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NANO GENERALIZED–SEMI CONTINUITY IN NANO TOPOLOGICAL SPACE

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ABSTRACT

In this paper, a new form of continuous maps called nano generalized–semi continuous maps has been introduced and their relations with various other forms of continuous maps are analysed. Further, nano generalized–semi closure and nano generalized–semi interior in nano topological spaces are analysed under continuous maps.

Keywords: *Nano gs- continuity, Nano gs–closed sets, Nano gs–open sets, Nano gs–closure, Nano gs – interior.*

1. INTRODUCTION

The notion of the generalized–semi closed sets by S.P.Arya *et.al* [1] and the generalized–semi continuous maps by R.Devi *et.al* [7] have led to the generalizations of continuous maps. The concept of generalized–semi homeomorphism was introduced and studied by Devi *et.al* [6]. The notion of nano topology was introduced by Lellis Thivagar [10] and he analysed a different form of continuous maps called nano continuous maps. Lellis Thivagar *et al.* [10] analysed the notion of nano homeomorphism in nano topological spaces.

2. PREMILINARIES

Definition 2.1: [5] A function $f:(X,\tau)\to(Y,\sigma)$ is called semi–continuous if $f^{-1}(V)$ is semi–open in (X,τ) for every open set V in (Y,σ).

Definition 2.2: [12] A function $f:(X,\tau)\to(Y,\sigma)$ is pre–continuous if $f^{-1}(V)$ is pre–closed in (X,τ) for every closed set V in (Y,σ).

Definition 2.3: [13] A function $f:(X,\tau)\to(Y,\sigma)$ is called α–continuous if $f^{-1}(V)$ is α–closed in (X,τ) for every closed set V in (Y,σ).

Definition 2.4: [2] A function $f:(X,\tau)\to(Y,\sigma)$ is called g–continuous if $f^{-1}(V)$ is g–closed in (X,τ) for every closed set V in (Y,σ).

Definition 2.5: [5] A function $f:(X,\tau)\to(Y,\sigma)$ is sg-continuous if f^{-1}(V) is sg-closed set in X for every closed set V of Y .

Definition 2.6: [8] A function $f:(X,\tau)\to(Y,\sigma)$ is gs–continuous if $f^{-1}(V)$ is gs–closed in (X,τ) for every closed set V in (Y,σ).

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Definition 2.7: [10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = U\{R(x): R(x) \subseteq X, x\epsilon U\}$ where R(x) denotes the equivalence class determined by $x\epsilon U$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(x): R(x) \cap X \neq \Phi, x\epsilon U\}$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not–X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property 2.8: [10] If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. 9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.9: [10] Let U be the universe, R be an equivalence relation on U and the Nano topology $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.5, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\Phi \in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. (U, $\tau_R(X)$) is called the Nano topological space. Elements of the Nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets with $[\tau_R(X)]^c$ being called Dual Nano topology of $\tau_R(X)$. If $\tau_R(X)$ is the Nano topology on U with respect to X , then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.10: [10] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.

(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest nano closed set containing A.

Remark 2.11: [10] Throughout this paper, U and V are non-empty, finite universes; $X \subseteq U$ and $Y \subseteq V$; U/R and V/R' denote the families of equivalence classes by equivalence relations R and R' respectively on U and V. $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ are the nano topological spaces with respect to X and Y respectively.

Definition 2.12: [3] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano semi-closure of A is defined as the intersection of all nano semi-closed sets containing A and is denoted by NsCl(A). NsCl(A) is the smallest nano semi-closed set containing A and NsCl(A) $\subseteq$A.

(ii) The nano semi-interior of A is defined as the union of all nano semi-open subsets of A and is denoted by NsInt(A). NsInt(A) is the largest nano semi open subset of A and NsInt(A) $\subseteq$A.

Definition 2.13: [3] A subset A of $(U, \tau_R(X))$ is called nano semi-generalized closed set (Nsg-closed) if NSCl(A)$\subseteq$ V and A $\subseteq$V and V is nano semi-open in $(U, \tau_R(X))$.

Definition 2.14: [1] If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano semi-generalized closure of A is defined as the intersection of all nano semi-generalized closed sets containing A and is denoted by NsgCl(A).

(ii) The nano semi-generalized interior of A is defined as the union of all nano semi-generalized open subsets of A and is denoted by NsgInt(A).

