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STRUCTURE OF FUZZY SOFT A-IDEAL IN BCI-ALGEBRA

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ABSTRACT

Molodtsov [1999] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Jun [2008] applied first the notion of soft set by Molodtsov to the theory of BCK/BCI-algebra. In this paper, the notion of soft a-ideals and a-idealistic soft BCI-algebra are introduced, and then investigated their basic properties. Using soft sets, characterizations of (fuzzy) a-ideals in BCI algebra are given. Relations between fuzzy a-ideals and a-idealistic soft BCI-algebras are provided.

Keywords: *Soft set (a-idealistic), soft BCI-algebra, soft ideal, and soft a-ideal.*

SECTION1 – INTRODUCTION

To solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionist) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets.

The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets developed by Zadeh [1965]. Maji et al. [2002] described the application of soft set theory to a decision making problem. Maji *et al.* [2003] also studied several operations on the theory of soft sets. However, all of these theories have their own difficulties which are pointed out in Molodstsov [1999]. Maji *et al.* [2002, 2003] and Molodtsov [1999] suggested that one reason for these difficulties may be due to the inadequacy of the parameterization tool of the theory. To overcome these difficulties, Molodtsov [1999] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly.

Chen et al. [2005] presented a new definition of soft set parameterization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. The algebraic structure of set theories dealing with uncertainties has been studied by some authors.

The author (together with colleagues) applied the fuzzy set theory to BCK-algebras (Jun [2008]; Jun & Park [2008]; Jun & X [2001a, 2001b, 2001c] Meng [1994];), BCC-algebras (Chen *et.al.* [2005]; Dudek & Stojokovic [2001];), fuzzy B-algebra (Jun et.al. [2002]), hyper BCK-algebras (Jun & Shim [2005]), MTL-algebras (Jun & Zhang [2005]), hemi rings (Jun & Oztark [2004]), implicative algebras (Jun [2001]), and lattice implication algebras (Chaudhry [1990]. Jun [2008] applied the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras. He introduced the notion of soft BCK/BCI-algebras and soft sub algebras, and then derived their basic properties.

Jun and Park [2008] dealt with the algebraic structure of BCK/BCI-algebras by applying soft set theory. They discussed the algebraic properties of soft sets in BCK/BCI-algebras. They introduced the notion of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They investigated relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras.

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In this paper, the notion of soft sets is applied by Molodtsov to a-ideals in BCI-algebras. The notion of soft a-ideal and a-idealistic are introduced in soft BCI-algebra, and then their basic properties are derived. Using soft sets, characterizations of (fuzzy) a-ideals are given in BCI-algebras, relation between fuzzy a-ideal and a-idealistic in soft BCI-algebra is provided.

SECTION 2 - BASIC RESULTS ON BCI-ALGEBRAS

(I). A BCK-algebra is an important class of logical algebras introduced by Iski and was extensively investigated by several researchers. Algebra (X; *, 0) of type (2, 0) satisfying the following axioms :(1) ((x * y) * (x * z) * (z * y)) = 0, (2) (x * (x * y)) * y) = 0; (3) x * x = 0,(4) x * y = 0 and y * x = 0 imply x = y. ∀x, y, z ∈ X. If a BCI-algebra X satisfies the following identity: (5) 0 * x = 0 for all x in X, then X is called a BCK-algebra. (II). In any BCK/BCI-algebra X, define a partial order by putting x ≤ y if and only if x *y = 0. (III). Every BCK/BCI-algebra's X satisfies: ∀x, y, z∈X, (x * y) * z = (x * z) *y.A non-empty subset S of a BCI-algebra X is a sub algebra of X if x * y ∈ S, ∀x, y ∈ S. (IV). A subset H of a BCI-algebra X is an ideal of X if it satisfies the following axioms: (I1) 0 ∈ H; (I2) ∀x ∈ X, y ∈ H and x * y ∈ H implies x ∈H.(V). Any ideal H of a BCI-algebra X satisfies the following implication: ∀x ∈ X, ∀y ∈ H &x ≤ y →x ∈ H. A subset H of a BCI-algebra X is called an a-ideal of X if it satisfies (I1) and (I3), ∀ x ∈ X, z ∈ X, y ∈ H and (x * z) *(0 * y) ∈ H →x *y∈ H. We know that every a-ideal of a BCI-algebra X is also an ideal of X.

