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β- Bi NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT

In this paper we introduce the notion of β- bi-near subtraction semigroup. Also we give characterizations of β- bi-near subtraction semigroup.

Mathematical subject classification: *06F35.*

Key words: *β bi-near subtraction semigroup, Boolean near subtraction semigroup, Weak commutative near subtraction semigroups, Nil near subtraction semigroup.*

1. INTRODUCTION

In 2007, Dheena [1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz [4]. In this paper we shall obtained equivalent conditions for regularity in terms of β- Bi near subtraction semigroup.

2. PRELIMINARIES

A non-empty subset X together with two binary operations "–" and "." is said to be subtraction semigroup If (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group (iii) x(y–z)=xy–xz and (x–y)z= xz–yz for every x, y, z∈X. A non-empty subset X together with two binary operations "–"and "." is said to be near subtraction semigroup if (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group and (iii) (x–y)z = xz–yz for every x, y, z∈X. A non-empty subset X is said to be **nil-near subtraction semigroup** if there exists a positive integer k>1 such that $a^k=0$ Which implies that xa=0 where $x=a^{k-1}$.

3. β-*BI NEAR SUBTRACTION SEMIGROUP*

Definition 3.1: A non-empty subset X together with two binary operations"-"and "." Is said to be β- **bi near subtraction semigroup**. Then X is the both boolean and weak commutative near subtraction semigroup

Example 3.2: Let X= {0, a, b, 1} in which "-" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	0	0	0
c	c	0	c	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then X is a β- bi near-subtraction semi group

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Lemma 3.3: If X is a β- bi near subtraction semigroup then xy=xyx for each x, y∈X.

Proof: Since X is β- bi near subtraction semigroup**.** Claim: xy=xyx for each x, y in X.

Let x, y in X. Since X is Boolean near subtraction semigroup**.** By definition, x^2 = x for all x in X and y^2 = y for all y in X. Now, xy = $(xy)^2$ = (xy) (xy) = xyyx = xy^2x = xyx.

Theorem 3.4: Let X be a β- bi near subtraction semigroup. Each of the following statement implies that X is a strong s_2- near subtraction semigroup.

(i) X is a zero symmetric
(ii) X is distributive
(iii) X is subcommutative.
(iv) X is of Type II
(v) X is commutative
(vi) aX=aXa for all a in X . (X is a P_1 near subtraction semigroup)

Proof:

(i) Since X is Boolean is isno nilpotent elements. Since $X=X_0$.Proposition 2.2.31 demands that X has (., IFP). Let a, b in X. Now (ab-aba)a=aba-aba^2=aba-aba (since X is Boolean)=0. That is, (ab-aba)a=0. By using (*, IFP) property we get, a(ab-aba)=0. ab(ab-aba) = 0....(I) and aba(ab-aba) = 0...(2). From (1) and (2) we get ab(ab-aba)-aba(ab-aba) = 0. That is, $(ab\text{-}ba)^2$ =0. Since X has no nilpotent element ab-aba = 0. Similarly we can prove aba-ab = 0. That is aba = ab. Hence X is strong s_2 near subtraction semigroup.
(ii) Since X is distributive X is zero symmetric. Therefore the result follows from (i).
(iii) Let a∈ X. Since X is subcommutative Xa=aX. Therefore for any x∈X, there exists y∈X such that ax=ya. Therefore axa= (ax)a=(ya)a=ya^2=ya [Since X is Boolean] = ax. Thus X is a Strong s_2 near subtraction semigroup.
(iv) Let X be a Type II near subtraction semigroup and X is also Boolean.. Let a, b in X. Then aba=aab=a^2b=ab. That is, aba=ab. Thus X is a strong $s_2$near subtraction semigroup.
(v) Let X be commutative. It is subcommutative also. Hence the proof (iii).
(vi) Let a∈ X. Since aX=aXa for any x∈X, there exists y∈X such that ax=aya. Therefore
(vii) axa= (ax)a=(aya)a=aya^2 =aya [Since X is Boolean] = ax. Thus X is a strong s_2 near subtraction semigroup.

Proposition 3.5: If X is a β- bi near subtraction semigroup then the following are true.

(i) ab and ba ∈ E for all a, b ∈X.
(ii) X is a p_1 near subtraction semigroup.

Proof: Since X is a β- bi near subtraction semigroup then xy=xyx for each x, y∈X.

(i) Let a, b∈X^*. Now $(ab)^2$=abab =a(ba)b=a(bab)=a(ba)=aba=ab. That is $(ab)^2$=ab. Consequently ab∈E for all a, b in X. In a similar fashion we get ba∈E for all a, b∈X.
(ii) Let z∈aX. Then there exists a'∈X such that z=aa'=aa'a∈aXa. That is, z∈aXa. Therefore aX⊂aXa...(1) Obviously aXa⊂aX...(2). From (1) and (2), we have aX=aXa. Thus, X is a p_1 near subtraction semigroup.

4. RESULTS ON β *BI NEAR SUBTRACTION SEMIGROUP*

Theorem 4.1: Homomorphic image of a β- bi near subtraction semigroup is also a β- bi near subtraction semigroup

Proof: Let T be a homomorphic image of X where Π: T→R is a epimorphism of a β- bi near subtraction semigroup. Let t∈T, then t=Π(r) for some r in R. Now, t^2=tt=Π(r) Π(r) =Π(r^2) =Π(r)=t. There t is Boolean near subtraction semigroup. Let t_1, t_2, t_3∈T, then t_1=Π(r_1), t_2=Π(r_2) and t_3=Π(r_3) for some r_1, r_2 r_3 in R. Also, $t_1 t_2 t_3$=Π(r_1) Π(r_2) Π(r_3) = $r_1r_2r_3$ = $r_1r_3r_2$ = Π(r_1) Π(r_3) Π(r_2) = $t_1 t_3 t_2$. Therefore T is weak Commutative. Hence, T is also a β- bi near subtraction semigroup.

Proposition 4.2: Let X be a s- β-bi near subtraction semigroup Then we have the following:

(i) Every X-system is invariant
(ii) Every left ideal of X is an ideal.

Proof

(i) If A is a X-system of X then XA ⊂ A(ie.,) an ∈ AX for all a∈X, n∈X we have an ∈ aX =Xa [Since X is sub-commutative]=Xa^2{Since X is left bipotent]. (ie.,) an ∈ Xa^2. Which implies that an=$n'a^2$ for some n'∈X. It follows that AX⊂A. Hence, X is invariant X-system.
(ii) Let A be a left ideal of X. Then XA ⊂ A. (ie.,) A is an X-system of X [by(i)] we have AX⊂A. Hence, X is an ideal.

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