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FUZZY SOFT UNION ACTION ON N-MODULE AND N-IDEAL STRUCTURES

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ABSTRACT

In this paper, we define a new concept, called soft union action (SU) on N- module structures on a fuzzy soft set. This new notions gathers fuzzy theory, soft set theory and near-ring modulo theory (N-module theory) together and it shows how a fuzzy soft set effects on N-module structure in the mean of union and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical N-module theoretic concepts for SU-action on N-module. Finally, we give the application of SU-actions on N-module theory.

Keywords: *Fuzzy set, soft set, fuzzy soft set, N-module SU-action, N-ideal SU-action, α-inclusion, pre-image, soft image.*

AMS mathematics subject classification: *03E70, 58E40,*

1. INTRODUCTION

Soft set theory was introduced in 1999 by Molodtsov [22] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 25, 28]. Moreover, Atagun and Sezgin [4] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Maji *et al.* [19] presented some definitions on soft sets and based on the analysis of several operations on soft sets Ali *et al.* [3] introduced several operations of soft sets and Sezgin and Atagun [26] studied on soft set operations as well. Furthermore, soft set relations and functions [5] and soft mappings [21] with many related concepts were discussed. The theory of soft set has also a wide-ranging applications especially in soft decision making as in the following studies: [6, 7, 23, 29]. In this paper, we define a new concept, called soft union action (SU) on N- module structures on a fuzzy soft set. This new notions gathers fuzzy theory, soft set theory and near-ring modulo theory (N-module theory) together and it shows how a fuzzy soft set effects on N-module structure in the mean of union and inclusion of sets. We then obtain its basic properties with illustrative examples and derive some analog of classical N-module theoretic concepts for SU-action on N-module. Finally, we give the application of SU-actions on N-module theory.

2. PRELIMINARIES

In this section, we recall some basic notions relevant to near-ring modules (N-modules) and fuzzy soft sets. By a near-ring, we shall mean an algebraic system (N, +, .), where
(N_1) (N, +) forms a group (not necessarily abelian)
(N_2) (N, .) forms a semi group and
(N_3) (x + y)z = xz + yz for all x, y, z ∈ N. (that is we study on right Near-ring modules)

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Throughout this paper, N will always denote right near-ring. A normal subgroup H of N is called a left ideal of N if n(s+h)-ns ∈ H for all n, s ∈ N and h ∈ I and denoted by $H \lhd_{\ell} N$. For a near-ring N, the zero-symmetric part of N denoted by N_0 is defined by $N_0 = \{n \in S / n0=0\}$.

Let (S, +) be a group and A: N×S →S, (n, s)→s. (S, A) is called N-module or near-ring module if for all x, y ∈ N, for all s ∈ S.

(i) x(ys) = (xy)s
(ii) (x+y)s = xs+ys. It is denoted by N^S. Clearly N itself is an N-module by natural operations. A subgroup T of N^S with NT⊆T is said to be N-sub module of S and denoted by $T \leq_N S$. A normal subgroup T of S is called an N-ideal of N^S and denoted by a near-ring, S and χ two N-modules. Then h: S→χ is called an N-homomorphism if s,δ ∈ S, for all n∈ N,
    (i) h(s+δ) = h(s)+h(δ) and
    (ii) h(ns) = nh(s).

For all undefined concepts and notions we refer to (24). From now on, U refers to on initial universe, E is a set of parameters P(U) is the power set of U and A, B, C⊆ E.

2.1. Definition [22]: A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A, G_A, H_A, respectively. For more details, we refer to [11, 17, 18, 26, 29, 7].

2.2. Definition [6]: The relative complement of the soft set F_A over U is denoted by F^r_A, where F^r_A: A → P(U) is a mapping given as $F^r_A(a) = U \setminus F_A(a)$, for all a ∈ A.

2.3. Definition [6]: Let F_A and G_B be two soft sets over U such that A ∩ B ≠ ∅,. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B$ = (H, C), where C = A∩B and for all c ∈ C, H(c) = F(c) ∩ G(c).