SECTION 3 - BASIC DEFINITIONS ON SOFT SETS

Definition 3.1 (Molodtsov, [1991]): Let U is an initial universe set and E is a set of parameters. Let P (U) denotes the power set of U and A subset E. A pair (F, A) is **a soft set** over U, where F is a mapping given by F: A → P (U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For ε ∈ A, F(ε) may be considered as the set of ε-approximate elements of the soft set (F, A). **Clearly, a soft set is not a set.**

Definition 3.2 ([Maji *et al.* [2002, 2003]): Let (F, A) and (G, B) be two soft sets over a common universe U. The **intersection** of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions: (i) C = A ∩ B, (ii) ∀e∈C, H (e) = F (e) or G (e), (as both are same sets). In this case, we write (F, A) ∩ (G, B) = (H, C).

Definition 3.3 ([Maji *et al.* [2002, 2003]): Let (F, A) and (G, B) be two soft sets over a common universe U. **The union** of (F, A) and (G, B) is defined to be the soft set (H, C) satisfying the following conditions: (i) C = A ∪ B, (ii) ∀e ∈ C, H (e) = F (e), if e ∈ A \ B, G (e), if e ∈ B \ A; F (e) ∪ G (e), if e ∈ A ∩ B. In this case, write it as (F, A) ∪ (G, B) = (H, C).

Definition 3.4 ([Maji *et al.* [2002, 2003]): If (F, A) and (G, B) are two soft sets over a common universe U, then (F, A) **AND** (G , B), denoted by (F, A)v^(G, B) is defined by (F, A)v^(G, B) = (H, (A X B), where H(α, β)= F(α) ∩ G(β), ∀ (α, β) ∈ A X B.

Definition 3.5 ([Maji *et al.* [2002, 2003]): If (F, A) and (G, B) are two soft sets over a common universe U, then`` (F, A) **OR** (G, B)" denoted by (F, A)v^(G, B) is defined by (F, A)v^(G, B) = (H, A X B), where H(α, β)= F(α) ∪G(β), ∀(α, β) ∈ A X B.

Definition 3.6 ([Maji *et. al.* [2002, 2003]): For two soft sets (F, A) and (G, B) over a common universe U. Then (F, A) is a **soft subset** of (G, B) denoted by (F, A) ⊂ (G, B), if it satisfies: (i) A ⊂ B, (ii) F (ε) and G (ε) are identical approximations for every ε ∈ A.

SECTION 4 - SOFT A-IDEAL

In what follows, let X and A be a BCI-algebra and a nonempty set, respectively, and R refers to an arbitrary binary relation between an element of A and an element of X (is a subset of A × X without otherwise specified. A set valued function F: A →P (U) can be defined as F(x) = {y in X: (x, y) in R} for all x∈ A. The pair (F, A) is a soft set over X

Definition 4.1: Let S is sub algebra of X. A subset I of X is called an ideal of X related to S (briefly, S-ideal of X), denoted by I Δ S, if it satisfies: (i) 0 ∈ I, (ii) ∀ x ∈ S, ∀ y ∈ I, (x * y) ∈ I implies x ∈ I.

Definition 4.2: Let S is sub algebra of X. A subset I of X is an a-ideal of X related to S (S-a-ideal of X), denoted by I Δ_aS, if it satisfies: (i) 0 ∈ I, (ii) ∀x, z ∈ S, ∀y ∈ I, (x * z) * (0* y) ∈ I implies x *y∈ I .

Example 4.3: Let X= {0, 1, 2, a, b} be a BCI-algebra with the following Cayley table:

*	0	1	2	a	b
0	0	0	0	0	0
1	1	0	1	1	1
2	2	2	0	2	2
a	a	a	a	0	a
b	b	b	b	b	0

Then S = {0, 1, 2, b} is a sub-algebra of X and I= {0, 2, b} is S-a-ideal of X. Note that every S-a-ideal of X is an S-ideal of X in BCK-algebra.

Definition 4.4: Let (F, A) be a soft set over X. Then (F, A) is a soft BCI-algebra over X if F(x) is sub algebra of X, $\forall x \in A$.

