FUZZY SOFT UNION ACTION ON N-MODULE AND N-IDEAL STRUCTURES

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ABSTRACT

In this paper, we define a new concept, called soft union action (SU) on N-module structures on a fuzzy soft set. This new notion gathers fuzzy theory, soft set theory and near-ring modulo theory (N-module theory) together and it shows how a fuzzy soft set effects on N-module structure in the mean of union and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical N-module theoretic concepts for SU-action on N-module. Finally, we give the application of SU-actions on N-module theory.

Keywords: Fuzzy set, soft set, fuzzy soft set, N-module SU-action, N-ideal SU-action, α-inclusion, pre-image, soft image.

AMS mathematics subject classification: 03E70, 58E40,

1. INTRODUCTION

Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [4] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji et al. [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali et al. [3] introduced several operations of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29]. In this paper, we define a new concept, called soft union action (SU) on N-module structures on a fuzzy soft set. This new notion gathers fuzzy theory, soft set theory and near-ring modulo theory (N-module theory) together and it shows how a fuzzy soft set effects on N-module structure in the mean of union and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical N-module theoretic concepts for SU-action on N-module. Finally, we give the application of SU-actions on N-module theory.

2. PRELIMINARIES

In this section, we recall some basic notions relevant to near-ring modules (N-modules) and fuzzy soft sets. By a near-ring, we shall mean an algebraic system (N, +, .), where

(N1) (N, +) forms a group (not necessarily abelian)
(N2) (N, .) forms a semi group and
(N3) (x + y)z = xz + yz for all x, y, z ∈ N. (that is we study on right Near-ring modules)
Throughout this paper, N will always denote right near-ring. A normal subgroup H of N is called a left ideal of N if n(s+h)-ns ∈ H for all n, s ∈ N and h ∈ I and denoted by H@N. For a near-ring N, the zero-symmetric part of N denoted by N₀ is defined by N₀ = {n ∈ S / n=0}.

Let (S, +) be a group and A: N×S→S, (n,s)→s. (S, A) is called N-module or near-ring module if for all x, y ∈ N, for all s ∈ S.
(i) x(ys) = (xy)s. It is denoted by N^S. Clearly N itself is an N-module by natural operations. A subgroup T of N^S with N^T is said to be N-sub module of S and denoted by T ⊆ S. A normal subgroup T of N is called an N-ideal of N^S and denoted by a near-ring, S and χ two N-modules. Then h: S→χ is called an N-homomorphism if s,δ ∈ S, for all n ∈ N.
(ii) h(s+δ) = h(s)+h(δ) and
(ii) h(ns) = nh(s).

For all undefined concepts and notions we refer to (24). From now on, U refers to on initial universe, E is a set of parameters P(U) is the power set of U and A, B, C ∈ E.

2.1. Definition [22]: A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by FA. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by FA, FB, FC, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by FA, GA, HA, respectively. For more details, we refer to [11, 17, 18, 26, 29, 7].

2.2. Definition [6]: The relative complement of the soft set FA over U is denoted by FA, where FA(a) = U \ FA(a), for all a ∈ A.

2.3. Definition [6]: Let FA and GB be two soft sets over U such that A ∩ B ≠ ∅. The restricted intersection of FA and GB is denoted by FA ∩ GB, and is defined as FA ∩ GB = (H, C), where C = A∩B and for all c ∈ C, H(c) = F(c) \ G(c).

2.4. Definition [6]: Let FA and GB be two soft sets over U such that A ∩ B ≠ ∅. The restricted union of FA and GB is denoted by FA∪GB, and is defined as FA∪GB = (H, C), where C = A∪B and for all c ∈ C, H(c) = F(c) ∪ G(c).

2.5. Definition [12]: Let FA and GB be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set ψ(FA) over U, where ψ(FA): A→P(U) is a set valued function defined by ψ(FA)(a) = U{F(a) | a ∈ A and ψ(a) = b}, if ψ⁻¹(b) ≠ ∅, = 0 otherwise for all b ∈ B. Here, ψ(FA) is called the soft image of FA under ψ. Moreover we can define a soft set ψ⁻¹(GB) over U, where ψ⁻¹(GB): A→P(U) is a set-valued function defined by ψ⁻¹(GB)(a) = G{ψ⁻¹(a)} for all a ∈ A. Then, ψ⁻¹(GB) is called the soft pre image (or inverse image) of GB under ψ.