Definition 2.15: [9] Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two nano topological spaces. Then a mapping $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano continuous on U if the inverse image of every nano open set in V is nano open in U.

Definition 2.16: [9] A function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called nano open if the image of every nano open set in $(U,\tau_R(X))$ is nano open in $(V,\tau_{R'}(Y))$.

Definition 2.17: [9] A function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called nano closed if the image of every nano closed set in $(U,\tau_R(X))$ is nano closed in $(V,\tau_{R'}(Y))$.

Definition 2.18: [4] A function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called nano g–closed if the image of every nano g–closed set in $(U,\tau_R(X))$ is nano g–closed in $(V,\tau_{R'}(Y))$.

Definition 2.19: [3] A mapping $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano semi – generalized continuous on U if the inverse image of every nano open set in V is nano sg–open in U.

Definition 2.20: [9] A bijection $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called nano homeomorphism if f is both nano continuous and nano open.

3. NANO GENERALIZED-SEMI CONTINUOUS MAPS

In this section, the concept of nano generalized–semi continuous maps is introduced and certain characterizations of these maps are discussed.

Definition 3.1: Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be any two nano topological spaces. Define a map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ such that the inverse image of every nano open subset in V is Ngs–open in U, then the map $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is called nano generalized–semi continuous (briefly Ngs–continuous) .

Theorem 3.2: A function $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous if and only if the inverse image of every nano closed set in $(V,\tau_{R'}(Y))$ is Ngs–closed in $(U,\tau_R(X))$.

Proof: Let $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be Ngs–continuous function and A be a nano closed set in $(V,\tau_{R'}(Y))$. That is, $V - A$ is nano open set in V. Since f is Ngs–continuous , the inverse image of every nano open set in V is Ngs–open in U. Hence $f^{-1}(V - A)$ is Ngs–open in U. That is, $f^{-1}(V - A) = f^{-1}(V) - f^{-1}(A) = U - f^{-1}(A)$ is Ngs–open in U. Hence $f^{-1}(A)$ is Ngs–closed in U. Thus the inverse image of every nano closed set in $(V,\tau_{R'}(Y))$ is Ngs–closed in $(U,\tau_R(X))$ if $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous.

Conversely, let the inverse image of every nano closed set in $(V,\tau_{R'}(Y))$ be Ngs–closed in $(U,\tau_R(X))$. Let B be a nano open set in V. Then $V - B$ is nano closed in V. By the given hypothesis, $f^{-1}(V - B)$ is Ngs–closed in U. That is, $f^{-1}(V - B) = f^{-1}(V) - f^{-1}(B) = U - f^{-1}(B)$ is Ngs–closed in U. Hence $f^{-1}(B)$ is Ngs–open in U. Thus the inverse image of every nano open set in $(V,\tau_{R'}(Y))$ is Ngs–open in $(U,\tau_R(X))$. Hence by definition, $f : (U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous.

Example: 3.3: Let $U = \{a,b,c,d\}$ be the universe with $U/R = \{\{a\},\{c\},\{b,d\}\}$ and let $X = \{a,b\} \subseteq U$. Then the nano open sets are $\tau_R(X) = \{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$. Ngs–open sets are $\{U,\varphi,\{a\},\{b\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Ngs–closed sets are

$\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{b,c\},\{c,d\},\{a,c\},\{b,d\},\{a,c,d\},\{b,c,d\},\{a,b,c\}\}$. Let $V=\{x,y,z,w\}$ be the universe with $V/R'=\{\{x\},\{w\},\{y,z\}\}$ and let $Y=\{x,z\}\subseteq V$. Then $\tau_{R'}(Y)=\{V,\phi,\{x\},\{x,y,z\},\{y,z\}\}$ Ngs–open sets are $\{V,\phi,\{x\},\{y\},\{z\},\{x,y\},\{x,z\},\{x,w\},\{y,z\},\{x,y,z\},\{x,y,w\},\{y,z,w\},\{x,z,w\}\}$. Ngs–closed sets are $\{V,\phi,\{x\},\{y\},\{z\},\{w\},\{y,w\},\{y,z\},\{x,w\},\{z,w\},\{y,z,w\},\{x,y,w\},\{x,z,w\}\}$. Now let us define a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=z,\ f(b)=x,\ f(c)=w,\ f(d)=y$. The inverse images are $f^{-1}(V)=U,\ f^{-1}(\phi)=\phi,\ f^{-1}(\{y,z\})=\{a,d\},\ f^{-1}(x)=\{b\},\ f^{-1}(\{x,y,z\})=\{a,b,d\}$. That is, the inverse image of every nano open set in V is Ngs–open in U. Thus the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ defined is Ngs–continuous.