2.4. Definition [6]: Let F_A and G_B be two soft sets over U such that A ∩ B ≠ ∅,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B$ = (H, C), where C = A∩B and for all c ∈ C, H(c) = F(c) ∪G(c).

2.5. Definition [12]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set ψ (F_A) over U, where ψ (F_A): B→P(U) is a set valued function defined by ψ (F_A)(b) =∪{F(a) | a ∈ A and ψ (a) = b}, if $\psi^{-1}(b) \neq \emptyset$, = 0 otherwise for all b ∈ B. Here, ψ (F_A) is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B)$: A → P(U) is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all a ∈ A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

2.6. Definition [13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A)$: B→P(U) is a set-valued function defined by

$\psi^\star(F_A)(b)$ =∩{F(a) | a ∈ A and ψ (a) = b}, if $\psi^{-1}(b) \neq \emptyset$,
= 0 otherwise for all b ∈ B. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

2.7 Definition [8]: Let f_A be a soft set over U and α be a subset of U. Then, lower α-inclusion of a soft set f_A, denoted by $f^{\alpha}A$, is defined as $f^{\alpha}A = \{x \in A: f_A(x) \subseteq \alpha\}$

3. SU-ACTION ON N-MODULE STRUCTURES AND N-IDEAL STRUCTURES WITH FUZZY VERSION

In this section, we first define fuzzy soft union action, abbreviated as fuzzy SU-action on N-module and N-ideal structures with illustrative examples. We then study their basic results with respect to soft set operation.

3.1 Definition: Let S be an N-module and f_s be a fuzzy soft set over U, then f_s is called fuzzy SU-action on N-module over U if it satisfies the following conditions;
(FS_UN-1) f_s (x+y) ⊆ f_s(x) U f_s(y)
(FS_UN-2) f_s (-x) ⊆ f_s(x)
(FS_UN-3) f_s (nx) ⊆ f_s(x)
For all x, y ∈ S and n ∈N.

3.1 Example: Consider the near-ring module N = {0, x, y, z}, be the near-ring under the operation defined by the following table:

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

.	0	x	y	z
0	0	0	0	0
x	x	x	x	x
y	0	0	0	0
z	x	x	x	x

Let S = N and S be the set of parameters and U = $\left\{\begin{bmatrix} a & a \\ 0 & a \end{bmatrix} / a, b \in Z_6\right\}$, 2×2 matrices with Z_6 terms, is the universal set. We construct a fuzzy soft set.
f_s (0) = $\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}\right\}$, f_s (x) = $\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 0 & 3 \end{bmatrix}\right\}$, f_s (y) = $\left\{\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\right\}$, and f_s (z) = $\left\{\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}\right\}$

Then one can easily show that the soft set f_s is a fuzzy SU-action on N-module.

3.1 Proposition: Let f_s be a fuzzy SU-action on N-module over U. Then, $f_s(0) \subseteq f_s(x)$ for all x ∈ S.

Proof: Assume that f_s is fuzzy SU-action over U. Then, for all x∈S, $f_s(0) = f_s(x-x) \subseteq f_s(x) \cup f_s(-x) = f_s(x) \cup f_s(x) = f_s(x)$.

3.1Theorem: Let S be a fuzzy SU-action on N-module and f_s be a fuzzy soft set over U. Then f_s is SU-action of N-module over U if and only if

(i) $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$
(ii) $f_s(nx) \subseteq f_s(x)$ for all x,y ∈ S and n ∈ N.

Proof: Suppose f_s is a fuzzy SU-action on N-module over U. Then, by definition-3.1, $f_s(xy) \subseteq f_s(y)$ and $f_s(x-y) \subseteq f_s(x) \cup f_s(-y) = f_s(x) \cup f_s(y)$ for all x, y ∈ S

Conversely, assume that $f_s(xy) \subseteq f_s(y)$ and $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$ for all x, y ∈ S.