2.6. Definition [13]: Let FA and GB be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set ψ(FA) over U, where ψ(FA): B→P(U) is a set-valued function defined by ψ(FA)(b) = U{F(a) | a ∈ A and ψ(a) = b}, if ψ⁻¹(b) ≠ ∅, = 0 otherwise for all b ∈ B. Here, ψ(FA) is called the soft anti image of FA under ψ.

2.7 Definition [8]: Let f_a be a soft set over U and α be a subset of U. Then, lower α-inclusion of a soft set f_a, denoted by f_a, is defined as f'_a = {x ∈ A: f_a(x) ⊆ α}

3. SU-ACTION ON N-MODULE STRUCTURES AND N-IDEAL STRUCTURES WITH FUZZY VERSION

In this section, we first define fuzzy soft union action, abbreviated as fuzzy SU-action on N-module and N-ideal structures with illustrative examples. We then study their basic results with respect to soft set operation.

3.1 Definition: Let S be an N-module and f_s be a fuzzy soft set over U, then f_s is called fuzzy SU-action on N-module over U if it satisfies the following conditions;
(FS(N-1)) f_s(x+y) ⊆ f_s(x) U f_s(y)
(FS(N-2)) f_s(-x) ⊆ f_s(x)
(FS(N-3)) f_s(nx) ⊆ f_s(x)

For all x, y ∈ S and n ∈ N.
3.1 Example: Consider the near-ring module $N = \{0, x, y, z\}$, be the near-ring under the operation defined by the following table:

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
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<td>y</td>
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<td>y</td>
<td>y</td>
<td>z</td>
<td>0</td>
<td>x</td>
</tr>
<tr>
<td>z</td>
<td>z</td>
<td>y</td>
<td>x</td>
<td>0</td>
</tr>
</tbody>
</table>

Let $S = N$ and $S$ be the set of parameters and $U = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} / a, b \in Z_2 \right\}$, 2X2 matrices with $Z_2$ terms, is the universal set. We construct a fuzzy soft set.

Then one can easily show that the soft set $\text{fs}_x$ is a fuzzy SU-action on N-module.

3.1 Proposition: Let $f_x$ be a fuzzy SU-action on N-module over U. Then, $f_x(0) \subseteq f_x(x)$ for all $x \in S$.

Proof: Assume that $f_x$ is fuzzy SU-action over U. Then, for all $x \in S$, $f_x(0) = f_x(x-x) \subseteq f_x(x)$ and $f_x(x) = f_x(x) \cup f_x(x) = f_x(x)$. Conversely, assume that $f_x(x) \subseteq f_x(x)$ and $f_x(x) \subseteq f_x(x)$ for all $x, y \in S$ and $n \in N$.

3.1 Theorem: Let $S$ be a fuzzy SU-action on N-module and $f_x$ be a fuzzy soft set over U. Then $f_x$ is SU-action of N-module over U if and only if

(i) $f_x(y-x) \subseteq f_x(x) \cup f_x(y)$

(ii) $f_x(nx) \subseteq f_x(x)$ for all $x, y \in S$.

Proof: Suppose $f_x$ is a fuzzy SU-action on N-module over U. Then, by definition-3.1, $f_x(xy) \subseteq f_x(y)$ and $f_x(x-y) \subseteq f_x(x) \cup f_x(-y) = f_x(x) \cup f_x(y)$ for all $x, y \in S$.

Conversely, assume that $f_x(xy) \subseteq f_x(y)$ and $f_x(x-y) \subseteq f_x(x) \cup f_x(y)$ for all $x, y \in S$.

If we choose $x=0$, then $f_x(0-y) = f_x(-y) \subseteq f_x(0) \cup f_x(y) = f_x(y)$ by proposition-3.1. Similarly $f_x(y) = f_x(-y) \subseteq f_x(y)$, thus $f_x(-y) = f_x(y)$ for all $y \in S$. Also, by assumption $f_x(x-y) \subseteq f_x(x) \cup f_x(-y) = f_x(x) \cup f_x(y)$. This complete the proof.