Definition 4.5: Let (F, A) be a soft BCI-algebra over X. A soft set (G, I) over X is a soft ideal of (F, A) denoted by (G,I) Δ (F, A), if it satisfies: (i) $I \subset A$, (ii) G(x) Δ F(x) $\forall x \in I$.

Definition 4.6: Let (F, A) be a soft BCI-algebra over X. A soft set (G, I) over X is a soft a-ideal of (F, A), denoted by (G, I) Δ_a (F, A) if it satisfies: (i) $I \subset A$, (ii) G (x) Δ_a F (x) $\forall x \in I$.

Let us illustrate this definition using the following example.

Example 4.7 (soft a-ideal): Consider a BCI-algebra X = {0, 1, 2, a, b} which is given in (4.3). Let (F, A) be a soft set over X, where A = {0, 1, 2, a}$\subset$ X and F: A$\rightarrow$ P (U) is a set -valued function defined by F(x) = {y$\in$ X: y*(y * x) $\in$ {0, 1},$\forall x \in A$. Then F (0) = F (1) =X, F (2) = {0, 1, a, b} and F (a) = f (0), which are sub algebras of X. Hence (F, A) is as soft BCI-algebra over X. Let I = {0, 1, 2}$\subset$ A and G:$\not\rightarrow$ P (U) be a set-valued function defined by G (x) = Z [{0, 1}], & if x = 2, [0] if x $\in$ {0, 1} where Z(0, 1) = {x $\in$ X :0*(0 * x) $\in$ {0, 1}. Then G(0) Δ_a F(0), G(1) Δ_a F(1) and G(2) Δ_a F(2). Hence (G, I) is a soft a-ideal of (F, A).

Note that every soft a-ideal is a soft ideal.

But the converse is not true as seen in the following example.

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	a
b	b	b	0	b	b
c	c	c	c	0	0
d	d	d	d	d	0

Example 4.8 (not soft a-ideal): Let X = {0, a, b, c, d} be a BCK-algebra, and hence a BCI-algebra, with the following Cayley table:

For A = X, define a set-valued function F: A$\rightarrow$ P(X) by F(x) = {y $\in$ X: y*(y*x)$\in${a, 0}}, $\forall x \in$ A. Then (F, A) is a soft BCI-algebra over X.

(1) Let (G, I) be a soft set over X, where I={a, b, c, d} and G: $I\!\!\!\rightarrow$P(X) is a set-value function defined by G(x) ={y $\in$ X: y*(y* x) $\in$ {0, d}}, $\forall x \in$ I. Then G(a)= {0, b, c, d} Δ X = F(a), G(b)={0, a, c, d} Δ{0, a, c, d}= F(b) and G (c)={0, a, b, d}Δ{ 0, a, b, d}= F(c), G(d) = {0, a ,b, d} Δ {0, a, b, c} F(d). Hence (G, I) is a soft ideal of (F, A). But (G, I) is not a soft a-ideal of (F, A), since (a *a)*(0*a) = 0$\in$ G (a) and a $\notin$ G (a).

(2) For I = {a, b, c, d}, let H: I$\mapsto$ P(X) be a set -valued function defined by H (x) = {0} $\cup$ {y $\in$ X: $x \le y$}, $\forall x \in$ I. Then H(a)= {0, a} Δ X = F(a),H(b)={0, b}Δ{0, a, c, d}= F(b) and H(c)={0, c} Δ{0, a, b, d}= F(c),H(d) ={0, d} Δ {0, a, b, c} =F(d). Therefore (H, I) is a soft ideal of (F, A). But (H, I) is not a soft a-ideal of (F, A) since. Since (b *b)*(0*b) = 0 $\in$ H (a) and b $\notin$ H (a).

Theorem 4.9: Let (F, A) be a soft BCI-algebra over X. Then (G_1, I_1) Δ_a (F, A), (G_2, I_2) Δ_a(F, A) $\Rightarrow$(G_1, I_1)$\cap$ (G_2, I_2) Δ_a (F, A) for any soft sets (G_1, I_1) and (G_2, I_2) over X.