If we choose x=0, then $f_s(0-y) = f_s(-y) \subseteq f_s(0) \cup f_s(y) = f_s(y)$ by proposition-3.1. Similarly $f_s(y) = f_s(-(-y)) \subseteq f_s(-y)$, thus $f_s(-y) = f_s(y)$ for all y ∈ S. Also, by assumption $f_s(x-y) \subseteq f_s(x) \cup f_s(-y) = f_s(x) \cup f_s(y)$. This complete the proof.

3.2. Theorem: Let f_s be a fuzzy SU-action on N-module over U.

(i) If $f_s(x-y) = f_s(0)$ for any x, y ∈ S ,then $f_s(x) = f_s(y)$.
(ii) $f_s(x-y) = f_s(0)$ for any x, y ∈ S, then $f_s(x) = f_s(y)$.

Proof: Assume that $f_s(x-y) = f_s(0)$ for any x, y ∈ S, then

$$f_s(x) = f_s(x-y+y) \subseteq f_s(x-y) \cup f_s(y)$$
$$= f_s(0) \cup f_s(y) = f_s(y)$$

and similarly,

$$f_s(y) = f_s((y-x)+x) \subseteq f_s(y-x) \cup f_s(x)$$
$$= f_s(-(y-x)) \cup f_s(x)$$
$$= f_s(0) \cup f_s(x) = f_s(x)$$

Thus, $f_s(x) = f_s(y)$ which completes the proof .Similarly, we can show the result (ii).

It is known that if S is an N-module, then (S, +) is a group but not necessarily abelian. That is, for any x, y ∈ S, x + y needs not be equal to y + x. However, we have the following:

3.3. Theorem: Let f_s be fuzzy SU-action on N-module over U and x ∈ S. Then,

$$f_s(x) = f_s(0) \Leftrightarrow f_s(x+y) = f_s(y+x) = f_s(y) \text{ for all } y \in S .$$

Proof: Suppose that $f_s(x+y) = f_s(y+x) = f_s(y)$ for all y ∈ S. Then, by choosing y = 0,

We obtain that $f_s(x) = f_s(0)$.

Conversely, assume that $f_s(x) = f_s(0)$. Then by proposition-3.1, we have

$$f_s(0) = f_s(x) \subset f_s(y), \forall\, y \in S \ldots\ldots\ldots\ldots\ldots\ldots.. (1)$$

Since f_s is fuzzy SU-action on N-module over U, then

$$f_s(x+y) \subseteq f_s(x) \cup f_s(y) = f_s(y), \forall y \in S.$$

Moreover, for all y ∈ S

$$f_s(y) = f_s((-x)+x)+y) = f_s(-x+(x+y)) \subseteq f_s(-x) \cup f_s(x+y)$$
$$= f_s(x) \cup f_s(x+y) = f_s(x+y)$$

Since by equation (1), $f_s(x) \subseteq f_s(y)$ for all y ∈ S and x, y ∈ S, implies that x+y ∈S. Thus, it follows that $f_s(x) \subseteq f_s(x+y)$.

So $f_s(x+y) = f_s(y)$ for all y ∈ S.

Now, let x ∈ S. Then, for all x, y ∈ S

$$f_s(y + x) = f_s(y+x+(y-y))$$
$$= f_s(y+(x+y)-y)$$
$$\subseteq f_s(y) \cup f_s(x+y) \cup f_s(y)$$
$$= f_s(y) \cup f_s(x+y) = f_s(y)$$

Since $f_s(x+y) = f_s(y)$. Furthermore, for all y ∈ S

$$f_s(y) = f_s(y+(x-x))$$
$$= f_s((y+x)-x)$$
$$\subseteq f_s(y+x) \cup f_s(x)$$
$$= f_s(y+x) \text{ by equation (1).}$$

It follows that $f_s(y+x) = f_s(y)$ and so $f_s(x+y) = f_s(y+x) = f_s(y)$, for all y ∈ S, which completes the proof.

3.4 Theorem: Let S be a near-field and f_s be a fuzzy soft set over U. If $f_s(0) \subseteq f_s(1) = f_s(x)$ for all $0 \neq x \in S$, then it is fuzzy SU-action on N-module over U.

