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ON THE CYCLIC GROUP GENERATED BY STRUCTURE EQUATION $F^{2k+1}+F=0$

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ABSTRACT

In this paper, we have studied the formation of a cyclic group generated by structure equation $F^{2k+1}+F=0$, *where k is a positive integer. Properties of some elements of* M_{4k} *have also been discussed.*

Key words: *Differnetiable manifold, complementary projectrion operators, cyclic group.*

1. INTRODUCTION

Let M^n be a differentiable manifold of class C^∞ and F be a (1, 1) tensor of class C^∞, satisfying

(1.1) $$F^{2K+1}+F=0$$

we define the operators l and m on M^n by

(1.2) $$l=-F^{2k}, \qquad m=I+F^{2k}$$

where I is the identify operator. From (1.1) and (1.2), we get

(1.3) $$l+m=I, \quad l^2=l,\ m^2=m, \quad lm=ml=0$$

$$Fl=lF=F, \quad mF=Fm=0,$$

$$F^r=\begin{cases} F^r & ,1\le r\le 2k \\ -F^{r-2k} & ,r>2k \end{cases}$$

Theorem 1.1: For F and m satisfying (1.1) and (1.2) respectively, the set

(1.4) $$M_{4k}=\left\{m\pm F^r \quad 1\le r\le 2k\right\}$$

is a cyclic group of order 4k, under the multiplication (composition) operation.

Proof: We have

(1.5) $$M_{4k}=\left\{m-F^{2k}, m-F^{2k-1}, \ldots. m-F,\ m+F, \ldots. m+F^{2k}\right\}$$

Let $m\pm F^r, m\pm F^s\in M_{4k}$, then $r\le 2k, s\le 2k \Rightarrow r+s-2k\le 2k$.

a) **Closure property** using (1.3), we have

(1.6) $$\left(m+F^r\right)\left(m+F^s\right)=\begin{cases} m+F^{r+s} & if \quad r+s\le 2k \\ m-F^{r+s-2k} & if \quad r+s>2k \end{cases}$$

(1.7) $$\left(m+F^r\right)\left(m-F^s\right)=\begin{cases} m-F^{r+s} & if \quad r+s\le 2k \\ m+F^{r+s-2k} & if \quad r+s>2k \end{cases}$$

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(1.8) $\left(m-F^{r}\right)\left(m-F^{s}\right)=\begin{cases} m+F^{r+s} & \text{if} \quad r+s\le 2k \\ m-F^{r+s-2k} & \text{if} \quad r+s>2k \end{cases}$

Thus the product of any two elements of M_{4k} is in M_{4k}

(b) **Associative property**: Since the multiplication of arbitrary functions obeys the associative law, therefore it holds for the elements of M_{4k} also

(c) **Existence of identity:** From (1.2), we have

(1.9) $m-F^{2k}=I \therefore m-F^{2k}$ is the identity element of M_{4k}

(d) **Existence of inverse:** For r< 2k Let $m+F^{r}\in M_{4k}$ then we claim that $\left(m+F^{r}\right)^{-1}=m-F^{2k-r}$ since with the help of (1.3)

(1.10) $\left(m+F^{r}\right)\left(m-F^{2k-r}\right)=m-F^{2k}=I$

Similarly

(1.11) $\left(m-F^{r}\right)^{-1}=m+F^{2k-r}$

Also

(1.12) $\left(m-F^{2k}\right)^{-1}=m-F^{2k}$

(1.13) $\left(m+F^{2k}\right)^{-1}=m+F^{2k}$

Thus each element in M_{4k} has its multiplicative inverse.

Hence M_{4k} is a group under multiplication moreover we have on using (1.3).

(1.14) $\left(m+F\right)^{1}=m+F,\left(m+F\right)^{2}=m+F^{2},\ldots.\left(m+F\right)^{2k}=m+F^{2k},$

$\left(m+F\right)^{2k+1}=m+F^{2k+1}=m-F,\left(m+F\right)^{2k+2}=m+F^{2k+2}$

$=m-F^{2},\ldots\left(m+F\right)^{4k}=m+F^{4k}=m-F^{2k}=I$

(1.15) $M_{4k}=<m+F>,0\left(m+F\right)=4k=0\left(M_{4k}\right)$

all the generators of M_{4k} are of the form $m+F^{t}$ where t is a positive integer relatively prime to 4k. Also

(1.16) $o\left(m+F^{r}\right)=o\left[\left(m+F\right)^{r}\right]=\dfrac{4k}{\left(4k,r\right)}$

where (a, b) denotes *gcd* of a and b.

