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FUZZY NORMAL SUBGROUP ACTING ON A GROUP

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ABSTRACT

In this paper, the concept of fuzzy group and fuzzy left (right) cosets acted by a group are introduced and discussed some of its properties.

Keywords: *Fuzzy group, group acting on fuzzy group, group acting on fuzzy normal group, their fuzzy normalizer, and fuzzy left (right) cosets.*

INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh[20]. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld [13] gave idea of fuzzy subgroups. Wanging Wu [1981] studied about normal fuzzy group. Dib & Hassan [1988] discussed algebraic properties on fuzzy normal group. Kuroki [1992] investigated algebraic properties on fuzzy congruence and fuzzy normal subgroup. Kuimar *et al.* [1992] studied fuzzy quotients on fuzzy normal subgroup.

Mukerjee [1991] a analysed properties on fuzzy normal subgroup and fuzzy cossets. Dobrista and Yahhyaeva [2002] found few algebraic results on homomorphism of fuzzy group. Morsi Hehad [1997] studied on normal fuzzy subgroup and fuzzy cossets of finite groups. Sidky and Mishref [1991] studied fuzzy cossets, cyclic fuzzy subgroup and fuzzy abelian group.

A.Solairaju and R.Nagarajan [16, 17] introduce and define a new algebraic structure of Q-fuzzy groups. In this paper, new algebraic structures of Q-fuzzy normal subgroups and Q-fuzzy left (right) cosets are introduced, and some of their properties are studied.

SECTION 2 – PRELIMINARIES

In this section, the fundamental definitions that are sited, used in the sequel.

Definition 2.1: A map μ: X→ [0, 1], where X is an arbitrary non-empty set is a fuzzy set in X.

Definition 2.2: Let G be any group. A mapping μ: G→ [0, 1] is a fuzzy group if
(FG1). $\mu(xy) \geq \min\{\mu(x),\ \mu(y)\}$ and (FG2). $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

Definition 2.3: Let (G, +) be a group, and S be a non-empty set. Then G acts on S if there exists a function $*: G \times S \to S$ (denoted * (g, s) = gs for all g ∈ G, and s ∈ S) such that es = s and (g + h)s = g(hs) for all s in S, and for all g, h in G.

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Definition 2.4: A group (G, +) with identity 0 acts on a fuzzy group A on a group (S, Δ) if (GAFS1) the group G acts on S [there exists a function $*: G \times S \rightarrow S$ with the conditions g ∗ (h∗ s) = (g + h) ∗ s and e ∗ s = s or all s in S, and for all g, h in G];

(GAFG2) $A(x * (s \Delta t) \geq \min\{A(x * s),\ A(x * t)\}$;

(GAFG3) $A((x + y) * s \geq \min\{A(x * s),\ A(y * s)\}$;

(GAFG4) $A(x * s^{-1}) \geq A(x * s)$ for all x, y ∈ G and s, t ∈ S.

Definition 2.5: A group (G, +) acts on a fuzzy normal group A on (S, Δ) if (i) (G, +) acts on a fuzzy group A on (S, Δ); (ii) $A(x * (t^{-1} \Delta s \Delta t)) \geq A(x * s)$ [hence $A(x * (t^{-1} \Delta s \Delta t)) \leq A(x * s)$] for all x in G, and s, t in S.

Definition 2.6: A group (G, +) acts on a fuzzy group A on (S, Δ). The upper level set of A is the set $A^{\alpha} = U(A, \alpha) = \{s \in S: A(x * s) \geq \alpha$ for all $x \in G\} = \{s \in S: inf_{x \in G} A(x * s) \geq \alpha\}$, where $\alpha \in [0, 1]$. Similarly the lower level set of A is the set $A_t = L(A, \alpha) = \{s \in S: inf_{x \in G} A(x * s) \leq \alpha\}$.