3.2. Theorem: Let $f_x$ be a fuzzy SU-action on N-module over U.

(i) If $f_x(x-y) = f_x(0)$ for any $x, y \in S$, then $f_x(x) = f_x(y)$.

(ii) $f_x(x+y) = f_x(0)$ for any $x, y \in S$, then $f_x(x) = f_x(y)$.

Proof: Assume that $f_x(x-y) = f_x(0)$ for any $x, y \in S$, then

$f_x(x) = f_x(x-y+y) \subseteq f_x(x-y) \cup f_x(y)$

and similarly

$f_x(y) = f_x(y-x+x) \subseteq f_x(y-x) \cup f_x(x)$

Thus, $f_x(x) = f_x(y)$ which completes the proof. Similarly, we can show the result (ii).

It is known that if $S$ is an N-module, then $(S, +)$ is a group but not necessarily abelian. That is, for any $x, y \in S$, $x + y$ needs not be equal to $y + x$. However, we have the following:

3.3. Theorem: Let $f_x$ be a fuzzy SU-action on N-module over U and $x \in S$. Then,

$f_x(x) = f_x(0) \iff f_x(x+y) = f_x(y+x) = f_x(y)$ for all $y \in S$.

Proof: Suppose that $f_x(x+y)$ = $f_x(y+x)$ = $f_x(y)$ for all $y \in S$. Then, by choosing $y = 0$,

We obtain that $f_x(x) = f_x(0)$.

Conversely, assume that $f_x(x) = f_x(0)$. Then by proposition-3.1, we have

$f_x(0) = f_x(x) \subseteq f_x(y), \forall y \in S$................. (1)
Since \( f_s \) is fuzzy SU-action on N-module over U, then
\[
\forall y \in S, \quad f_s(x+y) \subseteq f_s(x) \cup f_s(y) = f_s(y),
\]

Moreover, for all \( y \in S \)
\[
f_s(y) = f_s((-x)+y) = f_s(x+y-x) = f_s(x+y) \subseteq f_s(x) \cup f_s(y) = f_s(y) = f_s(x+y)
\]

Since by equation (1), \( f_s(x) \subseteq f_s(y) \) for all \( y \in S \) and \( x, y \in S \), implies that \( x+y \in S \). Thus, it follows that \( f_s(x) \subseteq f_s(x+y) \).

So \( f_s(x+y) = f_s(y) \) for all \( y \in S \).

Now, let \( x \in S \). Then, for all \( x, y \in S \)
\[
f_s(x+y) = f_s(y-x) = f_s(x+y-x) = f_s(x+y)
\]

Since \( f_s(x+y) = f_s(y) \). Furthermore, for all \( y \in S \)
\[
f_s(y) = f_s(y-x) = f_s(x+y-x) = f_s(x+y-y) = f_s(y-x)
\]

It follows that \( f_s(x+y) = f_s(y) \) and so \( f_s(x+y) = f_s(y) = f_s(x) \), for all \( y \in S \), which completes the proof.

3.4 Theorem: Let \( S \) be a near-field and \( f_s \) a fuzzy soft set over U. If \( f_s(0) \subseteq f_s(1) = f_s(x) \) for all \( 0 \neq x \in S \), then it is fuzzy SU-action on N-module over U.

Proof: Suppose that \( f_s(0) \subseteq f_s(1) = f_s(x) \) for all \( 0 \neq x \in S \). In order to prove that it is fuzzy SU-action on N-module over U, it is enough to prove that \( f_s(x-y) \subseteq f_s(x) \cup f_s(y) \) and \( f_s(nx) \subseteq f_s(x) \).