Proof: Suppose that $f_s(0) \subseteq f_s(1) = f_s(x)$ for all $0 \neq x \in S$. In order to prove that it is fuzzy SU-action on N-module over U,it is enough to prove that $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$ and $f_s(nx) \subseteq f_s(x)$.

Let x, y ∈ S. Then we have the following cases:

Case-1: Suppose that $x \neq 0$ and $y = 0$ or $x = 0$ and $y \neq 0$. Since S is a near-field, so it follows that nx=0 and $f_s(nx) = f_s(0)$. Since $f_s(0) \subseteq f_s(x)$, for all x∈S, so $f_s(nx) = f_s(0) \subseteq f_s(x)$, and $f_s(nx) = f_s(0) \subseteq f_s(y)$. This imply $f_s(nx) \subseteq f_s(x)$.

Case-2: Suppose that $x \neq 0$ and $y \neq 0$. It follows that $nx \neq 0$. Then, $f_s(nx) = f_s(1) = f_s(x)$ and $f_s(nx) = f_s(1) = f_s(y)$, which implies that $f_s(nx) \subseteq f_s(x)$.

Case-3: suppose that x= 0 and y = 0, then clearly $f_s(nx) \subseteq f_s(x)$. Hence $f_s(nx) \subseteq f_s(x)$, for all x, y ∈ S.

Now, let x, y ∈ S. Then x-y=0 or $x-y \neq 0$. If x-y = 0, then either x=y=0 or $x \neq 0$, $y \neq 0$ and x=y.

But, since $f_s(x-y) = f_s(0) \subseteq f_s(x)$, for all x ∈ N, it follows that $f_s(x-y) = f_s(0) \subseteq f_s(x) \cup f_s(y)$.

If $x - y \neq 0$, then either $x \neq 0$, $y \neq 0$ and $x \neq y$ or $x \neq 0$ and y = 0 or x = 0 and $y \neq 0$.

Assume that $x \neq 0$, $y \neq 0$ and $x \neq y$. This follows that

$$f_s(x-y) = f_s(1) = f_s(x) \subseteq f_s(x) \cup f_s(y).$$

Now, let $x \neq 0$ and y = 0. Then $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$. Finally, let x = 0 and $y \neq 0$.

Then, $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$. Hence $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$, for all x, y ∈ S.

Thus, f_s is fuzzy SU-action on N-module over U.

3.5 Theorem: Let f_s and f_T be two fuzzy SU - action on N-module over U. Then $f_s \wedge f_T$ is fuzzy soft SU-action on N-module over U.

Proof: let (x_1, y_1), $(x_2, y_2) \in S \times T$.Then

$$\begin{aligned} f_{S\wedge T}\left((x_1,y_1)-(x_2,y_2)\right) &= f_{S\wedge T}\,(x_1-x_2,\, y_1-y_2) \\ &= f_S\,(x_1-x_2) \cap f_T\,(y_1-y_2) \\ &\subseteq (f_S\,(x_1) \cup f_S\,(x_2)) \cap (f_T\,(y_1) \cup f_T\,(y_2)) \\ &= (f_S\,(x_1) \cup f_T\,(y_1)) \cap (f_S\,(x_2) \cup f_T\,(y_2)) \\ &= f_{S\wedge T}(x_1,y_1) \cap f_{S\wedge T}(x_2,y_2) \end{aligned}$$

and

$$\begin{aligned} f_{S\wedge T}\left((n_1,n_2),(x_2,y_2)\right) &= f_{S\wedge T}(n_1x_2,n_2y_2) \\ &= f_S(n_1x_2) \cap f_T\,(n_2y_2) \\ &\subseteq f_S(x_2) \cap f_T\,(y_2) \\ &= f_{S\wedge T}(x_2,y_2) \end{aligned}$$

Thus $f_s \wedge f_T$ is fuzzy SU-action on N-module over U.

Note that $f_s \vee f_T$ is not fuzzy SU-action on N-module over U.