SECTION 3: FUZZY NORMAL SUBGROUPS ACTING ON A GROUP

Lemma 3.1: A group (G, +) acts on a fuzzy group A of (S, Δ) such that each element of G has its own inverse. Then (i) A (x∗ 1) ≥ A (x ∗ s), (ii) A (x ∗ 1) ≥ A (x ∗ s) and A (0 ∗ 1) ≥ A (x ∗ s) for all s in S and x in G where 0 is the identity of G, and 1 is the identity element of S.

Proof: Since A is a fuzzy group on (S, Δ).

Now

$$\begin{aligned} A(x * 1) &= A((x * (s \Delta s^{-1})) \\ &\geq \min\{A(x * s), A(x * s^{-1})\} \\ &= \min\{A(x * s), A(x * s)\} \\ &= A(x * s) \end{aligned}$$

In addition,

$$\begin{aligned} A(0 * s) &= A((x + (-x) * (s \Delta 1)) \\ &\geq \min\{A(x * s), A((-x) * 1)\} \\ &= \min\{A(x * s), A(x * 1)\} \\ &= A(x * s) \end{aligned}$$

$$\begin{aligned} A(0 * 1) = A((x + (-x)) * (s \Delta s^{-1})) &\geq \min\{A(x * s), A((-x) * s^{-1})\} \\ &= \min\{A(x * s), A(x * s^{-1})\} \\ &\geq \min\{A(x * s), A(x * s)\} \\ &= A(x * s) \end{aligned}$$

Theorem 3.2: A group (G, +) acts on a fuzzy set A on (S, Δ). Then a group (G, +) acts on a fuzzy group A on (S, Δ) if and only if all the level subsets $U(A, \alpha) = A^{\alpha}$ and $L(A, \alpha) = A_{\alpha}$ are subgroup of S acted by (G,+) where α in [0, 1].

Proof: A group (G, +) acts on a fuzzy subgroup A under (S, Δ).

The upper level subset of A is the set $U(A, \alpha) = \{s \in S: inf_{x \in G} A(x * s) \geq \alpha\}$.

Let u, v in U (A, α). Then $inf_{x \in G} A(x * u) \geq \alpha$ and $inf_{x \in G} A(x * v) \geq \alpha$.

$$\begin{aligned} \text{Now } A(x * (u \Delta v^{-1})) &= A(x * (u \Delta v^{-1})) \\ &\geq \min\{A(x * u),\ A(x * v^{-1})\} \\ &\geq \min\{A(x * u),\ A(x * v)\} \text{ for all x in G.} \end{aligned}$$

Then $inf_{x \in G} A(x * (u \Delta v^{-1}) \geq \min\{A(x * u),\ A(x * v)\}$ for all x, y in G.

$$\begin{aligned} inf_{x \in G} A(x * (u \Delta v^{-1})) &\geq \min\{inf_{x \in G} A(x * u),\ inf_{x \in G} A(x * v)\} \text{ for all u, v in S} \\ &\geq \min\{\alpha, \alpha\} = \alpha \end{aligned}$$

Therefore $(u \Delta v^{-1})$ is in U (A, α), and so U (A, α) is a subgroup of (S, Δ). The group (G, +) acts on U (A, α), under (S, Δ), Similarly the group (G, +) acts on L(A, α), under (S, Δ),

Conversely, assume that all the level subsets $U(A, \alpha) = A^{\alpha}$ and $L(A, \alpha) = A_{\alpha}$ are subgroup of S acted by (G,+) where t in [0, 1].

Let u, v in A^α, and x, y in G. Since A^α is a subgroup of (S, Δ), then u Δ v is in A^α

Then $inf_{x\in G}$A(x ∗ (u Δ v)) ≥ α = min{α, α}= min{$inf_{x\in G}$A(x ∗ u), $in\ f_{x\in G}$A(x ∗ v)}.

Then A(x ∗ (u Δ v)) ≥ min{A(x ∗ u), A(x ∗ v)} for all x in G, and u, v in S.

Similarly A ((x +y) ∗ u) ≥ min{A(x ∗ u), A(x∗u)}.

A(x ∗ u^{-1})) ≥ A(x ∗ u), and A(x^{-1}∗ u) ≥ A(x ∗ u) for all x, y in G, and u, v in S.