Proof: Using (3.2), we can write (G_1, I_1) Δ (G_2, I_2) = (G , I), where I = $I_1 \Delta I_2$ and G(x)= G_1(x) or G_2(x) for all x $\in$ I. Obviously, I$\subset$ A and G: I $\rightarrow$ P(X) is a mapping. Hence (G, I) is a soft set over X. Since (G_1, I_1) Δ_a (F, A) and

(G_2, I_2) Δ_a (F, A), it knows that G (x) = $G_1(x)$ Δ_a F(x) or G (x) = $G_2(x)$ Δ_a F(x),$\forall x \in$ I. Hence (G_1, I_1) Δ (G_2, I_2) = (G, I) Δ_a (F, A). This completes the proof.

Corollary 4.10: Let (F, A) be a soft BCI-algebra over X. For any soft sets (G, I) and (H, I) over X. Then (G, I) Δ_a (F, A), (H, I) Δ_a (F, A) $\rightarrow$ (G, I) $\cap$ (H, I) Δ_a (F, A).

Proof: Straightforward.

Theorem 4.11: Let (F, A) be a soft BCI-algebra over X, For any soft sets (G, I) and (H, J) over X in which I and J are disjoint, then (G, I) Δ_a (F, A), (H, J) Δ_a(F, A) $\rightarrow$(G, I) (H, J) Δ_a (F, A).

Proof: Assume that (G, I) Δ_a (F, A) and (H, J) Δ_a (F, A). By (3.3), we can write (G, I) $\cup$ (H, J) = (K, U), where U = I $\cup$ J and for every x $\in$ U. K(x) = G (x), if x $\in$ I\ J; H (x), if x $\in$ J \ I; G (x) $\cup$ H (x), if x $\in$ I $\cap$ J. Since I $\cap$ J =0; either x $\in$ I \ J or x $\in$ J \ I for all x $\in$ U. If x $\in$ I \ J, then H(x) = G (x) Δ_a = F(x) since (G, I) Δ_a (F, A). If x $\in$J\I, then K(x) = H(x) Δ_a F(x), since. (H, J) Δ_a (F, A). Thus H(x)Δ_a F(x),$\forall x \in$U, and (G, I) Δ (H, J)=(H, U) Δ_a (F, A),

Example 4.12 (Union of soft a-ideals): If I and J are not disjoint in (4.11), the conclusion of (4.11) does not follow. Let X = {0, 1, a, b, c} be a BCI-algebra with the following Cayley table:

*	0	1	a	b	c
0	0	0	a	b	c
1	1	0	a	b	c
a	a	a	0	a	a
b	b	b	a	0	a
c	c	c	c	a	0

For A = {0, 1} $\subset$ X, let F: A$\rightarrow$ P(X) be a set -valued function defined by F(x) = {y $\in$ x: y *x =y}, $\forall x \in$ A. Then F (0) = X and F (1) = {0, a, b, c}, which are sub algebras of X, and hence (F, A) is a soft BCI-algebra over X. If we take I = A and define a set-valued function G: I$\rightarrow$ P(X) by G(x) = {y $\in$ X: x *(x *y) $\in$ {0, b}}, $\forall x \in$ I, then we obtain that G (0)= {0, 1, b} Δ_a F(0) and G (1)={0, 1, b} Δ_a F(1) , This means that (G, I)Δ_R (F, A).

Now, consider J = {0} which is not disjoint with I, and let H: J$\rightarrow$P(X) be a set-valued function defined by H (x) ={y $\in$ X: x *(x *y) $\in$ {0, c}}, $\forall x \in$ J. Then H (0) = {0, 1, c} Δ_a= F (0), and so (H, J) Δ_a (F, A). But if (H, U) = (G, I) $\cup$ (H, J), then H(0)= G (0) $\cup$ H(0) = {0, 1, b, c}, which is not an a-ideal of X related to F(0) since (a *0)*(b*0)= a$\in$H(0) and a $\in$ H(0). Hence (H, U) = (G, I) $\cup$ (H, J) is not a soft a-ideal of (F, A).

SECTION 5 - A-IDEALISTIC SOFT BCI-ALGEBRA

Definition 5.1: Let (F, A) be a soft set over X. Then (F, A) is an idealistic soft BCI-algebra over X if F(x) is an ideal of X, $\forall x \in$ A.

Definition 5.2: Let (F, A) be a soft set over X. Then (F, A) is a-idealistic soft BCI-algebra over X if F(x) is a-ideal of X,$\forall x \in$ A.