Let \( x, y \in S \). Then we have the following cases:

Case-1: Suppose that \( x \neq 0 \) and \( y = 0 \) or \( x = 0 \) and \( y \neq 0 \). Since \( S \) is a near-field, so it follows that \( nx=0 \) and \( f_s(nx) = f_s(0) \). Since \( f_s(0) \subseteq f_s(x) \), for all \( x \in S \), so \( f_s(nx) = f_s(0) \subseteq f_s(x) \), and \( f_s(nx) = f_s(0) \subseteq f_s(y) \). This implies \( f_s(nx) \subseteq f_s(x) \).

Case-2: Suppose that \( x \neq 0 \) and \( y \neq 0 \). It follows that \( nx \neq 0 \). Then, \( f_s(nx) = f_s(1) = f_s(x) \) and \( f_s(nx) = f_s(1) = f_s(y) \), which implies that \( f_s(nx) \subseteq f_s(x) \).

Case-3: Suppose that \( x = 0 \) and \( y = 0 \), then clearly \( f_s(nx) \subseteq f_s(x) \). Hence \( f_s(nx) \subseteq f_s(x) \), for all \( x, y \in S \).

Now, let \( x, y \in S \). Then \( x+y \neq 0 \). If \( x+y = 0 \), then either \( x = y = 0 \) or \( x \neq 0 \), \( y \neq 0 \) and \( x \neq y \).

But, since \( f_s(x-y) = f_s(0) \subseteq f_s(x) \), for all \( x \in N \), it follows that \( f_s(x-y) = f_s(0) \subseteq f_s(x) \cup f_s(y) \).

If \( x - y \neq 0 \), then either \( x \neq 0 \), \( y \neq 0 \) and \( x \neq y \) or \( x \neq 0 \) and \( y = 0 \) or \( x = 0 \) and \( y \neq 0 \).

Assume that \( x \neq 0 \), \( y \neq 0 \) and \( x \neq y \). This follows that
\[
f_s(x-y) = f_s(1) = f_s(x) \subseteq f_s(x) \cup f_s(y).
\]

Now, let \( x \neq 0 \) and \( y = 0 \). Then \( f_s(x-y) \subseteq f_s(x) \cup f_s(y) \). Finally, let \( x = 0 \) and \( y \neq 0 \).

Then, \( f_s(x-y) \subseteq f_s(x) \cup f_s(y) \). Hence \( f_s(x-y) \subseteq f_s(x) \cup f_s(y) \), for all \( x, y \in S \).

Thus, \( f_s \) is fuzzy SU-action on N-module over U.

3.5 Theorem: Let \( f_s \) and \( f_T \) be two fuzzy SU action on N-module over U. Then \( f_s \wedge f_T \) is fuzzy soft SU-action on N-module over U.

Proof: Let \( (x_1, y_1), (x_2, y_2) \in S \times T \). Then
Note that.

3.2 Example: Assume \( U = p_3 \) is the universal set. Let \( S = Z_3 \) and \( H = \{ \begin{pmatrix} a & a \\ b & b \end{pmatrix} / a, b \in Z_3 \} \) 2x2 matrices with \( Z_3 \) terms, be set of parameters. We define fuzzy SU-action on N-module \( f_S \) over \( U = p_3 \) by

\[
\begin{align*}
&f_S(0) = p_3 \\
&f_S(1) = \{(1),(1 2),(1 3 2)) \\
&f_S(2) = \{(1),(1 2),(1 2 3),(1 3 2)}
\end{align*}
\]

We define fuzzy SU-action on N-module \( f_H \) over \( U = p_3 \) by

\[
\begin{align*}
&f_H \left( \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right) = p_3 \\
&f_H \left( \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) = \{(1),(1 2),(1 3 2)}
\end{align*}
\]

Note that \( f_S f_H \) is not fuzzy SU-action on N-module over \( U \).

3.2 Definition: Let \( f_S, g_T \) be fuzzy SU-action on N-module over \( U \). Then product of fuzzy SU-action on N-module \( f_S \) and \( g_T \) is defined as \( f_S \times g_T = h_{S\times T} \), where \( h_{S\times T}(x,y) = f_S(x) \times g_T(y) \) for all \((x, y) \in S \times T\).

3.6 Theorem: If \( f_S \) and \( g_T \) are fuzzy SU-action on N-module over \( U \). Then so is \( f_S \times g_T \) over \( U \times U \).