The group (G, +) acts the fuzzy subgroup A of (S, Δ).

In fact, there exists alternative proof as follows: The given group (G, +) acts every level cut-sets (lower and upper) of a fuzzy subgroup A under (S, Δ) by assumption. It follows that U(A, k_1) and U(A, k_2) are subgroups on (S, Δ), and U(A, k_1) ⊆ U(A, k_2) if $k_1 \geq k_2$ where k_1, k_2 are in [0, 1]. Each upper level set is contained in another upper level set, and their union is a subgroup of (S, Δ). In fact, all upper level sets form an increasing sequence of subgroups of (S, Δ), and hence A = $\bigcup_{\alpha\in[0,1]} A_{U)A,\alpha)}$ is a fuzzy subgroup of (S, Δ). So the group (G, +) acts on A under (S, Δ) since it acts on each upper level set under (S, Δ).

Theorem 3.3: Let G be a group and μ be a fuzzy subset of G acting on S. Then the group (G, +) acts on a fuzzy normal subgroup A under S if and only if all the level subsets U (A, α) = A^α and L (A, α) = A_α are normal subgroup of S acted by (G,+) where α in [0, 1].

Proof: Let (G, +) act on a fuzzy normal subgroup A under S. Every level subset A^α of A is a subgroup of G acting on A where αis in [0, 1] by (3.1).

Let x be in G, and s, t be in A^α. So $inf_{x\in G}$A(x ∗ s) ≥ α and $inf_{x\in G}$A (x ∗ t) ≥ α. Also $inf_{x\in G}$A (x ∗ (t Δ s Δt^{-1}) ≥ $inf_{x\in G}$A (x ∗ s) ≥ α, and therefore t Δ s Δ t^{-1} is in A^α. Thus each A^α is a normal subgroup of A. Further (G, +) acts on each A^α (upper or lower set) under S.

Conversely, all the level subsets U (A, α) = A^α and L (A, α) = A_α are normal subgroup of S acted by (G, +) where α in [0, 1]. Clearly (G, +) act on a fuzzy subgroup A under S by (3.1).

Let x in G, and s, t in U(A, α). Then A(x ∗ s) ≥ α. Further t Δ s Δ t^{-1}∈U (A, α) since U (A, α) is normal. It follows that A(x ∗ (t Δ s Δ t^{-1})) ≥ α = A(x ∗ s). Hence the group (G, +) acts on a fuzzy normal subgroup A under S.

SECTION 4 – NORMALIZER AND ITS COSET OF A FUZZY GROUP

Definition 4.1: Let a group (G, +) act on a fuzzy subgroup A under S. Let N(A) = {s∈ S: A (x ∗ (sΔ tΔs^{-1})) = A(x ∗t) for all x in G, and t in S} Then N(A) is the normalizer of A under S.

Theorem 4.2: Let a group (G, +) act on a fuzzy subgroup A under (S, Δ). Then (i) (G, +) acts on the subgroup N(A) of S; (ii) (G, +) act on fuzzy normal subgroup A under S ⇔ N(A) = G; (iii) (G, +) acts on a fuzzy normal subgroup A under (N(A), Δ).

Proof: (i).Let a, b be in N(A). Then A (x ∗ (aΔ tΔa^{-1})) = A(x ∗ t), and A (x ∗ (bΔ tΔ b^{-1})) = A(x ∗ t) for all x in G, and t in S.

Now A (x ∗ [(aΔb)ΔtΔ$(a\Delta b)^{-1}$)]) = A (x ∗ [(aΔb) Δ t Δ (b^{-1}Δ a^{-1})])
= A (x ∗ [aΔ (bΔ t Δ b^{-1}) Δ a^{-1}])
= A (x ∗ [(b Δ t Δ b^{-1})])
= A (x ∗ t).

Now A (x ∗ (b^{-1})Δ t Δ b]) = A (x ∗(bΔ t^{-1}Δ b^{-1}))
= A (x ∗t^{-1})
= A (x ∗t)

Thus (a Δ b) ∈ N(A), and b^{-1}∈ N(A).