Theorem 3.4: If the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano continuous, then it is Ngs–continuous but not conversely.

Proof: Let the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be nano continuous on U. Also, every nano closed set is Ngs–closed but not conversely. Since f is nano continuous on $(U,\tau_R(X))$, the inverse image of every nano closed set in $(V,\tau_{R'}(Y))$ is nano closed in $(U,\tau_R(X))$. Hence the inverse image of every nano closed set in V is Ngs–closed in U and so $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngs–continuous.

Conversely, all Ngs–closed sets are not nano closed and hence a Ngs–continuous map need not be nano continuous which can be seen from the following example.

Example 3.5: Let $U=\{a,b,c,d\}$ be the universe with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let $X=\{a,b\}$. Then $\tau_R(X)=\{U,\phi,\{a\},\{a,b,d\},\{b,d\}\}$ which are nano open sets. The nano closed sets are $\{U,\phi,\{b,c,d\},\{c\},\{a,c\}\}$. The nano semi–closed sets are $\{U,\phi,\{b,c,d\},\{a,c\},\{b,d\},\{c\},\{a\}\}$. The nano semi–open sets are $\{U,\varphi,\{a\},\{a,c\},\{b,d\},\{a,b,d\},\{b,c,d\}\}$. Thus, Ngs– closed sets are $\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},\{a,c\},\{b,c\},\{b,d\},\{c,d\}\}$. The Ngs–open sets are $\{U,\phi,\{a\},\{b\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$.

Theorem 3.6: If the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ng–continuous, then f is Ngs–continuous but not conversely.

Proof: Since $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ng–continuous, the inverse image $f^{-1}(A)$ of a nano open set A in $(V,\tau_{R'}(Y))$ is Ng–open in $(U,\tau_R(X))$. Hence, $f^{-1}(A)$ is Ngs–open in $(U,\tau_R(X))$. Hence the function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngs–continuous.

The converse of the Theorem 3.6 need not be true in general as can be seen from the following example.

Example 3.7: In Example 3.5, let us define a map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as $f(a)=y,\ f(b)=x,\ f(c)=z,\ f(d)=w$. Here the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngs–continuous but $f^{-1}(\{y,z\})=\{a,d\}$, is not Ng–open in $(U,\tau_R(X))$ for the nano open set $\{y,z\}$ in $(V,\tau_{R'}(Y))$. Also $f^{-1}(\{x,y,z\})=\{a,b,d\}$ is not Ng–open in $(U,\tau_R(X))$ for the nano open set $\{x,y,z\}$ in $(V,\tau_{R'}(Y))$. Thus the map $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is not Ng–continuous even though the map is Ngs–continuous.

Theorem 3.8: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is Ngs–continuous if and only if $f(NgsCl(A))\subseteq NCl(f(A))$ for every subset A of $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be Ngs–continuous. Let $A \subseteq U$ and thus $f(A) \subseteq V$. Hence $NCl(f(A))$ is nano closed in V. Since f is Ngs–continuous, $f^{-1}(NCl(f(A))$ is also Ngs–closed in $(U,\tau_R(X))$. Since $f(A) \subseteq NCl(f(A))$, it follows that $A \subseteq f^{-1}(NCl(f(A))$. Hence $f^{-1}(NCl(f(A))$ is a Ngs–closed set containing A. As $NgsCl(A)$ is the smallest Ngs–closed set containing A, it follows that $NgsCl(A) \subseteq f^{-1}(NCl(f(A))$ which implies $f(NgsCl(A)) \subseteq NCl(f(A))$.

Conversely, let $f(NgsCl(A)) \subseteq NCl(f(A))$ for every subset A of $(U,\tau_R(X))$. Let F be a nano closed set in $(V,\tau_{R'}(Y))$. Now $f^{-1}(F) \subseteq U$ and hence, $f(NgsCl(f^{-1}(F))) \subseteq NCl(f(f^{-1}(F))) = NCl(F)$. It follows that $NgsCl(f^{-1}(F)) \subseteq f^{-1}(NCl(F))$ and thus $NgsCl(f^{-1}(F)) \subseteq f^{-1}(F) \subseteq NgsCl(f^{-1}(F))$. Hence $NsgCl(f^{-1}(F)) = f^{-1}(F)$ which implies that $f^{-1}(F)$ is Ngs–closed in U for every nano closed set F in V. That is the map $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Nsg–continuous.