Proof: By definition-3.2, let \( f_S \times g_T = h_{S\times T} \), where \( h_{S\times T}(x,y) = f_S(x) \times g_T(y) \) for all \((x, y) \in S \times T\). Then for all \((x_1, y_1), (x_2, y_2) \in S \times T \) and \((n_1, n_2) = N \times N \).

\[
\begin{align*}
&h_{S\times T} \left( (x_1, y_1) - (x_2, y_2) \right) = h_{S\times T} \left( x_1-x_2, y_1-y_2 \right) \\
&= f_S(x_1-x_2) \times g_T(y_1-y_2) \\
&\subseteq (f_S(x_1) \cup f_S(x_2)) \times (g_T(y_1) \cup g_T(y_2)) \\
&= (f_S(x_1) \times g_T(y_1)) \cup (f_S(x_2) \times g_T(y_2)) \\
&= h_{S\times T}(x_1, y_1) \cup h_{S\times T}(x_2, y_2)
\end{align*}
\]

\[
\begin{align*}
&h_{S\times T} \left( (n_1, n_2)(x_2, y_2) \right) = h_{S\times T}(n_1x_2n_2y_2) \\
&= f_S(n_1x_2) \times g_T(n_2y_2) \\
&\subseteq f_S(n_1x_2) \times g_T(y_2) \\
&= h_{S\times T}(x_2, y_2)
\end{align*}
\]

Hence \( f_S \times g_T = h_{S\times T} \) is fuzzy SU-action on N-module over \( U \).

3.7 Theorem: If \( f_S \) and \( h_S \) are fuzzy SU-action on N-module over \( U \), then so is \( f_S \cap h_S \) over \( U \).

Proof: Let \( x, y \in S \) and \( n \in N \) then

\[
\begin{align*}
(f_S \cap h_S)(x-y) &= f_S(x-y) \cap h_S(x-y) \\
&\subseteq (f_S(x) \cup f_S(y)) \cap (h_S(x) \cup h_S(y)) \\
&= (f_S(x) \cap h_S(x)) \cup (f_S(y) \cap h_S(y)) \\
&= (f_S \cap h_S)(x) \cup (f_S \cap h_S)(y)
\end{align*}
\]
\[(f_S \cap h_S)(nx) = f_S(nx) \cap h_S(nx) \subseteq f_S(x) \cap h_S(x) = (f_S \cap h_S)(x)\]

Therefore, \((f_S \cap h_S)\) is fuzzy SU-action on N-module over U.

4. SU-ACTION ON N-IDEAL STRUCTURES

4.1 Definition: Let S be an N-module and \(f_S\) be a fuzzy soft set over U. Then \(f_S\) is called fuzzy SU-action on N-ideal of S over U if the following conditions are satisfied:

1. \(f_S(x + y) \subseteq f_S(x) \cup f_S(y)\)
2. \(f_S(-x) = f_S(x)\)
3. \(f_S(x + y - x) \subseteq f_S(y)\)
4. \(f_S(n(x + y) - nx) \subseteq f_S(y)\) for all \(x, y \in S\) and \(n \in N\).

Here, note that \(f_S(x + y) \subseteq f_S(x) \cup f_S(y)\) and \(f_S(-x) = f_S(x)\) imply \(f_S(x - y) \subseteq f_S(x) \cup f_S(y)\)

4.1 Example: Consider the near –ring \(N=\{0, x, y, z\}\) with the following tables

\[
\begin{array}{c|cccc}
+ & 0 & x & y & z \\
\hline
0 & 0 & x & y & z \\
x & x & 0 & y & z \\
y & y & z & 0 & x \\
z & z & y & x & 0 \\
\end{array}
\quad
\begin{array}{c|cccc}
. & 0 & x & y & z \\
\hline
0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & x \\
y & 0 & x & y & y \\
z & 0 & x & y & z \\
\end{array}
\]

Let \(S=N\) be the parameters and \(U=\text{D}_2\), dihedral group, be the universal set. We define a fuzzy soft set \(f_S\) over U by

\(f_S(0) = \text{D}_2\), \(f_S(x) = \{e, b, ba\}\), \(f_S(y) = \{a, b\}\), \(f_S(z) = \{b\}\).