Therefore, the normalizer N(A) of A is a subgroup of (S, Δ), and so the group (G, +) acts on the normalizer N(A) under (S, Δ). (ii). Clearly N(A) ⊆ S.

Suppose G acts on a normal fuzzy subgroup A under S. Thus $s \in S$ implies that $A(x * (s \Delta t \Delta s^{-1})) = A(x * t)$ for all x in G, and t in S. Then $s \in N(A)$, and so $S \subseteq N(A)$. Hence S = N(A).

Conversely, let N(A) = S. Clearly $A(x * (s \Delta t \Delta s^{-1})) = A(x * t)$ for all x in G, and s, t in S. Hence (G, +) acts on the fuzzy normal subgroup A under (S, Δ).

From (ii), (G, +) acts on a fuzzy normal subgroup N(A) under (S, Δ).

Definition 4.3: Define ${}_xf: G \times S \to G \times S$ be a function defined by ${}_xf(a, s) = (xa, s)$ [similarly define $f_x(a, s) = (ax, s)$] for all x in G. Let a group (G, +) act on a fuzzy subgroup A under (S, Δ). A G-fuzzy left (right) coset ${}_xA(A_x)$ of A is defined ${}_xA(y, s) = A(x^{-1}y, s)$ and $A_x(y, s) = A(yx^{-1}, s)$ for all (y, s) in G × S.

Theorem 4.4: Let a group (G, +) act on a fuzzy subgroup A under (S, Δ). Then the following conditions are equivalent for each s, t in S.

(i) $A(x * (t \Delta s \Delta t^{-1})) \geq A(x * s)$;
(ii) $A(x * (t \Delta s \Delta t^{-1})) = A(x * s)$
(iii) $A(x * (t \Delta s)) \geq A(x * (s \Delta t))$
(iv) ${}_xA = A_x$
(v) ${}_xA_{x-1} = A$

Proof: It is obvious.

Theorem 4.5: If a group (G, +) acts a fuzzy subgroup A under (S, Δ), then G acts on the fuzzy subgroup uAu^{-1} under (S, Δ) for all u in S.

Proof: Let a group (G, +) act a fuzzy subgroup A under (S, Δ).Let a group (G, +) act a fuzzy subgroup uAu^{-1} under (S, Δ) by the operation ∗ where u is in S.

Then

(ii) $$\begin{aligned} uAu^{-1}((x + y) * s) &= Au^{-1}((x + y) * (u^{-1}\Delta s)) \\ &= A((x + y) * (u^{-1}\Delta s\Delta u)) \\ &\geq \min\{A(x * (u^{-1}\Delta s\Delta u)), A(y * (u^{-1}\Delta s\Delta u))\} \\ &= \min\{uAu^{-1}(x * s), uAu^{-1}(y * s)\} \end{aligned}$$

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(iii) $$\begin{aligned} uAu^{-1}(x * (s \Delta t)) &= Au^{-1}(x * (u^{-1}\Delta s\Delta t\Delta u)) \\ &= A(x * (u^{-1}\Delta s\Delta u)\Delta(u^{-1}\Delta t \Delta u)) \\ &\geq \min\{A(x * (u^{-1}\Delta s\Delta u)), A(x * (u^{-1}\Delta t\Delta u))\} \\ &= \min\{tAt^{-1}(x * s), tAt^{-1}(x * s)\} \end{aligned}$$

(iv) $$\begin{aligned} uAu^{-1}(x * s^{-1}) &= Au^{-1}(x * (u^{-1}\Delta s^{-1})) \\ &= A(x * (u^{-1}\Delta s^{-1}\Delta u)) \\ &= A(x * (u^{-1}\Delta s\Delta u)^{-1}) \\ &= A(x * (u^{-1}\Delta s\Delta u)) \\ &= uAu^{-1}(x * s) \end{aligned}$$

Hence the group (G, +) acts on the fuzzy subgroup uAu^{-1} under (S, Δ).

