



ON THE CYCLIC GROUP GENERATED BY STRUCTURE EQUATION $F^{2k+1} + F = 0$

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ABSTRACT

In this paper, we have studied the formation of a cyclic group generated by structure equation $F^{2k+1} + F = 0$, where k is a positive integer. Properties of some elements of M_{4k} have also been discussed.

Key words: Differentiable manifold, complementary projection operators, cyclic group.

1. INTRODUCTION

Let M^n be a differentiable manifold of class C^∞ and F be a $(1, 1)$ tensor of class C^∞ , satisfying

$$(1.1) \quad F^{2k+1} + F = 0$$

we define the operators l and m on M^n by

$$(1.2) \quad l = -F^{2k}, \quad m = I + F^{2k}$$

where I is the identify operator. From (1.1) and (1.2), we get

$$(1.3) \quad l + m = I, \quad l^2 = l, \quad m^2 = m, \quad lm = ml = 0$$

$$Fl = lF = F, \quad mF = Fm = 0,$$

$$F^r = \begin{cases} F^r & , 1 \leq r \leq 2k \\ -F^{r-2k} & , r > 2k \end{cases}$$

Theorem 1.1: For F and m satisfying (1.1) and (1.2) respectively, the set

$$(1.4) \quad M_{4k} = \{m \pm F^r \quad 1 \leq r \leq 2k\}$$

is a cyclic group of order $4k$, under the multiplication (composition) operation.

Proof: We have

$$(1.5) \quad M_{4k} = \{m - F^{2k}, m - F^{2k-1}, \dots, m - F, m + F, \dots, m + F^{2k}\}$$

Let $m \pm F^r, m \pm F^s \in M_{4k}$, then $r \leq 2k, s \leq 2k \Rightarrow r + s - 2k \leq 2k$.

a) **Closure property** using (1.3), we have

$$(1.6) \quad (m + F^r)(m + F^s) = \begin{cases} m + F^{r+s} & \text{if } r + s \leq 2k \\ m - F^{r+s-2k} & \text{if } r + s > 2k \end{cases}$$

$$(1.7) \quad (m + F^r)(m - F^s) = \begin{cases} m - F^{r+s} & \text{if } r + s \leq 2k \\ m + F^{r+s-2k} & \text{if } r + s > 2k \end{cases}$$

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$$(1.8) \quad (m - F^r)(m - F^s) = \begin{cases} m + F^{r+s} & \text{if } r + s \leq 2k \\ m - F^{r+s-2k} & \text{if } r + s > 2k \end{cases}$$

Thus the product of any two elements of M_{4k} is in M_{4k}

(b) **Associative property:** Since the multiplication of arbitrary functions obeys the associative law, therefore it holds for the elements of M_{4k} also

(c) **Existence of identity:** From (1.2), we have

$$(1.9) \quad m - F^{2k} = I \therefore m - F^{2k} \text{ is the identity element of } M_{4k}$$

(d) **Existence of inverse:** For $r < 2k$ Let $m + F^r \in M_{4k}$ then we claim that $(m + F^r)^{-1} = m - F^{2k-r}$ since with the help of (1.3)

$$(1.10) \quad (m + F^r)(m - F^{2k-r}) = m - F^{2k} = I$$

Similarly

$$(1.11) \quad (m - F^r)^{-1} = m + F^{2k-r}$$

Also

$$(1.12) \quad (m - F^{2k})^{-1} = m - F^{2k}$$

$$(1.13) \quad (m + F^{2k})^{-1} = m + F^{2k}$$

Thus each element in M_{4k} has its multiplicative inverse.

Hence M_{4k} is a group under multiplication moreover we have on using (1.3).

$$(1.14) \quad (m + F)^1 = m + F, (m + F)^2 = m + F^2, \dots, (m + F)^{2k} = m + F^{2k}, \\ (m + F)^{2k+1} = m + F^{2k+1} = m - F, (m + F)^{2k+2} = m + F^{2k+2} \\ = m - F^2, \dots, (m + F)^{4k} = m + F^{4k} = m - F^{2k} = I$$

$$(1.15) \quad M_{4k} = \langle m + F \rangle, O(m + F) = 4k = O(M_{4k})$$

all the generators of M_{4k} are of the form $m + F^t$ where t is a positive integer relatively prime to $4k$.

Also

$$(1.16) \quad o(m + F^r) = o[(m + F)^r] = \frac{4k}{(4k, r)}$$

where (a, b) denotes gcd of a and b .