Then, one can show that \(f_S\) is fuzzy SU-action on N-ideal of S over U.

4.2 Example: Consider the near –ring \(N=\{0, 1, 2, 3\}\) with the following tables

\[
\begin{array}{c|cccc}
+ & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 1 & 2 & 3 \\
1 & 1 & 2 & 3 & 0 \\
2 & 2 & 3 & 0 & 1 \\
3 & 3 & 0 & 1 & 2 \\
\end{array}
\quad
\begin{array}{c|cccc}
. & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 0 \\
x & 0 & 1 & 0 & 1 \\
y & 0 & 3 & 0 & 3 \\
z & 0 & 2 & 0 & 2 \\
\end{array}
\]

Let \(S=N\) be the set of parameters and \(U=\text{Z}^+\) be the universal set. We define a fuzzy soft set \(f_S\) over U by

\(f_S(0) = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 17\}\)
\(f_S(1) = f_S(3) = \{1, 3, 5, 7, 9, 11\}\)
\(f_S(2) = \{1, 5, 7, 9, 11\}\)

Since \(f_S(2.3 + 1) - 2.3 = f_S(2) - 2.3 = f_S(3) - 3 = f_S(0) \not\subseteq f_S(1)\)

Therefore, \(f_S\) is not fuzzy SU-action on N-ideal over U.

It is known that if N is a zero- symmetric near-ring, then every N-ideal of S is also N-module of S. Here, we have an analog for this case.

4.1 Theorem: Let N be a zero- symmetric near-ring. Then, every fuzzy SU-action on N-ideal is fuzzy SU-action on N-module over U.

Proof: Let \(f_S\) be an fuzzy SU-action on N-ideal on S over U. Since \(f_S(n(x+y)-nx) \subseteq f_S(y)\), for all \(x, y \in S\), and \(n \in N\), in particular for \(x=0\), it follows that \(f_S(n(0+y)-n.0) = f_S(ny-0) = f_S(y) \subseteq f_S(y)\).

Since the other condition is satisfied by definition-4.1, \(f_S\) is fuzzy SU-action on N-ideals of S over U.

4.2 Theorem: Let \(f_S\) be fuzzy SU-action on N-ideal of S and \(f_T\) be fuzzy SU-action on N-ideal of T over U. Then \(f_S \cap f_T\) is fuzzy SU-action on N-ideal of \(S \times T\) over U.
5.1 Theorem: If \( f_x \) is fuzzy SU-action on N-ideal of S and \( f_y \) be fuzzy SU-action on N-ideal of T over U, then \( f_x \times f_y \) is fuzzy SU-action on N-ideal over \( U \times U \).

5.4 Theorem: If \( f_x \) and \( h_x \) are two fuzzy SU-action on N-modules of S over U, then \( f_x \cap h_x \) is Fuzzy SU-action on N-ideal over U.

5. APPLICATION OF FUZZY SU-ACTION ON N-MODULE

In this section, we give the applications of fuzzy soft image, soft pre-image, lower \( \alpha \)-inclusion of fuzzy soft sets and N-module homomorphism with respect to fuzzy SU-action on N-modules and N-ideals.

5.1 Theorem: If \( f_x \) is fuzzy SU-action on N-ideal of S over U, then \( S^f = \{ x \in S / f_x(x) = f_x(0) \} \) is a N-ideal of S.

Proof: It is obvious that \( 0 \in S^f \) we need to show that (i) \( x-y \in S^f \), (ii) \( s+x-s \in S^f \) and (iii) \( n(s+x) - ns \in S^f \) for all \( x, y \in S^f \) and \( n \in \mathbb{N} \) and \( s \in S \).

If \( x, y \in S^f \), then \( f_x(x) = f_x(0) \). By proposition-3.1, \( f_x(0) \subseteq f_x(x-y), f_x(0) \subseteq f_x(s+x-s), \) and \( f_x(0) \subseteq f_x(n(s+x)-ns) \) for all \( x, y \in S^f \) and \( n \in \mathbb{N} \) and \( s \in S \).