Theorem 4.6: If a group (G, +) acts on a fuzzy normal subgroup A under (S, Δ), then G acts on the fuzzy **normal** subgroup uAu^{-1} under Sfor all $u \in S$.

Proof: Let a group (G, +) acts on a fuzzy normal subgroup A under (S, Δ). Then G acts on the fuzzy subgroup uAu^{-1} under S for all $u \in S$.

Now $$\begin{aligned} uAu^{-1}(x * (t^{-1}\Delta s \Delta t) &= A(x * (u^{-1}\Delta (t^{-1}\Delta s \Delta t) \Delta u)) \\ &= A(x * u^{-1}\Delta (t^{-1}\Delta s \Delta t) \Delta u)) \\ &= A(x * (t \Delta u)^{-1}\Delta s \Delta (t \Delta u)) \\ &= A(x * s) \\ &= A(x * (u^{-1}\Delta s \Delta u) \quad \text{since u is in S} \\ &= uAu^{-1}(x * s) \text{ for all t in S} \end{aligned}$$

Hence G acts on the fuzzy normal subgroup uAu^{-1} under S for all $u \in S$.

SECTION 5: BASIC NORMAL PROPERTIES ON INTERSECTION

Theorem 5.1: Let a group (G, +) act on two fuzzy subgroups A and B on (S, Δ). Then the group G acts on the fuzzy group A ∩ B on (S, Δ).

Proof: Let a group (G, +) act on two fuzzy subgroups A and B on (S, Δ). It is given that A and B are fuzzy groups on the group (S, Δ). It follows that A ∩ B is a fuzzy group on (S, Δ).

Since the group (G, +) acts S, then there exists a map $*$: G × S → S such that g $*$ (h$*$ s) = (g + h) $*$ s and e $*$ s = s or all s in S, and for all g, h in G which gives GAFG1.

Let $x, y \in G$ and $s \in S$.

(GAFG2) $(A \cap B)\big(x * (s \,\Delta\, t)\big) = \min\{A\big(x * (s \,\Delta\, t)\big), B(x * (s \,\Delta\, t\,))\}$
$\geq \min\{\min\{A(x * s), A(x * t)\}, \min\{B(x * s), B(x * t)\}\}$
$= \min\{\min\{A(x * s), B(x * s)\}, \min\{A(x * t), B(x * t)\}\}$
$= \min\{(A \cap B)(x * s), (A \cap B)(x * t)\}$

(GAFG3) $(A \cap B)\big((x + y) * s\big) = \min\{A\big((x + y) * s\big), B((x + y) * s)\}$
$\geq \min\{\min\{A(x * s), A(y * s)\}, \min\{B(x * s), B(y * s)\}\}$
$= \min\{\min\{A(x * s), B(x * s)\}, \min\{A(y * s), B(y * s)\}\}$
$= \min\{(A \cap B)(x * s), (A \cap B)(y * s)\}$

(GAFG4) $(A \cap B)\,(x * s^{-1}) = \min\{A\,(x * s^{-1}), B\,(x * s^{-1})\}$
$\geq \min\{A(x * s), B(x * s)\}$
$= A \cap B)(x * s).$

Thus the group (G, +) acts on the fuzzy group A ∩ B on the group (S, Δ).

Corollary 5.2: If a group (G, +) acts each member in the family $\{A_i\}_{i\varepsilon A}$ of fuzzy groups under S, then a group (G, +) acts on the fuzzy group $\cap A_i$ under S.

Proof: It is obvious.

Theorem 5.3: Let a group (G, +) act on two fuzzy normal subgroups A and B on (S, Δ). Then the group G acts on the fuzzy **normal** group (A ∩ B) on (S, Δ).

Proof: Let a group (G, +) act on two fuzzy normal subgroups A and B on (S, Δ).

According to (5.2), the group (G, +) acts on a fuzzy normal subgroup (A ∩ B) under S.