Theorem 1.2:

Let $p,q\in M_{4k}$ were

(1.17) $p=m+F^{k}, \quad q=m-F^{k}$, then

(1.18) (i) $o\left(p\right)=o\left(q\right)=4$

(ii) $pq=I,\ p^{-1}=q=p^{3},\ q^{-1}=p=q^{3},\ p^{2}=q^{2}$

(iii) $p^{2}-p-q+I=0=q^{2}-p-q+I$

(iv) $pm=qm=p^{2}m=q^{2}m=m$

Proof: from (1.16) taking r=k we have

(1.19) $o\left(p\right)=o\left(m+F^{k}\right)=o\left[\left(m+F\right)^{k}\right]=\dfrac{4k}{\left(4k,k\right)}=\dfrac{4k}{k}=4$

etc, the other parts follow similarly

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Remark: Let

(1.20) $L_{4k} = \left\{ l - F^{2k}, l - F^{2k-1}, ... \ l - F, l + F, ... \ l + F^{2k} \right\}$

Since by (1.2), $l + F^{2k} = o$ Thus L_{4k} is not a group under multiplication.

Ex.1 Let $k = 1 \Rightarrow 2k = 2,$ the structure equation is

(1.21) $F^3 + F = 0$

(1.22) $l = -F^2, \quad m = I + F^2$

(1.23) $M_4 = \left\{ I = m - F^2, m - F, m + F, m + F^2 \right\}$

The Cayley table for M_4 is

	$m-F^2$	$m-F$	$m+F$	$m+F^2$
$m-F^2$	$m-F^2$	$m-F$	$m+F$	$m+F^2$
$m-F$	$m-F$	$m+F^2$	$m-F^2$	$m+F$
$m+F$	$m+F$	$m-F^2$	$m+F^2$	$m-F$
$m+F^2$	$m+F^2$	$m+F$	$m-F$	$m-F^2$

From this table we have

(1.24) $\left(m + F\right)^{-1} = m - F,$

$\left(m + F^2\right)^{-1} = m + F^2$

$\left(m - F^2\right)^{-1} = m - F^2$

(1.25) $o\left(m + F\right) = 4, \quad o\left(m - F\right) = 4,$

$o\left(m + F^2\right) = 2, \quad o\left(m - F^2\right) = 1$

The only subgroups of M_4 are

(1.26) $H_1 = \left\{ m - F^2 \right\}, \ H_2 = \left\{ m - F^2, m + F^2 \right\}, H_3 = M_4$

Ex. 2: Let $k = 2 \Rightarrow 2k = 4$. The structure equation is

(1.27) $F^5 + F = o,$

(1.28) $l = -F^4, \quad m = I + F^4$

(1.29) $M_8 = \left\{ m - F^4, m - F^3, m - F^2, m - F, \right.$

$\left. m + F, m + F^2, m + F^3, m + F^4 \right\}$

The Caylay table for M_8 is

	$m-F^4$	$m-F^3$	$m-F^2$	$m-F$	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$
$m-F^4$	$m-F^4$	$m-F^3$	$m-F^2$	$m-F$	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$
$m-F^3$	$m-F^3$	$m-F^2$	$m-F$	$m+F^4$	$m-F^4$	$m+F$	$m+F^2$	$m+F^3$
$m-F^2$	$m-F^2$	$m-F$	$m+F^4$	$m+F^3$	$m-F^3$	$m-F^4$	$m+F$	$m+F^2$
$m-F$	$m-F$	$m+F^4$	$m+F^3$	$m+F^2$	$m-F^2$	$m-F^3$	$m-F^4$	$m+F$
$m+F$	$m+F$	$m-F^4$	$m-F^3$	$m-F^2$	$m+F^2$	$m+F^3$	$m+F^4$	$m-F$
$m+F^2$	$m+F^2$	$m+F$	$m-F^4$	$m-F^3$	$m+F^3$	$m+F^4$	$m-F$	$m-F^2$
$m+F^3$	$m+F^3$	$m+F^2$	$m+F$	$m-F^4$	$m+F^4$	$m-F$	$m-F^2$	$m-F^3$
$m+F^4$	$m+F^4$	$m+F^3$	$m+F^2$	$m+F$	$m-F$	$m-F^2$	$m-F^3$	$m-F^4$

The inverses and orders of each element can be calculated easily.

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