Since \( f_x \) is fuzzy SU-action on N-ideal of S over U, then for all \( x, y \in S^f \) and \( n \in \mathbb{N} \) and \( s \in S \).

(i) \( f_x(x-y) \subseteq f_x(0) \) and \( f_x(x-y) \subseteq f_x(s+x-s) \) and \( f_x(n(s+x)-ns) \subseteq f_x(0) \).

Hence \( f_x(x-y) = f_x(0), f_x(s+x-s) = f_x(0) \) and \( f_x(n(s+x)-ns) = f_x(0) \), for all \( x, y \in S^f \) and \( n \in \mathbb{N} \) and \( s \in S \).

Therefore \( S^f \) is N-ideal of S.

5.2 Theorem: Let \( f_x \) be fuzzy soft set over U and \( \alpha \) be a subset of U such that \( \emptyset \supseteq \alpha \subset f_x(0) \). If \( f_x \) is fuzzy SU-action on N-ideal over U, then \( f_x^{\subseteq \alpha} \) is an N-ideal of S.

Proof: Since \( f_x(0) \subseteq \alpha \), then \( 0 \notin f_x^{\subseteq \alpha} \) and \( 0 \neq f_x^{\subseteq \alpha} \subseteq S \). Let \( x, y \in f_x^{\subseteq \alpha} \), then \( f_x(x) \subseteq \alpha \) and \( f_x(y) \subseteq \alpha \). We need to show that

(i) \( x-y \in f_x^{\subseteq \alpha} \)
(ii) \( s+x-s \in f_x^{\subseteq \alpha} \)
(iii) \( n(s+x) - ns \in f_x^{\subseteq \alpha} \) for all \( x, y \in f_x^{\subseteq \alpha} \) and \( n \in \mathbb{N} \) and \( s \in S \).

Since \( f_x \) is fuzzy SU-action on N-ideal over U, it follows that

(i) \( f_x(x-y) \subseteq f_x(x) \cup f_x(y) \subseteq \alpha \)
(ii) \( f_x(s+x-s) \subseteq f_x(x) \subseteq \alpha \) and
(iii) \( f_x(n(s+x)-ns) \subseteq f_x(x) \subseteq \alpha \). Thus, the proof is completed.

5.3. Theorem: Let \( f_x \) and \( f_y \) be fuzzy soft sets over U and \( \chi \) be an N-isomorphism from S to T.

If \( f_x \) is fuzzy SU-action on N-ideal of S over U, then \( \chi(f_x) \) is fuzzy SU-action on N-ideal of T over U.

Proof: Let \( \delta_1 \), \( \delta_2 \) and \( n \in \mathbb{N} \). Since \( \chi \) is surjective, there exists \( s_1, s_2 \in S \) such that \( \chi(s_1) = \delta_1 \) and \( \chi(s_2) = \delta_2 \). Then

\[
(g_f)(\delta_1 - \delta_2) = U \{ f_x(s) / s \in S, \chi(s) = \delta_1 - \delta_2 \} = U \{ f_x(s) / s \in S, s = \chi^{-1}(\delta_1 - \delta_2) \} = U \{ f_x(s) / s \in S, s = \chi^{-1}(\chi(s_1 - s_2)) = s_1 - s_2 \} = U \{ f_x(s_1 - s_2) / s_1 \in S, \chi(s_1) = \delta_1, i = 1, 2, \ldots \} = U \{ f_x(s_1 - s_2) / s_1 \in S, \chi(s_1) = \delta_1, i = 1, 2, \ldots \} = U \{ f_x(s_1) / s_1 \in S, \chi(s_1) = \delta_1 \} \cup \{ f_x(s_2) / s_2 \in S, \chi(s_2) = \delta_2 \} = U \{ f_x(s_1) / s_1 \in S, \chi(s_1) = \delta_1 \} \cup \{ f_x(s_2) / s_2 \in S, \chi(s_2) = \delta_2 \} = (\chi(f_x))(\delta_1) \cup (\chi(f_x))(\delta_2)
\]