Theorem 3.9: Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two nano topological spaces where $X \subseteq U$ and $Y \subseteq V$. Then $\tau_{R'}(Y) = \{V,\phi,L_{R'}(Y),U_{R'}(Y),B_{R'}(Y)\}$ and its basis is given by $B_{R'} = \{V,L_{R'}(Y),B_{R'}(Y)\}$. A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous if and only if the inverse image of every member of $B_{R'}$ is Ngs–open in U.

Proof: Let the map $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be Ngs–continuous on U. Let $B \in B_{R'}$. Then B is nano open in V. Since f is Ngs–continuous, $f^{-1}(B)$ is Ngs–open in U and hence $f^{-1}(B) \in \tau_R(X)$. Hence the inverse image of every member of $B_{R'}$ is Ngs–open in U.

Conversely, let the inverse image of every member of $B_{R'}$ be Ngs–open in U. Let G be nano open in V. Now $G = \bigcup\{B : B \in B_1\}$ where $B_1 \subset B_{R'}$. Then $f^{-1}(G) = f^{-1}[\bigcup\{B : B \in B_1\}] = \bigcup\{f^{-1}(B) : B \in B_1\}$ where each $f^{-1}(B)$ is Ngs–open in U and their union which is $f^{-1}(G)$ is also Ngs–open in U. Hence the inverse image of a nano open set in V is Ngs–open in U and thus $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous on U.

Theorem 3.10: A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous if and only if $f^{-1}(NInt(B)) \subseteq NgsInt(f^{-1}(B))$ for every subset B of $(V,\tau_{R'}(Y))$.

Proof: Let $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be Ngs–continuous. By the given hypothesis, let $B \subseteq V$. Then $NInt(B)$ is nano open in V. As f is Ngs–continuous, $f^{-1}(NInt(B))$ is Ngs–open in $(U,\tau_R(X))$. Hence it follows that $NgsInt(f^{-1}(NInt(B))) = f^{-1}(NInt(B))$. Also for $B \subseteq V$, $NInt(B) \subseteq B$ always. Then, $f^{-1}(NInt(B)) \subseteq f^{-1}(B)$. It follows that $NgsInt(f^{-1}(NInt(B))) \subseteq NgsInt(f^{-1}(B))$, and hence $f^{-1}(NInt(B)) \subseteq NgsInt(f^{-1}(B))$.

Conversely, let $f^{-1}(NInt(B)) \subseteq NgsInt(f^{-1}(B))$ for every subset B of V. Let B be nano open in V and hence $NInt(B) = B$. Given $f^{-1}(NInt(B)) \subseteq NgsInt(f^{-1}(B))$, i.e., $f^{-1}(B) \subseteq NgsInt(f^{-1}(B))$. Also, $NgsInt(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence it follows that $f^{-1}(B) = NgsInt(f^{-1}(B))$ which implies that $f^{-1}(B)$ is Ngs–open in U for every subset B of V. Therefore, $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous.

Theorem 3.11: A function $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is Ngs–continuous if and only if $NgsCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset B of $(V,\tau_{R'}(Y))$.

Proof: Let $B \subseteq V$ and $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ be Ngs–continuous. Then $NCl(B)$ is nano closed in $(V,\tau_{R'}(Y))$ and hence $f^{-1}(NCl(B))$ is Ngs–closed in $(U,\tau_R(X))$.

Therefore, $NgsCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Since $B \subseteq NCl(B)$, then $f^{-1}(B) \subseteq f^{-1}(NCl(B))$, i.e., $NgsCl(f^{-1}(B)) \subseteq NgsCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Hence $NgsCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$.

Conversely, let $NgsCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset $B \subseteq V$. Now, let B be a nano closed set in $(V, \tau_{R'}(Y))$, then $NCl(B) = B$. By the given hypothesis, $NgsCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ and hence $NgsCl(f^{-1}(B)) \subseteq f^{-1}(B)$. But we also have $f^{-1}(B) \subseteq NgsCl(f^{-1}(B))$ and hence $NgsCl(f^{-1}(B)) = f^{-1}(B)$. Thus $f^{-1}(B)$ is Ngs–closed set in $(U, \tau_R(X))$ for every nano closed set B in $(V, \tau_{R'}(Y))$. Hence $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Ngs–continuous.