3.2 Example: Assume U $= p_3$ is the universal set. Let S $= Z_3$ and H $= \left\{\begin{bmatrix} a & a \\ b & b \end{bmatrix} / a,b \in Z_3\right\}$ 2×2 matrices with Z_3 terms, be set of parameters. We define fuzzy SU-action on N-module f_S over U= p_3 by

$f_S(0) = p_3$
$f_S(1) = \{(1), (1\ 2), (1\ 3\ 2)\}$
$f_S(2) = \{(1), (1\ 2), (1\ 2\ 3), (1\ 3\ 2)\}$

We define fuzzy SU-action on N-module f_H over U= p_3 by

$f_H\left\{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\right\} = p_3$
$f_H\left\{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}\right\} = \{(1), (1\ 2), (1\ 3\ 2)\}$

Note that $f_s \vee f_T$ is not fuzzy SU-action on N-module over U.

3.2 Definition: Let f_S, g_T be fuzzy SU-action on N-module over U. Then product of fuzzy SU-action on N-module f_S and g_T is defined as $f_S \times g_T = h_{S\times T}$, where $h_{S\times T}(x,y) = f_S\,(x) \times g_T(y)$ for all $(x, y) \in S \times T$.

3.6 Theorem: If f_S *and* g_T are fuzzy SU-action on N-module over U. Then so is $f_S \times g_T$ over U×U.

Proof: By definition-3.2, let $f_S \times g_T = h_{S\times T}$, where $h_{S\times T}(x,y) = f_S\,(x) \times g_T(y)$ for all $(x, y) \in S \times T$. Then for all $(x_1, y_1), (x_2, y_2) \in S \times T$ and (n_1,n_2)=N×N.

$$\begin{aligned} h_{S\times T}\left((x_1,y_1)-(x_2,y_2)\right) &= h_{S\times T}\,(x_1-x_2,\, y_1-y_2) \\ &= f_S\,(x_1-x_2) \times g_T\,(y_1-y_2) \\ &\subseteq (f_S\,(x_1) \cup f_S\,(x_2)) \times (g_T\,(y_1) \cup g_T\,(y_2)) \\ &= (f_S\,(x_1) \times g_T\,(y_1)) \cup (f_S\,(x_2) \times g_T\,(y_2)) \\ &= h_{S\times T}(x_1,y_1) \cup h_{S\times T}(x_2,y_2) \end{aligned}$$

$$\begin{aligned} h_{S\times T}\left((n_1,n_2)(x_2,y_2)\right) &= h_{S\times T}(n_1x_2,n_2y_2) \\ &= f_S(n_1x_2) \times g_T\,(n_2y_2) \\ &\subseteq f_S(x_2) \times g_T\,(y_2) \\ &= h_{S\times T}(x_2,y_2) \end{aligned}$$

Hence $f_S \times g_T = h_{S\times T}$ is fuzzy SU-action on N-module over U.

3.7. Theorem: If f_S and h_S are fuzzy SU-action on N-module over U, then so is $f_S \tilde{\cap} h_S$ over U.

Proof: Let $x, y \in$ s and $n \in N$ then

$$\begin{aligned} (f_S \tilde{\cap} h_S)\,(x-y) &= f_S(x-y) \cap h_S\,(x-y) \\ &\subseteq (f_S(x) \cup f_S\,(y)) \cap (h_S\,(x) \cup h_S\,(y)) \\ &= (f_S(x) \cap h_S(x)) \cup (f_S\,(y) \cap h_S\,(y)) \\ &= (f_S \tilde{\cap} h_S)\,(x) \cup (f_S \tilde{\cap} h_S)(y) \end{aligned}$$

$$(f_S \tilde{\cap} h_S)(nx) = f_S(nx) \cap h_S(nx)$$
$$\subseteq f_S(x) \cap h_S(x)$$
$$= (f_S \tilde{\cap} h_S)(x)$$

Therefore, $(f_S \tilde{\cap} h_S)$ is fuzzy SU-action on N-module over U.