Theorem 1.2:

Let $p, q \in M_{4k}$ were

$$(1.17) \quad p = m + F^k, \quad q = m - F^k, \text{ then}$$

$$(1.18) \quad (i) \quad o(p) = o(q) = 4$$

$$(ii) \quad pq = I, p^{-1} = q = p^3, q^{-1} = p = q^3, p^2 = q^2$$

$$(iii) \quad p^2 - p - q + I = 0 = q^2 - p - q + I$$

$$(iv) \quad pm = qm = p^2m = q^2m = m$$

Proof: from (1.16) taking $r=k$ we have

$$(1.19) \quad o(p) = o(m + F^k) = o[(m + F)^k] = \frac{4k}{(4k, k)} = \frac{4k}{k} = 4$$

etc, the other parts follow similarly

Remark: Let

$$(1.20) \quad L_{4k} = \{l - F^{2k}, l - F^{2k-1}, \dots, l - F, l + F, \dots, l + F^{2k}\}$$

Since by (1.2), $l + F^{2k} = o$ Thus L_{4k} is not a group under multiplication.

Ex.1 Let $k = 1 \Rightarrow 2k = 2$, the structure equation is

$$(1.21) \quad F^3 + F = 0$$

$$(1.22) \quad l = -F^2, \quad m = I + F^2$$

$$(1.23) \quad M_4 = \{I = m - F^2, m - F, m + F, m + F^2\}$$

The Cayley table for M_4 is

	$m-F^2$	$m-F$	$m+F$	$m+F^2$
$m-F^2$	$m-F^2$	$m-F$	$m+F$	$m+F^2$
$m-F$	$m-F$	$m+F^2$	$m-F^2$	$m+F$
$m+F$	$m+F$	$m-F^2$	$m+F^2$	$m-F$
$m+F^2$	$m+F^2$	$m+F$	$m-F$	$m-F^2$

From this table we have

$$(1.24) \quad (m + F)^{-1} = m - F,$$

$$(m + F^2)^{-1} = m + F^2$$

$$(m - F^2)^{-1} = m - F^2$$

$$(1.25) \quad o(m + F) = 4, \quad o(m - F) = 4,$$

$$o(m + F^2) = 2, \quad o(m - F^2) = 1$$

The only subgroups of M_4 are

$$(1.26) \quad H_1 = \{m - F^2\}, \quad H_2 = \{m - F^2, m + F^2\}, \quad H_3 = M_4$$

Ex. 2: Let $k = 2 \Rightarrow 2k = 4$. The structure equation is

$$(1.27) \quad F^5 + F = o,$$

$$(1.28) \quad l = -F^4, \quad m = I + F^4$$

$$(1.29) \quad M_8 = \{m - F^4, m - F^3, m - F^2, m - F, m + F, m + F^2, m + F^3, m + F^4\}$$

The Cayley table for M_8 is

	$m-F^4$	$m-F^3$	$m-F^2$	$m-F$	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$
$m-F^4$	$m-F^4$	$m-F^3$	$m-F^2$	$m-F$	$m+F$	$m+F^2$	$m+F^3$	$m+F^4$
$m-F^3$	$m-F^3$	$m-F^2$	$m-F$	$m+F^4$	$m-F^4$	$m+F$	$m+F^2$	$m+F^3$
$m-F^2$	$m-F^2$	$m-F$	$m+F^4$	$m+F^3$	$m-F^3$	$m-F^4$	$m+F$	$m+F^2$
$m-F$	$m-F$	$m+F^4$	$m+F^3$	$m+F^2$	$m-F^2$	$m-F^3$	$m-F^4$	$m+F$
$m+F$	$m+F$	$m-F^4$	$m-F^3$	$m-F^2$	$m+F^2$	$m+F^3$	$m+F^4$	$m-F$
$m+F^2$	$m+F^2$	$m+F$	$m-F^4$	$m-F^3$	$m+F^3$	$m+F^4$	$m-F$	$m-F^2$
$m+F^3$	$m+F^3$	$m+F^2$	$m+F$	$m-F^4$	$m+F^4$	$m-F$	$m-F^2$	$m-F^3$
$m+F^4$	$m+F^4$	$m+F^3$	$m+F^2$	$m+F$	$m-F$	$m-F^2$	$m-F^3$	$m-F^4$

The inverses and orders of each element can be calculated easily.

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