The following theorem establishes a criteria for Ngs–continuous functions in terms of inverse image of nano interior of a subset of $(V, \tau_{R'}(Y))$.

Theorem 3.12: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces with respect to $X \subseteq U$ and $Y \subseteq V$ respectively. Then for any function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$, the following are equivalent.

(i) f is Ngs–continuous.

(ii) The inverse image of every nano closed set in V is Ngs–closed in $(U, \tau_R(X))$.

(iii) $f(NgsCl(A)) \subseteq NCl(f(A))$ for every subset A of $(U, \tau_R(X))$.

(iv) The inverse image of every member of $B_{R'}$ is Ngs–open in $(U, \tau_R(X))$.

(v) $NgsCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset B of $(V, \tau_{R'}(Y))$.

Proof of the Theorem 3.12 is obvious.

Remark 3.13: The composition of two Ngs–continuous functions need not be Ngs–continuous and this can shown by the following example.

Example 3.14: Let $(U, \tau_R(X))$, $(V, \tau_{R'}(Y))$ and $(W, \tau_{R''}(Z))$ be three nano topological spaces and let $U = V = W = \{a, b, c, d\}$, then the nano open sets are $\tau_R(X) = \{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$, $\tau_{R'}(Y) = \{V, \phi, \{b\}, \{a, b, c\}, \{a, c\}\}$ and $\tau_{R''}(Z) = \{W, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}$. Define a map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = c, f(b) = b, f(c) = a, f(d) = d$ and another map $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ by $g(a) = a, g(b) = b, g(c) = c, g(d) = d$, an identity map. Then f and g are Ngs–continuous but their composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is not Ngs–continuous because $(g \circ f)^{-1}(\{c, d\}) = f^{-1}[g^{-1}(\{c, d\})] = f^{-1}(\{c, d\}) = \{a, d\}$ is not Ngs–closed in $(U, \tau_R(X))$ for every nano closed set $\{c, d\}$ in $(W, \tau_{R''}(Z)$. Hence the composition of two Ngs–continuous functions need not be Ngs–continuous.

Theorem 3.15: If the map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nsg–continuous, then it is Ngs–continuous but not conversely.

Proof: Let A be a nano open set in $(V, \tau_{R'}(Y))$. As the map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nsg–continuous, $f^{-1}(A)$ is Nsg–open in $(U, \tau_R(X))$. Then, $f^{-1}(A)$ is Ngs–open in $(U, \tau_R(X))$ and hence the map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ which is Nsg–continuous is Ngs–continuous.

The converse of the Theorem 3.15 need not be true as seen from the following example.

Example 3.16: Let $U = \{a, b, c\}$ be the universe with $U/R = \{\{a\}, \{b, c\}\}$ and let $X = \{a\}$. Then $\tau_R(X) = \{U, \phi, \{a\}\}$ which are nano the open sets. Ngs–open sets are $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}\}$. Nsg–open sets are $\{U, \phi, \{a\}, \{a, c\}, \{a, b\}\}$. Let $V = \{x, y, z\}$ with $V/R' = \{\{y\}, \{x, z\}\}$ and let $Y = \{y\}$.

Then the nano open sets are $\tau_{R'}(Y) = \{V, \phi, \{y\}\}$. Ngs –open sets are $\{V, \phi, \{y\}, \{z\}, \{y, z\}, \{x, y\}, \{x\}\}$. Nsg –open sets are $\{V, \phi, \{y\}, \{y, z\}, \{x, y\}\}$. Define a map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ by $f(a) = x$, $f(b) = z$, $f(c) = y$. The map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nsg –continuous as the inverse image of every nano open set in $(V, \tau_{R'}(Y))$ is Nsg –open in $(U, \tau_R(X))$. But the inverse image $f^{-1}(\{y\}) = \{c\}$ is not Nsg –open in $(U, \tau_R(X))$ for the nano open set $\{y\}$ in $(V, \tau_{R'}(Y))$. Hence the map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ which is Ngs –continuous is not Nsg –continuous.

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