Research Journal of Pure Algebra -1(7), 2011, Page: 164-166
Available online through www.rjpa.info ISSN 2248–9037

The Congruence of k-Curves

[1]H. Khosravi*, [2]H. Golmakani and [3]Elias. Faryad

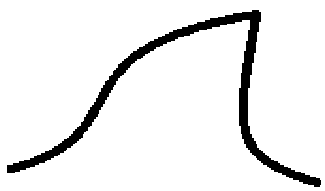
[123]Department of Mathematics, Faculty of Science, Mashhad Branch, Islamic Azad University, Mashhad, 91735-413, Iran

Email: Khosravi@mshdiau.ac.ir, H.golmakani@mshdiau.ac.ir, Elias.Faryad@yahoo.com

(Received on: 16-10-11; Accepted on: 28-10-11)

ABSTRACT

In this paper we introduce the conditions about congruence of K-curves then by using of this conditions stated congruence of curves and surfaces. We prove the conditions about congruence of two curves in R^2 and R^3 in different case and discuss congruence of two curves by using the concept of isometry, translation and rotation without using torsion and curvature.And we study half-curves that created the graphs of their conjunctions as follows;



Our purpose of invariant curve is half-curve or segment of curve that less than limited in one side.(see [3,7])

Keywords: *Congruence Theory, Isometry, Rotation, Translation*

1. INTRODUCTION:

In elementary differential geometry the condition about congruence of two curves has been considered. And with the extension of congruence of curves discuss of surface congruence stated in the world in the last time. In second reference the theorem of congruence of two curves in $\mathbf{R}^2$ and $\mathbf{R}^3$ are express as follows:

$\mathbf{R}$ is the set of real numbers, $\mathbf{I}$, $\mathbf{I_1}$ and $\mathbf{I_2}$ are an interval in $\mathbf{R}$, $\delta\mathbf{I}$ is boundary of interval $\mathbf{I}$, $\boldsymbol{\tau}$ is torsion of a curve, $\boldsymbol{\kappa}$ is curvature a curve, $\mathbf{v}$ is the parallel vector of a curve, E^2 is Euclidian plane and $\mathbf{E}^3$ is Euclidian space.

Definition1.1: The two curves α and β: $I \to \mathbf{E}^3$ are congruent if there is an isometry F: $\mathbf{E}^3 \to \mathbf{E}^3$ such that $F(\beta(t))=\alpha(t)$ for every t in I **.** (See [1, 3])

Theorem 1.2: If α and β: $I \to \mathbf{E}^3$ are two curves such that $\|\alpha'(t)\| = \|\beta'(t)\| = 1$ for every t in I , $\boldsymbol{\kappa}_\alpha = \boldsymbol{\kappa}_\beta$ and $\boldsymbol{\tau}_\alpha = \pm\, \boldsymbol{\tau}_\beta$ then α and β are congruent. (see [3,5])

Theorem 1.3: Suppose α and β: $I \to \mathbf{E}^3$ are two curves such that

$\|\alpha'(t)\| = \|\beta'(t)\| = 1$: for every t in I, $\boldsymbol{\kappa}_\alpha = \boldsymbol{\kappa}_\beta > 0$, $\boldsymbol{\tau}_\alpha = \pm\, \boldsymbol{\tau}_\beta$

and $v_\alpha = v_\beta > 0$,then α and β are congruent. (see [2,3])

Theorem 1.4: Suppose α and β: $I \to \mathbf{E}^3$ are two arbitrary curves.

Let E_1, E_2 and E_3 is a frame field on α and F_1, F_2 and F_3 is a frame field on β,

if $\alpha'.\, E_i = \beta'.\, F_i$ $(1\le i \le 3)$ and $E_i'.\, E_j = F_i'.\, F_j$ $(1\le i, j \le 3)$, then α and β are congruent. (see [3])

**Corresponding author: [1]H. Khosravi*, *E-mail: Khosravi@mshdiau.ac.ir*

Theorem 1.5: Suppose α and β: I $\rightarrow$ $\mathbf{E}^2$ are two curves such that

$\|\alpha'(t)\| = \|\beta'(t)\| = 1$: for every t in I.

Then α and β in $\mathbf{E}^2$ are congruent if and only if $\kappa_\alpha = \pm\, \kappa_\beta$. (see [3,4,6])

2. THE RESULTS:

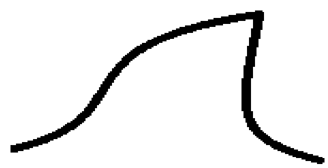
Definition 2.1: (Junction Point) Suppose $\alpha : I_1 \rightarrow R^2$ and $\beta : I_2 \rightarrow R^2$ are two segment curves or half-curves. In this case we say the unique point $p \in \delta I_1 \cap \delta I_2$ is junction point if $\alpha(p) = \beta(p)$.