Now $(A \cap B)\,(x * (t^{-1}\Delta\, s\, \Delta\, t)) = \min\{A(x * (t^{-1}\Delta\, s\, \Delta\, t)), B((x * (t^{-1}\Delta\, s\, \Delta\, t))\}$
$= \min\{A(x * s), B((x * s)\}$
$= (A \cap B)\,(x * s).$

Then the group G acts on the fuzzy normal group (A ∩ B) on (S, Δ).

Remark 5.4: If μ_i, $i \in \Delta$ is a fuzzy normal subgroup of G acting on S, then $\underset{i\in\Delta}{\cap}\mu_i$ is a fuzzy normal subgroup of G acting on S.

Definition 5.5: Let (G, +) and (H, +') be two groups both acting on (S, Δ). The mapping $f: G \times Q \to H \times Q$ is said to be a group homomorphism under S if

(i) $f: G \to H$ is a group homomorphism

(ii) $f(x, pq) = (x, f(p) \Delta f(q)$, for all $x \in G$ and p, $q \in S$.

Theorem 5.6: Let (G, +) and (H, +') be two groups both acting on (S, Δ), and $f: G \times S \to H \times S$ is a group-homomorphismunder S. Then

(i) If μ is a fuzzy normal subgroup of H acting on S, then $f^{-1}(\mu)$ is a fuzzy normal subgroup of G acting on S.

(ii) If f is an epimorphism and μ is a fuzzy normal subgroup of G acting on S, then $f(\mu)$ is a fuzzy normal subgroup of H acting on S.

Proof: Let (G, +) and (H, +') be two groups both acting on (S, Δ), and $f: G \times S \to H \times S$ is a grouphomomorphism under S. Clearly $f: G \to H$ is a group homomorphism.

(i) Let μ be a fuzzy normal subgroup of H acting on S.

Now for all $x, y \in G$, we have

$$f^{-1}(\mu)(x*(s\Delta t)) = \mu(f(x*(s\Delta t))$$
$$= \mu((x*(f(s)\Delta f(t))$$
$$\geq \min\{\mu((x*(f(s)), \mu((x*(f(t))\}$$
$$= \min\{f^{-1}(\mu)(x*s), f^{-1}(\mu)(x*t)\}$$

$$f^{-1}(\mu)((x+'y)*s) = \mu(f((x+'y)*s)$$
$$= \mu(((x+'y)*(f(s)))$$
$$\geq \min\{\mu((x*f(s)),\ \mu(y*(f(s))\}$$
$$= \min\{f^{-1}(\mu)(x*s), f^{-1}(\mu)(x*s)\}$$

$$f^{-1}(\mu)(x*s^{-1})) = \mu(f((x*s^{-1})))$$
$$= \mu((x*(f(s^{-1})))$$
$$\geq \mu((x*(f(s)))$$
$$= f^{-1}(\mu)(x*s).$$

Hence $f^{-1}(\mu)$ is a fuzzy normal subgroup of G acting on S.

(ii). Let μ be a fuzzy **normal** subgroup of G acting on S. Then $f(\mu)$ is a fuzzy subgroup of H acting on S.

Now for all $u, v \in H$, we have

$$f(\mu)(u*(s\Delta t\Delta s^{-1})) = \sup_{f(y)=s\Delta t\Delta s^{-1}} \mu(u*y) = \sup_{f(x)=s;f(y)=t} \mu(u*y)$$
$$= \sup_{f(y)=t} \mu(u*y) = f(\mu)(u*t)$$

(Since f is an epimorphism)

Hence $f(\mu)$ is a fuzzy normal subgroup of H acting on S.

Definition 5.7: Let λ and μ be two fuzzy subsets of G acting on S. The product of λ and μ is defined to be the fuzzy subset $\lambda\mu$ of G is given by

$$\lambda\mu(x*s) = \sup_{yz=x} \min(\lambda(y*s), \mu(z*s)),\ x \in G.$$

Theorem 5.8: If λ & μ are fuzzy normal subgroups of G acting on S, then $\lambda\mu$ is a fuzzy normal subgroup of G acting on S.