4. SU-ACTION ON N-IDEAL STRUCTURES

4.1 Definition: Let S be an N-module and f_S be a fuzzy soft set over U. Then f_S is called fuzzy SU-action on N-ideal of S over U if the following conditions are satisfied:

(i) $f_s(x+y) \subseteq f_s(x) \cup f_s(y)$
(ii) $f_s(-x) = f_s(x)$
(iii) $f_s(x+y-x) \subseteq f_s(y)$
(iv) $f_s(n(x+y)-nx) \subseteq f_s(y)$ for all $x, y \in$ S and $n \in$ N.

Here, note that

$f_s(x+y) \subseteq f_s(x) \cup f_s(y)$ and $f_s(-x) = f_s(x)$ imply $f_s(x-y) \subseteq f_s(x) \cup f_s(y)$

4.1 Example: Consider the near –ring N={0, x, y, z} with the following tables

+	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

.	0	x	y	z
0	0	0	0	0
x	0	0	0	x
y	0	x	y	y
z	0	x	y	z

Let S=N be the parameters and U= D_2 , dihedral group, be the universal set. We define a fuzzy soft set f_s over U by $f_s(0) = D_2$, $f_s(x) = \{e, b, ba\}$, $f_s(y) = \{a, b\}$, $f_s(z) = \{b\}$.

Then, one can show that f_s is fuzzy SU-action on N-ideal of S over U.

4.2 Example: Consider the near –ring N={0, 1, 2, 3} with the following tables

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

.	0	x	y	z
0	0	0	0	0
x	0	1	0	1
y	0	3	0	3
z	0	2	0	2

Let S=N be the set of parameters and U= Z^+ be the universal set. We define a fuzzy soft set f_s over U by

$f_s(0) = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 17\}$
$f_s(1) = f_s(3) = \{1, 3, 5, 7, 9, 11\}$
$f_s(2) = \{1, 5, 7, 9, 11\}$

Since $f_s(2.(3+1) - 2.3) = f_s(2.1 - 2.3) = f_s(3-3) = f_s(0) \nsubseteq f_s(1)$

Therefore, f_s is not fuzzy SU-action on N-ideal over U.

It is known that if N is a zero- symmetric near-ring, then every N-ideal of S is also N-module of S. Here, we have an analog for this case.

4.1 Theorem: Let N be a zero- symmetric near-ring. Then, every fuzzy SU-action on N-ideal is fuzzy SU-action on N-module over U.

Proof: Let f_s be an fuzzy SU-action on N-ideal on S over U. Since f_s(n(x+y)-nx) ⊆ f_s(y), for all x, y ∈ S, and n∈N, in particular for x=0, it follows that f_s(n(0+y)-n.0) = f_s(ny-0)= f_s(y)⊆ f_s(y).

Since the other condition is satisfied by definition-4.1, f_s is fuzzy SU-action on N-ideals of S over U.

4.2 Theorem: Let f_s be fuzzy SU-action on N-ideal of S and f_T be fuzzy SU-action on N-ideal of T over U. Then $f_s \wedge f_T$ is fuzzy SU-action on N-ideal of S×T over U.

4.3 Theorem: If f_s is fuzzy SU-action on N-ideal of S and f_T be fuzzy SU-action on N-ideal of T over U, then $f_s \times f_T$ is fuzzy SU-action on N-ideal over U×U.

4.4 Theorem: If f_s and h_s are two fuzzy SU-action on N-modules of S over U, then $f_S \tilde{\cap} h_S$ is Fuzzy SU-action on N-ideal over U.

5. APPLICATION OF FUZZY SU-ACTION ON N-MODULE

In this section, we give the applications of fuzzy soft image, soft pre-image, lower α-inclusion of fuzzy soft sets and N-module homomorphism with respect to fuzzy SU-action on N-modules and N-ideals.