In simple words we say the points that created by conjunction of two half-curves or segment curves are junction points.

Definition 2.2: (Broken k-Curve) A continuous curve is called broken k-curved if it has k corner point and without closed region in its graph.

Definition 2.3: (Invariant 2-Curve) If α and β are two invariant curves such that of their conjunction in junction point that created a broken 1-curve or broken 2-curve, we say it invariant 2-curves and we show it with $\overline{\alpha\beta}$ notation.

In simple words invariant 2-curves are the same curved as follows;



Definition 2.4: (k-Curves by n) Suppose α is a curve we say α is k-curve by n if formed by k times invariant curves and we can see less than n times invariant 2- curves in its graph.

Definition 2.5: (invariant k-curve (k>2)) If $\alpha_1,\ldots,\alpha_k$ are k invariant curves and $\overline{\alpha_i\alpha_{i+1}}$ (i =1,…, k-1) are invariant 2-curves such that P_i is junction point for $\overline{\alpha_i\alpha_{i+1}}$,then broken (k-1)-curves $\overline{\alpha_1\alpha_2 \ldots \alpha_k}$ is called an invariant k-curves ,or , The α curve formed by k times invariant curves and its graph has (k-1) times invariant 2-curves such that every two invariant 2-curves with disjoint junction point are common at most in one invariant curve.

Corollary 2.6: Every k-curve by n is not an invariant k-curve.

For example the following figure is a 3-curve by 3 but is not an invariant 3-curve.

Corollary 2.7: Every invariant k-curve is continuous. It is clear by definition of invariant k-curve.

Theorem 2.8: The union of two isometry is an isometry. (see [3,8])

Theorem 2.9: (Invariant k-Curves Conformity) Suppose α and β are two invariant k-curves that α includes from k curves $\{\alpha_1,\ldots,\alpha_k\}$ and β includes from k-curves $\{\beta_1,\ldots,\beta_k\}$ then α on β is congruent if ; there is a translation F such that $F(\alpha) = \beta$ or there is a rotation F such that $F(\alpha)=\beta$ or there is an isometry F such that $F(\alpha)=\beta$.

Proof: If the invariant curves of α_i (i=1, 2, 3... k) congruent on β_i by isometry G, then it is enough to put translation F equal G. (By the last theorem the union of two isometry is isometry).

Theorem 2.10: (khosravi-faryad theorem) Every two same shapes in $\mathbf{R}^2$ are congruent by at most of two translations and one rotation.

Proof: Suppose S_1 and S_2 are two same shapes in $\mathbf{R}^2$ such that are in different places. We will show that S_1 and S_2 are congruent by at most two translations and one rotation. We congruence the centroid of $S_1$and S_2 by at most of two translations.

Let θ is the angle of between smallest surrounded rectangles $S_1$and S_2.

Now with rotation around in this point to measure of θ, the congruence is possible.

In other words every point of surrounded rectangle of S_1 input to point of surrounded rectangle S_2 with rotation to measure of θ, especially the region of S_1 input to S_2.

Lemma 2.11: Every two same rectangular parallelepiped with the same centroid in $\mathbf{R}^3$ are congruent at most by two rotations.

Proof: Suppose A and B are two same rectangular parallelepiped in $\mathbf{R}^3$ with the same centroid.

Suppose S_1 is a perpendicular sheet on four parallel ribs a_1 ,a_2 ,a_3 and a_4 from A.

There are four parallel ribs b_1, b_2, b_3 and b_4 in B such that have the same length with a_1, a_2, a_3 and a_4 i.e. $|a_i| = |b_i|$ (i=1, 2, 3, 4).

Let S_2 is a perpendicular sheet on b_1, b_2, b_3 and b_4**,** with a rotation to measure angle of between two sheet normal vectors S_1 and S_2 around centroid this two sheets are congruent together.

Now located A and B in one direction i.e. a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 are parallel together.

Now we consider four rectangles of perpendicular ribs on a_i (i =1, 2, 3, 4) from A and perpendicular ribs on b_i (i =1,2,3,4) from B. These rectangles have the same center two by two. A and B are congruent with together by a rotation to measure angle of between these two rectangles around their centroid.

Theorem 2.12: (khosravi-faryad theorem) Every two same shapes in $\mathbf{R}^3$ are congruent by at most three translations and two rotations.

Proof: Suppose $S_1$and S_2 are the same shapes in $\mathbf{R}^3$and locates in different places.

At first by three translation, the centroid of S_1 and S_2 are congruent. Then by last lemma there are at most two rotations such that smallest surrounded rectangular parallelepiped of two shapes S_1 and S_2 are conformed, so S_1 and S_2 will be congruent.

Conjecture: Every two same shapes in $\mathbf{R}^n$ are at most congruent by n translation and (n-1) rotation.

3. ACKNOWLEDGMENTS:

The authors thank the research council of Mashhad Branch, (Islamic Azad University) for support.

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