Proof: Let λ & μ be two fuzzy normal subgroups of G acting on S. Thus $\lambda\mu$ is a fuzzy group acting on S. Further

(i) $$(\lambda\mu)((x+y)*s) = \sup_{\substack{x_1+y_1=x \\ x_2+y_2=y}} \min\{\lambda(x_1+y_1*s), \mu(x_2+y_2*s)\}$$
$$\geq \sup_{\substack{x_1+y_1=x \\ x_2+y_2=y}} \min\{\min\{\lambda(x_1*s), \lambda(y_1*s)\}, \min\{\mu(x_2*s)\mu(y_2*s)\}\}$$
$$\geq \min_{\substack{x_1+y_1=x \\ x_2+y_2=y}} \{\sup \min\{\lambda(x_1*s), \lambda(y_1*s)\}, sup \min\{\mu(x_2*s)\mu(y_2*s)\}\}$$

(i) $\lambda\mu(x*(s\,\Delta\,t)) \geq \min\{(\lambda\mu)(x*s), (\lambda\mu)(x*s)\}$

(ii) $\lambda\mu\,(x*s^{-1}) \geq \lambda\mu\,(x*s)$

(iii) $\lambda\mu\,(x*(t\,\Delta\,s\,\Delta\,t^{-1}) = \lambda\mu\,(x*s)$ for all x, y in G, and s, t in S.

Hence $\lambda\mu$ is a normal fuzzy subgroup of G acting on S.

Example 1: The group $GL_n(R)$ acts on vectors in R^n the usual way that a matrix can be multiplied with a (Column) vector: $A.V = AV$., In this action, the orgin O is fixed by every A while other vectors get moved around (as A varies). The axioms of a group action are properties of matrix- vector multiplication: $I_n v = v$ and $A(Bv) = (AB)v$.

Example 2: The group S_n acts on polynomials $f(T_1, \ldots, T_n)$by permuting the variables:

$$\sigma\,.\,f(T_1, \ldots, T_n) = f\left(T_{\sigma(1)}, \ldots, T_{\sigma(n)}\right) - (1)$$

For example $(23)\,.\,(T_2+T_3^2) = T_3+T_2^2$ and $(12)\,.\left((23)\,.\,(T_2+T_3^2)\right) = (12).(T_2+T_3^2) = T_3+T_1^2$ and (12) (13). $(T_2+T_3^2) = (123).(T_2+T_3^2) = T_3+T_1^2$

It is obvious that $(1).f = f$. To check that $\sigma.(\sigma' . f) = (\sigma\sigma') . f$ for all σ and σ' in S_n so (1) is a group action, we compute

$$\begin{aligned}\sigma.(\sigma' . f(T_1, \dots, T_n)) &= \sigma . f(T_{\sigma'(1)}, \dots, T_{\sigma'(n)}) \\ &= f\left(T_{\sigma(\sigma'(1))}, \dots, T_{\sigma(\sigma'(n))}\right) \\ &= f(T_{(\sigma\sigma')(1)}, \dots, T_{(\sigma\sigma')(n)}) \\ &= (\sigma\sigma') . f(T_1, \dots, T_n)\end{aligned}$$

Lagrange's study of this group action (ca.1770) marked the first systematic use of symmetric groups in algebra. Lagrange wanted to understand why nobody had found an analogue of the quadratic formula for roots of a polynomial of degree greater than four. While Lagrange was not completely successful, he found in this group action that there are some different features in the cases $n \leq 4$ and $n = 5$.

Since f and $\sigma . f$ have the same degree, and if f is homogeneous then $\sigma . f$is homogeneous, this action of S_n can be restricted to the set of polynomials in n variables with a fixed degree or the set of homogeneous polynomials in $n$variables with a fixed degree. An example is the set of homogeneous linear polynomials$\{a_1T_1 + \dots + a_nT_n\}$, where $\sigma(c_1T_1 + \dots + c_nT_1) = c_1T_{\sigma(1)} + \dots + c_nT_{\sigma(n)} = c_{\sigma^{-1}(1)}T_1 + \dots + c_{\sigma^{-1}(n)}T_n$

Example 3: Let S_n act on R^n by permuting the coordinates: for $\sigma \in S_n$ and $v = (c_1, \dots, c_n) \in R^n$, set $\sigma . v = (c_{\sigma(1)}, \dots, c_{\sigma(n)})$.