5.1 Theorem: If f_s is fuzzy SU-action on N-ideal of S over U, then S^f={x ∈S / f_s(x)= $f_s(0)$} is a N-ideal of S.

Proof: It is obvious that 0 ∈ S^f we need to show that (i) x-y ∈ S^f, (ii) s+x-s ∈ S^f and (iii) n(s+x)-ns ∈ S^f for all x, y ∈ S^f and n∈N and s∈S.

If x,y∈ S^f, then f_s(x)= f_s(y) $= f_s$(0). By proposition-3.1, f_s(0) ⊆ f_s(x-y), f_s(0) ⊆ f_s(s+x-s), and f_s(0) ⊆ f_s(n(s+x)-ns) for all x, y ∈ S^f and n∈N and s∈S.

Since f_s is fuzzy SU-action on N-ideal of S over U, then for all x, y ∈ S^f and n∈N and s ∈ S.

(i) f_s(x-y) ⊆ f_s(x)∪ f_s(y) = f_s(0).
(ii) f_s(s+x-s)⊆ f_s(x) =f_s(0).
(iii) f_s(n(s+x)-ns) ⊆ f_s(x) =f_s(0).

Hence f_s(x-y) = f_s(0), f_s(s+x-s) = f_s(0) and f_s(n(s+x)-ns) =f_s(0), for all x, y ∈ S^f and n∈N and s ∈ S.

Therefore S^f is N-ideal of S.

5.2 Theorem: Let f_s be fuzzy soft set over U and α be a subset of U such that $\emptyset \supseteq \alpha \supseteq f_s(0)$. If f_s is fuzzy SU-action on N-ideal over U, then $f_s^{\subseteq\alpha}$ is an N-ideal of S.

Proof: Since f_s(0) ⊆ α, then 0 ∈ $f_s^{\subseteq\alpha}$ and $\emptyset \neq f_s^{\subseteq\alpha} \supseteq$ S. Let x, y ∈ $f_s^{\subseteq\alpha}$, then f_s(x) ⊆ α and f_s(y) ⊆ α. We need to show that

(i) x-y ∈ $f_s^{\subseteq\alpha}$
(ii) s+x-s ∈ $f_s^{\subseteq\alpha}$
(iii) n(s+x)-ns ∈ $f_s^{\subseteq\alpha}$ for all x,y ∈ $f_s^{\subseteq\alpha}$ and n∈N and s∈S.

Since f_s is fuzzy SU-action on N-ideal over U, it follows that

(i) f_s(x-y) ⊆ f_s(x)∪ f_s(y) ⊆ $\alpha \cup \alpha = \alpha$,
(ii) f_s(s+x-s)⊆ f_s(x) ⊆ α and
(iii) (iii)f_s(n(s+x)-ns) ⊆ f_s(x) ⊆ α. Thus, the proof is completed.

5.3. Theorem: Let f_s and f_T be fuzzy soft sets over U and χ be an N-isomorphism from S to T.

If f_s is fuzzy SU-action on N-ideal of S over U, then $\chi(f_s)$ is fuzzy SU-action on N-ideal of T over U.

Proof: Let δ_1, δ_2 and n∈ N. Since χ is surjective, there exists s_1, s_2 ∈ S such that χ(s_1) = δ_1 and χ(s_2)= δ_2. Then

$$\begin{aligned}(\chi f_s)(\delta_1 - \delta_2) &= \cup\{f_s(s) / s \in S, \chi(s) = \delta_1 - \delta_2\}\\ &= \cup\{f_s(s) / s \in S, s = \chi^{-1}(\delta_1 - \delta_2)\}\\ &= \cup\{f_s(s) / s \in S, s = \chi^{-1}(\chi(s_1 - s_2)) = s_1 - s_2\}\\ &= \cup\{f_s(s_1 - s_2) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\dots\}\\ &\subseteq \cup\{f_s(s_1) \cup f_s(s_2)) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\dots\}\\ &= \left(\{\cup\{f_s(s_1)/s_1 \in S, \chi(s_1) = \delta_1\}\} \cup \{\cup\{f_s(s_2)/s_2 \in S, \chi(s_2) = \delta_2\}\}\right)\\ &= (\chi(f_s))(\delta_1) \cup (\chi(f_s))(\delta_2)\end{aligned}$$

