1_2^{1,0} & 0 \\ 0 & 0 & 1_2^{1,0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $1_k^{i,j}$ express the relation between vertices and edges such that (I, j, k) express the tangle vertex analytically (I number of fiber number of rolls, k number of curves).

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