Example 5.3: Consider a BCI-algebra X = {0, 1, 2, a, b} which is given in (4.3). Let (F, A) be a soft set over X, where A = X and F: A$\rightarrow$ P(X) is a set -valued function defined by F(x) = Z {0, 1}, if x $\in$ {2, a, b}, X, if x $\in$ {0, 1}, where Z {0, 1} ={x $\in$ X: 0 *(0 *x) $\in${0, 1}}.

Then (F, A) is an a-idealistic soft BCI-algebra over X. For any element x of a BCI-algebra X, the order of x is defined o(x)=min { n $\in$ N :0*x^{n}= 0},where 0*x^{n} =(. . . {0 *x}*x . . .) *x in which x appear n-times.

Example 5.4 (not a-idealistic soft BCI-algebra): Let X = {0, a, b, c, d, e, f, g} and consider the following Cayley table:

*	0	a	b	c	d	e	f	g
0	0	0	0	0	g	g	g	g
a	a	0	0	0	f	g	g	g
b	b	b	0	0	e	f	g	g
c	c	b	a	0	d	e	f	g
d	d	e	f	g	0	0	0	0
e	e	f	g	g	a	0	0	0
f	f	g	g	g	b	b	0	0
g	g	g	g	g	c	b	a	0

Then (x;*, 0) is a BCI-algebra. Let (F, A) be a soft set over X, where A = {a, b, c}$\subset$ X and F: A $\rightarrow$ P(X) is a set-valued function defined as follows. F(x) = {y $\in$X: o(x) = o(y)}, $\forall$x $\in$ A. Then F (a) = F (b) = F(c) = {0, a, b, c} is an a-ideal of X. Hence (F, A) is an a-idealistic soft BCI-algebra over X. But, if we take B = {a, b, d, f} $\subset$ X and define a set-valued function G: B$\rightarrow$ P(X) by G (x)={0} $\cup${y$\in$ X: o(x)= o(y)}, $\forall$x $\in$ B, then (G, B) is not ana-idealistic soft BCI-algebra over X since G (d)={0, d, e, f, g} is not a a-ideal of X.

Example 5.5: Every a-idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X. Consider a BCI-algebra X = {0, a, b, c} with the following Cayley table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Let A=X and F: A$\rightarrow$P(X) is a set -valued function defined as follows F(x) = {0, x}, $\forall$x$\in$A. Then F(0)=(0); F(a)={0, a}; F(b)= {0, b} and F(c)={0, c} which are ideals of X. Hence (F, A) is an idealistic soft BCI-algebra over X (see [17]). Note that F(x) is an a-ideal of X for all x$\in$A. Hence (F, A) is a a-idealistic soft-BCI-algebra over X. Obviously, **every a-idealistic soft BCI-algebra over X is an idealistic soft BCI-algebra over X,** but the converse is not true in the following example

Example 5.6: (an idealistic soft BCI-algebra over X need not be an a-idealistic soft BCI-algebra over X) Consider a BCI-algebra X= Y x Z, where {Y, *, 0} is a BCI-algebra and (Z, -, 0) is the ad joint BCI-algebra of the additive group (Z, +, 0) of integers. Let F: X $\rightarrow$ P(X) be a set-valued function defined as follows f{y, n} = Y x N_0, if n in $N_{0,}$ {0, 0}, otherwise $\forall$(y, n)$\in$ X, where N_0 is the set of all non-negative integers. Then (F, X) is an idealistic soft BCI-algebra over X (see [17]).But it is not an a-idealistic soft BCI-algebra over X since {(0, 0)} may not be an a-ideal of X.

Theorem 5.7: Let (F, A) and (F, B) be soft sets over X where B$\subset$A$\subset$X. If (F, A) is an a-idealistic soft BCI-algebra over X, then so is (F, B).

Proof: Straightforward.

The converse of (5.7) is not true in the following example.

Example 5.8: Consider an a-idealistic soft BCI-algebra (F, A) over X which is described in (5.4). If we take B = {a, b, c, d} $\supseteq$ A, then (F, B) is not a a-idealistic soft BCI-algebra over X since F (d) = {d, e, f, g} is not a a-ideal of X.