Also

$$\begin{aligned}(\chi f_s)(\delta_1 + \delta_2 - \delta_1) &= \cup\{f_s(s) / s \in S, \chi(s) = \delta_1 + \delta_2 - \delta_1\}\\ &= \cup\{f_s(s) / s \in S, s = \chi^{-1}(\delta_1 + \delta_2 - \delta_1)\}\\ &= \cup\{f_s(s) / s \in S, s = \chi^{-1}(\chi(s_1 + s_2 - s_1)) = s_1 + s_2 - s_1\}\\ &= \cup\{f_s(s_1 + s_2 - s_1) / s_i \in S, \chi(s_i) = \delta_i, i = 1,2,\dots\}\\ &\subseteq \cup\{f_s(s_2)/s_2 \in S, \chi(s_2) = \delta_2\}\\ &= (\chi(f_s))(\delta_2)\end{aligned}$$

Furthermore, (χf_s) $(n(\delta_1+\delta_2)-n\delta_1) = \cup \{f_s(s) / s\in S, \chi(s) = n(\delta_1+\delta_2)-n\delta_1\}$
$= \cup \{f_s(s) / s\in S, s = \chi^{-1}(n(\delta_1+\delta_2)-n\delta_1)\}$
$= \cup \{f_s(s) / s\in S, s = n(s_1+s_2)-ns_1\}$
$= \cup \{f_s(n(s_1+s_2)-ns_1) / s_i \in S , \chi(s_i) = \delta_i, i =1,2,\dots\}$
$\subseteq \cup \{f_s(s_2)/s_2 \in S, \chi(s_2) = \delta_2\}$
$= (\chi(f_s))(\delta_2)$.

Hence $\chi(f_s)$ is fuzzy SU-action on N-ideal of T over U.

5.4 Theorem: Let f_s and f_T be fuzzy soft sets over U and χ be an N-isomorphism from S to T.

If f_T is fuzzy SU-action on N-ideal of T over U, then $\chi^{-1}(f_T)$ is fuzzy SU-action on N-ideal of S over U.

Proof: Let s_1, s_2 ∈S and n∈ N. Then
$(\chi^{-1}(f_T))(s_1 - s_2) = f_T(\chi(s_1 - s_2))$
$= f_T(\chi(s_1)-\chi(s_2))$
$\subseteq f_T(\chi(s_1)) \cup f_T(\chi(s_2))$
$= (\chi^{-1}(f_T))(s_1) \cup (\chi^{-1}(f_T))(s_2)$.

Also $(\chi^{-1}(f_T))(s_1+s_2-s_1) = f_T(\chi(s_1+s_2-s_1))$
$= f_T(\chi(s_1)+\chi(s_2)-\chi(s_1))$
$\subseteq f_T(\chi(s_2)) = (\chi^{-1}(f_T))(s_2)$

Furthermore, $(\chi^{-1}(f_T))(n(s_1+s_2)-ns_1) = f_T(\chi(n(s_1+s_2)-ns_1))$
$= f_T(n(\chi(s_1)+\chi(s_2))-n\chi(s_1))$
$\subseteq f_T(\chi(s_2)) = (\chi^{-1}(f_T))(s_2)$

Hence, $(\chi^{-1}(f_T))$ is fuzzy SU-action on N-ideal of S over U.

CONCLUSION

In this paper, we have defined a new type of N-module action on a fuzzy soft set, called fuzzy SU-action on N-module by using the soft sets. This new concept picks up the soft set theory, fuzzy theory and N-module theory together and therefore, it is very functional for obtaining results in the mean of N-module structure. Based on this definition, we have introduced the concept of fuzzy SU-action on N-ideal. We have investigated these notions with respect to soft image, soft pre-image and lower α-inclusion of soft sets. Finally, we give some application of fuzzy SU-action on N-ideal to N-module theory. To extend this study, one can further study the other algebraic structures such as different algebra in view of their SU-actions.

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