Also \( (g_f)(\delta_1 + \delta_2 - \delta_1) = U \{ f_x(s) / s \in S, \chi(s) = \delta_1 + \delta_2 - \delta_1 \} = U \{ f_x(s) / s \in S, s = \chi^{-1}(\delta_1 + \delta_2 - \delta_1) \} = U \{ f_x(s) / s \in S, s = \chi^{-1}(\chi(s_1 + s_2) - s_1 - s_2) = s_1 - s_2 \} = U \{ f_x(s_1 + s_2 - s_1) / s_1 \in S, \chi(s_1) = \delta_1, i = 1, 2, \ldots \} = U \{ f_x(s_1) / s_1 \in S, \chi(s_1) = \delta_1 \} \cup \{ f_x(s_2) / s_2 \in S, \chi(s_2) = \delta_2 \} = (\chi(f_x))(\delta_1) \cup (\chi(f_x))(\delta_2) = (\chi(f_x))(\delta_2)\)
Furthermore, \((\chi f_s) (n(\delta_1 + \delta_2) - n\delta_1) = \cup \{f_s(s) / s \in S, \chi(s) = n(\delta_1 + \delta_2) - n\delta_1\}
= \cup \{f_s(s) / s \in S, s = \chi^{-1}(n(\delta_1 + \delta_2) - n\delta_1)\}
= \cup \{f_s(s) / s \in S, s = n(s_1 + s_2) - ns_1\}
= \cup \{f_s(n(s_1 + s_2) - ns_1) / s_1 \in S, \chi(s_1) = \delta_i, i = 1, 2, \ldots\}
\subseteq \cup \{f_s(s_2) / s_2 \in S, \chi(s_2) = \delta_2\}
= (\chi f_s)(\delta_2).

Hence \(\chi f_s\) is fuzzy SU-action on N-ideal of \(T\) over \(U\).

5.4 Theorem: Let \(f_s\) and \(f_T\) be fuzzy soft sets over \(U\) and \(\chi\) be an N-isomorphism from \(S\) to \(T\).

If \(f_T\) is fuzzy SU-action on N-ideal of \(T\) over \(U\), then \(\chi^{-1}(f_T)\) is fuzzy SU-action on N-ideal of \(S\) over \(U\).

Proof: Let \(s_1, s_2 \in S\) and \(n \in N\). Then
\[
(\chi^{-1}(f_T))(s_1 - s_2) = f_T(\chi(s_1 - s_2))
= f_T(\chi(s_1) - \chi(s_2))
\subseteq f_T(\chi(s_1)) \cup f_T(\chi(s_2))
= (\chi^{-1}(f_T))(s_1) \cup (\chi^{-1}(f_T))(s_2).
\]

Also
\[
(\chi^{-1}(f_T))(s_1 + s_2 - s_1) = f_T(\chi(s_1 + s_2 - s_1))
= f_T(\chi(s_1) + \chi(s_2) - \chi(s_1))
\subseteq f_T(\chi(s_2)) = (\chi^{-1}(f_T))(s_2).
\]

Furthermore,
\[
(\chi^{-1}(f_T))(n(s_1 + s_2) - ns_1) = f_T(\chi(n(s_1 + s_2) - ns_1))
= f_T(n(\chi(s_1) + \chi(s_2)) - n\chi(s_1))
\subseteq f_T(\chi(s_2)) = (\chi^{-1}(f_T))(s_2).
\]

Hence, \(\chi^{-1}(f_T)\) is fuzzy SU-action on N-ideal of \(S\) over \(U\).

CONCLUSION

In this paper, we have defined a new type of N-module action on a fuzzy soft set, called fuzzy SU-action on N-module by using the soft sets. This new concept picks up the soft set theory, fuzzy theory and N-module theory together and therefore, it is very functional for obtaining results in the mean of N-module structure. Based on this definition, we have introduced the concept of fuzzy SU-action on N-ideal. We have investigated these notions with respect to soft image, soft pre-image and lower \(\alpha\)-inclusion of soft sets. Finally, we give some application of fuzzy SU-action on N-ideal to N-module theory. To extend this study, one can further study the other algebraic structures such as different algebra in view of their SU-actions.

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