Theorem 5.9: Let (F, A) and (G, B) be two a-idealistic soft BCI-algebras over X. If A $\cap$ B $\neq$ 0, then the intersection (F, A) $\cap$ (G, B) is an a-idealistic soft BCI-algebra over X.

Proof: Using (3.2), we can write (F, A) $\cap$ (G, B) = (H, C), where C = A$\cap$ B and H (x) = F(x) or G (x) $\forall$x $\in$ C. Note that H: C$\rightarrow$ P(X) is a mapping, and therefore (H, C) is a soft set over X. Since (F, A) and (G, B) are a -idealistic soft BCI-algebras over X, it follows that H (x) = F(x) is an a-ideal of X, or H (x) = G (x) is a a-ideal of X,$\forall$ x $\in$ C. Hence (H, C) = (F, A) $\cap$(G, B) is a-idealistic soft BCI-algebra over X.

Corollary 5.10: Let (F, A) and (G, A) be two a-idealistic soft BCI-algebras over X. Then their intersection (F, A) $\cap$(G, A) is an a-idealistic soft BCI-algebra over X.

Proof: Straightforward.

Theorem 5.11: Let (F, A) and (G, B) be two a-idealistic soft BCI-algebras over X. If A and B are disjoint, then the union (F, A) $\cup$ (G, B) is an a-idealistic soft BCI-algebra over X.

Proof: Using (3.3), write this as (F, A) $\cup$ (G, B) = (H, C), where C = A $\cup$B and for every x$\in$C, H (x) = F(x), if x $\in$ A\ B; G(x), if x $\in$ B\A; F(x) $\cup$ G (x), if x $\in$A $\cap$ B. Since A $\cap$ B =0; either x $\in$ A\ B or x $\in$B\ A,$\forall$x $\in$ C. If x$\in$A \ B, then H (x) = F(x) is an a-ideal of X since (F, A) is an a-idealistic soft BCI-algebra over X. If x $\in$B \ A, then H (x) = G(x) is an a-ideal of X since (G, B) is an a-idealistic soft BCI-algebra over X. Hence. (H, C) = (F, A) $\cup$ (G, B) is an a-idealistic soft BCI-algebra over X.

Theorem 5.12: If (F, A) and (G, B) are a-idealistic soft BCI-algebras over X, then (F, A) $\cap$ (G, B) is an a-idealistic soft BCI-algebra over X.

Proof: By (3.4), it follows that (F, A) Λ (G, B) = {H, A x B}, where H (x, y) = F(x)$\cap$ G (y) $\forall$(x, y) $\in$ A$\times$ B. Since F(x) and G (y) are a-ideals of X, the intersection F(x) $\cap$ G (y) is also an a-ideal of X. Hence H (x, y) is an a-ideal of X for all (x, y) $\in$A $\times$ B, and therefore (F, A)Λ (G, B) = (H, A $\times$ B) is an a-idealistic soft BCI-algebra over X.

Definition 5.13: A a-idealistic soft BCI-algebra (F, A) over X is said to be trivial (resp., whole) if F(x) = {0} (resp., F(x) = X),$\forall$x$\in$A.

Example 5.14(Trivial and whole a-idealistic soft BCI-algebras): Let X be a BCI-algebra which is given in Example 5.5, and let F: X$\to$P(X) be a set -valued function defined by F(x) = {0} $\cup${y $\in$ X: o(x) = o(y)}; $\forall$x $\in$ X. Then F (0) = {0} and F (a) = F (b) = F(c) = X. We can check that {0} Δ_a X and X Δ_aX. Hence (F, {0}) is a trivial a-idealistic soft BCI-algebra over X and (F, X\ {0} is a whole a-idealistic soft BCI-algebra over X. The proofs of the following three lemmas are straight forward, so they are omitted.

Lemma 5.15: Let f: X $\to$ Y is an onto homomorphism of BCI-algebras. If I is an ideal of X, then f (I) is an ideal of Y.

Lemma 5.16: Let f: X $\to$Y is an isomorphism of BCI-algebras. If I is an a-ideal of X, then f (I) is an a-ideal of Y. Let f: X$\to$Y is a mapping of BCI -algebras. For a soft set (F, A) over X, (f (F), A) is a soft set over Y where f (F): A $\to$P(Y) is defined by f (F) (x) = f (F(x)), $\forall$x $\in$ A.