For example, let $n = 3, \sigma = (12)$, and $\sigma' = (23)$. Then $\sigma' . (c_1, c_2, c_3) = (c_1, c_2, c_3)$. To compute $\sigma . (\sigma' . (c_1, c_2, c_3)) = \sigma . (c_1, c_3, c_2)$ we must be careful; by definition an element of S_n is applied to a vector whose indices are written as $1, \dots, n$ in that order. So write $(c_1, c_3, c_2) = ((d_1, d_2, d_3))$. Then

$$\sigma . (\sigma' . (c_1, c_2, c_3)) = \sigma . (c_1, c_3, c_2) = \sigma . (d_1, d_2, d_3) = (d_2, d_1, d_3) = (c_3, c_1, c_2)$$

And

$$(\sigma\sigma') . (c_1, c_2, c_3) = (123) . (c_1, c_2, c_3) = (c_2, c_3, c_1)$$

Which does not agree with $\sigma.(\sigma' . (c_1, c_2, c_3))$! So our so-called action of S_n on R^n is not a group action based on our definition.

Let’s make a general calculation to see what is going wrong. For σ and σ' in S_n, and $v = (c_1, \dots, c_n)$ in R^n, we compute $\sigma . (\sigma' . v)$ by setting $d_i = c_{\sigma'(i)}$, so $\sigma' . v = (d_1, \dots, d_n)$.

Then

$$\begin{aligned}\sigma . (\sigma' . v) &= (d_{\sigma(1)}, \dots, d_{\sigma(n)}) \\ &= \left(c_{\sigma'(\sigma(1))}, \dots, c_{\sigma'(\sigma(n))}\right) \\ &= \left(c_{(\sigma'\sigma)(1)}, \dots, c_{(\sigma'\sigma)(n)}\right) \\ &= (\sigma'\sigma) . v\end{aligned}$$

And the other of composition of permutations is backwards!

To make things turn out correctly, we should redefine the effect of S_n on R^n to use the inverse: $\sigma . v = (c_{\sigma^{-1}(1)}, \dots, c_{\sigma^{-1}(n)})$. Then $\sigma . (\sigma' . v) = (\sigma'\sigma) . v$ and we have a group action of S_n on R^n, which in fact is essentially the action of S_n from the previous example on homogeneous linear polynomials.

Example 4: Let G be a group acting on the set X, and S be any set. Write map (X, s) for the set of all functions $f: X \to S$. We can make G act on map (X, S) by

$$(g . f)(x) = f(g^{-1}x)$$

It is left to the reader to check that is an action of G on map(X, S), and to see why we need g^{-1} on the right side rather than g.

If $G = S_n$, $X = \{1, \dots, n\}$ with the natural $S_n - action$, and $S = R$, the map $(X, S) = R^n$: writing down a vector $v = (c_1, \dots, c_n)$ amounts to listing the coordinates, and the list of coordinated in order is a function $f: \{1,2, \dots, n\} \to R$ where $f(i) = c_i$. Therefore the condition $(g . f)(i) = f(g^{-1}i)$ amounts to saying$g . (c_1, \dots, c_n) = (c_{g^{-1}(1)}, \dots, c_{g^{-1}(n)})$, which is precisely the action of S_n on R^n in the previous example.

There are three basic ways we will make an abstract group G act: left multiplication of G on itself, conjugation of G on itself, and left multiplication of G on a coset space G/H. All of these will now be described.

CONCLUSION

In this article we have discussed group acting on fuzzy Normal Subgroups, group acting on fuzzy Normalizer and group acting on fuzzy subgroups under homomorphism. Interestingly, it has been observed that group acting fuzzy concept adds an another dimension to the defined fuzzy normal subgroups. This concept can further be extended for new results.

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