Lemma 5.17: Let f: X$\to$ Y is an isomorphism of BCI-algebras. If (F, A) is an a-idealistic soft BCI-algebra μover X, then (f (F), A) is an a-idealistic soft BCI-algebra over Y.

Theorem 5.18: Let f: X $\to$Y is an isomorphism of BCI-algebras and let (F, A) be an a-idealistic soft BCI-algebra over X.

(1) If F(x) $\subseteq$ kern (f) for all x $\in$A, then (f (F), A) is a trivial a-idealistic soft BCI-algebra over Y.
(2) If (F, A) is whole, then (f (F), A) is a whole a-idealistic soft BCI-algebra over Y.

Proof:

(1) Assume that F(x) $\subseteq$ kern (f), $\forall$x $\in$A. Then f (F) (x) = f (F(x)) = {0y} for all x $\in$A. Hence (f (F), A) is a trivial a-idealistic soft BCI-algebra over Y by (5.17) and (5.13).
(2) Suppose that (F, A) is whole. Then F(x) = X,$\forall$x $\in$A, and so {f (F) x) = f (F(x)) = f(X) = Y, $\forall$x $\in$ A. It follows from Lemma 5.17 and Definition 5.13 that (f (F), A) is a whole a-idealistic soft BCI-algebra over Y.

Definition 5.19: A fuzzy μ in X is a fuzzy a-ideal of X if it satisfies the following assertions:
(i) ($\forall$ x $\in$X) (μ(0) $\geq$ μ(x), (ii) ($\forall$x, y, z $\in$ X) (μ(x*y) $\geq$ min {μ{(x * z)*(0 *y)), μ(z)})

Lemma 5.20: A fuzzy set μ in X is a fuzzy a-ideal of X if and only if it satisfies:
($\forall$ t $\in$[0, 1])(U (μ ; t) $\neq$ 0 $\Rightarrow$U (μ ; t) is a a-ideal of X)

Theorem 5.21: For every fuzzy a-ideal $\mu$of X, there exists an a-idealistic soft BCI-algebra (F, A) over X.

Proof: Let μ be a fuzzy a-ideal of X. Then U (μ; t) = {x $\in$ X |μ(x) $\geq$ t} is an a-ideal of X for all t$\in$ Im (μ). If we take A = Im (μ) and consider a set-valued function F: A$\to$ P(X) given by$ F (t) = U (μ; t),$\forall$t$\in$ A , then F(f, A) is an a-idealistic soft BCI-algebra over X.

Conversely, the following theorem is straightforward.

Theorem 5.22: For any fuzzy set μ in X, if an a-idealistic soft BCI-algebra (F, A) over X is given by A = Im (μ) and F (t) = U (μ; t), $\forall$t $\in$ A, then is a fuzzy a-ideal of X.

Proof: Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = Im (μ) and F: A$\to$ P(X) is a set - valued function defined by ($\forall$ t $\in$ A) (F (t) ={x$\in$ X: |μ(x) + t > 1} t). Then there exists t $\in$ A such that F (t) is not an a-ideal of X as seen in the following example.

Example 5.23 nor a-ideal): For any BCI-algebra X, define a fuzzy set μ in X by μ(0) = t_0 < 0.5 and μ(x) =1 – $t_{0,}$ $\forall$x $\neq$ 0. Let A = Im (μ) and F: A$\to$ P(X) is a set-valued function given by (5.2). Then F (1 – t_0) = X \{0}, which is not a-ideal of X.

Theorem 5.24: Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = [0, 1] and F: A $\rightarrow$ P(X) is given by (5.2). Then the following assertions are equivalent: (1) μ is a fuzzy a-ideal of X,(2) For every t $\in$A with F (t)$\neq$ 0, F (t) is an a-ideal of X.

Proof: Assume that μ is a fuzzy a-ideal of X. Let t $\in$ A be such that F (t) $\neq$ 0. If we select x$\in$ F(t), then $\mu(0) + t \geq \mu(x) + t > 1$ and so 0 $\in$ F (t). Let t $\in$ A and x, y, z $\in$A be such that y $\in$ F (t) and (x*z)*(0 *y)$\in$ F (t). Then μ (y) +t > 1 and μ ((x *z) *(0 *y)) t > 1. Since μ is a fuzzy a-ideal of X, it follows that
μ (x* y) + t $\geq$ min {μ((x *z)*(0 *y)), μ (z)} + t = min {μ((x *z)*(0 *y)) +t, μ(z) + t}>1.

So that x$\in$ F (t), Hence F (t) is an a-ideal of X, $\forall$t $\in$ A with F (t) $\neq$ 0.

Conversely, suppose that (2) is valid. If there exists an$\in$X such that $\mu(0) < \mu(a)$, then we can select $t_a \in$ A such that $\mu(0) + t_{a \leq} 1 < \mu(a) + t_a$. It follows that a$\in$ F (t_a) and 0 $\neq$ F (t_a), which is a contradiction. Hence $\mu(0) \geq \mu(x)$, $\forall$x $\in$ X. Now, assume that μ (a *b) <min {μ((a *c)*(0 *c)), μ(b)}, for some a, b, c $\in$ X. Then $\mu(a) + S_0 \leq 1<$ min μ ((a *c)*(0 *b)), $\mu(c)\}+S_0$ for some $S_0 \in$A, which implies that (a *c)*(0*c) $\in$ F (S_0) and b$\in$F (S_0), but a$\notin$F (S_0). This is a contradiction. Therefore $\mu(x) \geq$ min {μ((x *z)*(0 *y)), μ(z)}, $\forall$x, y, z $\in$X, and thus μ is a fuzzy a-ideal of X.

Corollary 5.25: Let μ be a fuzzy set in X such that μ (x) > 0.5 for some x $\in$ X, and let (F, A) be a soft set over X in which A= {t $\in$ Im(μ)|t > 0.5} and F: A$\rightarrow$ P(X) is given by (5.2). If μ is a fuzzy a-ideal of X, then (F, A) is an a-idealistic soft BCI-algebra over X.

Proof: Straightforward.

Theorem 5.26: Let μ be a fuzzy set in X and let (F, A) be a soft set over X in which A = (0.5, 1] and F: A$\rightarrow$ P(X) is defined by ($\forall$ t $\in$ A) (F (t) = U (μ; t)). Then F (t) is an a-ideal of X for all t $\in$ A with F (t)$\neq$ 0 if and only if
(1) ($\forall$ x $\in$ X) (Max {μ(0), 0.5} $\geq$ μ(x)); (2) ($\forall$x, y, z $\in$ X) (max {μ(x *y), 0.5} $\geq$ min {μ {(x*z)*(0 *y), μ(z)}).

Proof: Assume that F(t) is an a-ideal of X for all t$\in$ A with F(t) $\neq$0. If there exists $X_0 \in$X such that max {μ(0), 0.5} < μ(X_0), then we can select $t_0 \in$ A such that max{μ(0), 0.5} < $t_0 \leq \mu(X_0)$. It follows that $\mu(0) < t_0$ so that $X_0 \in$ F (t_0) and 0$\notin$F (t_0). This is a contradiction, and so (1) is valid. Suppose that there exist a, b, c $\in$ X such that max {μ(a), 0.5} < min (μ(a *c)*(0 *c)), μ(b)}.Then max {μ(a), 0.5} < $u_0 \leq$ Min {μ((a *c)*(0*b)), μ(b)}, for some u0$\in$A. Thus (a *c)*(0 *b)$\in$F (u_0) and b$\in$ F (u_0) but a $\notin$ F(u_0). This is a contradiction, and so (2) is valid.

Conversely, suppose that (1) and (2) are valid. Let t$\in$A with F(t)$\neq$0, for any x$\in$F(t), it gives that Max {μ(0), 0.5} $\geq$ μ(x) $\geq$ t > 0.5 and so $\mu(0) \geq$ t, (ie) 0 $\in$ F (t). Let x, y, z $\in$X be such that y $\in$ F (t) and (x *z)*(0 *y)$\in$ F (t). Thenμ(y) $\geq$ t and μ(x *z)*(0* y))>t. It follows from the second condition that Max {μ(x), 0.5} $\geq$min {μ((x * z)*(z * y)), μ(y)}$\geq$ t >0.5, so that $\mu(x) \geq$ t, i.e., x $\in$ F (t). Therefore F (t) is an R-ideal of X, $\forall$t $\in$ A with F (t) $\neq$ 0.

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