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CERTAIN THRESHOLDS ON FLEXIBLE FUZZY SUBGROUPS WITH FLEXIBLE FUZZY ORDER

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ABSTRACT

In this paper, we introduce further theory of fuzzy subgroups with flexible fuzzy order. The purpose of this paper is to generalize new definition of flexible fuzzy groups with threshold and using this definition to study some properties for this subject.

Keywords: *Fuzzy subset, Fuzzy group, Flexible fuzzy group, Flexible fuzzy normal subgroups.*

SECTION-1: INTRODUCTION

Zadeh′s classical paper [16] of 1965 introduced the concepts of fuzzy sets and fuzzy set operations. Foster [5] combined the structure of a fuzzy topological space. The study of the fuzzy algebraic structures started with the introduction of concepts of fuzzy subgroups and fuzzy (left, right) ideals in the pioneering paper of Rosenfeld [11]. Anthony and Sherwood [1] redefined fuzzy subgroups using the concept of triangular norm. Several mathematicians [4,6,8] followed the Rosenfeld approach in investigating fuzzy algebra where given ordinary algebraic structure on a given set X is assumed then introducing the fuzzy algebraic structure as a fuzzy subset A of X satisfying some suitable conditions. Throughout this paper, G will denote a group and "e" will denote its identity element. Let sup, inf, card, min, max will denote the supermum, infimum, cardinality, minimum, maximum respectively. To know more of this subject, it is possible to return to Doctorate thesis of Mourad Massa'deh (Damascus University 2008). Many researchers Abd – Allahetal. [2], Chengyi [3], Dib and Hassan [4], Tang and Zhang [15], Syransu and Ruy [13], Massa'deh [10] studied the properties of groups and subgroups by the definition of fuzzy sub groups. In this paper, we generalize new definition of flexible fuzzy groups and using this definition to study some properties for this subject.

SECTION-2: PRELIMINARIES AND BASIC CONCEPTS

In this section, we recall some basic notions relevant to group theory.

2.1 Definition [16]: Let X be a set. Then a mapping $\mu: X \rightarrow [0, 1]$ is called a fuzzy subset of X.

2.2 Definition [11]: Let G be any group. A mapping $\mu: G \rightarrow [0, 1]$ is called a upper fuzzy group of G if

(i) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$,

(ii) $\mu(x^{-1}) \leq \mu(x)$ for all $x, y \in G$

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2.3 Definition [12]: A fuzzy set A is called fuzzy group of G if (FG1): $A(xy) \geq \min\{A(x),A(y)\}$(FG2): $A(x^{-1}) = A(x, q)$

2.1 Proposition: If μ is a fuzzy group of a group G having identity e, then
(i) $\mu(x^{-1})=\mu(x)$
(ii) $\mu(e) \leq \mu(x)\ \forall x\in G$

2.4 Definition [12]: Let μ be a fuzzy group of G. then "μ" is called a fuzzy normal group if
$\mu (xy) = \mu (yx)\ \forall\ x, y \in G$

2.5 Definition: Let X be a set. Then a mapping $\mu: X\rightarrow M^*([0, 1])$ is called flexible fuzzy subset of X, where $M^*([0, 1])$ denotes the set of all non empty subset [0, 1]

2.6 Definition: Let X be a non empty set and A and B be two flexible fuzzy subsets of X. Then the intersection of A and B denoted by $A\cap B$ and defined by
$A\cap B= \{\min\{a, b\}/\ a\in A(x), b\in B(x)\}$ for all $x\in X$. The union of A and B and denoted by $A\cup B$ and defined by $A\cup B= \{\max\{a, b\}/\ a\in A(x), b\in B(x)\}$ for all $x\in X$.

2.7 Definition: Let X be a groupoid that is a set which is closed under a binary relation denoted multiplicatively. A mapping is called flexible fuzzy groupoid if for all x, y ∈ X, following conditions hold:
(i) $\min \mu(xy) \leq S\{ \min \mu(x), \min \mu(y)\}$
(ii) $\max \mu(xy) \leq S\{ \max \mu(x), \max \mu(y)\}$

2.8 Definition: Let G be a group. A mapping $\mu: G\rightarrow M^*([0,1]$ is called a threshold flexible fuzzy subgroup of G if for all x, y ∈G, following conditions hold:
(i) $\min \mu(xy, \lambda) \leq S\{ \min \mu(x), \min \mu(y), \Phi\}$
(ii) $\max \mu(xy, \lambda) \leq S\{ \max \mu(x), \max \mu(y), \Phi\}$
(iii) $\min \mu(x^{-1},\lambda) \leq \min \{\mu(x), \Phi\}$
(iv) $\max \mu(x^{-1},\lambda\} \leq \max \{\mu(x) ,\Phi\}$

2.1Example: Let G = {e, a, b, c} be the kelin's four group. Define multiplication in G as follows

•	e	a	b	c
e	e	e	e	e
a	a	a	a	a
b	e	e	e	b
c	a	a	a	e

Then (G, •) is a group. Define a flexible fuzzy subset $\mu: G\rightarrow M^*([0, 1])$ by $\mu (e) = 0.42$, $\mu(a) = \mu(c) = 0.4$, $\mu(b) = 0.44$. Then μ is flexible fuzzy group of G.

Note: In definition * if $\mu: G\rightarrow [0, 1]$ then $\mu(x)\ \forall\ x\in G$ are real points in [0, 1] and also
$\min(\mu(x), \lambda) = \max (\mu(x),\Phi) = \mu(x), x\in G$.

Thus definition * reduces to definition of Rosenfield's upper fuzzy groups. So flexible fuzzy subgroup is a generalization of Rosenfeld's fuzzy group.

SECTION-3: PROPERTIES OF THRESHOLD OF FLEXIBLE FUZZY GROUPS

In this section, we discuss some properties of flexible fuzzy subset to group theory.

3.1Proposition: A flexible fuzzy subset μ of a group G is threshold of flexible fuzzy subgroup if and only if for all x, y ∈G followings are hold
(i) $\min \mu(xy^{-1},\lambda) \leq S\{\min \mu(x),\min \mu(y), \Phi\}$,
(ii)$\max \mu(xy^{-1},\lambda) \leq S\{\max \mu(x),\max \mu(y), \Phi\}$

Proof: At first let μ be threshold of flexible fuzzy subgroup of G and x, y ∈G. Then
$\min \mu(xy^{-1}, \lambda) < S \{\min \mu(x), \min \mu(y^{-1}), \Phi\} = S\{\min \mu(x), \min \mu(y), \Phi\}$ and
$\max \mu(xy^{-1}, \lambda) < S \{\max \mu(x), \max \mu(y^{-1}), \Phi\} = S\{\max \mu(x), \max \mu(y), \Phi\}$

Conversely, let μ be flexible fuzzy subset of G and given conditions hold.

Then for all $x \in G$, we have

$\min(\mu(e), \lambda) = \min\{\mu(xx^{-1}), \lambda\} \leq S\{\min \mu(x), \min \mu(x), \Phi\} = \min\{\mu(x), \Phi\}$ (1)

$\max(\mu(e), \lambda) = \max\{\mu(xx^{-1}), \lambda\} \leq S\{\max \mu(x), \max \mu(x), \Phi\} = \max\{\mu(x), \Phi\}$ (2)

So, $\min\{\mu(x^{-1}), \lambda\} = \min\{\mu(ex^{-1}), \lambda\} \leq S\{\min \mu(e), \min \mu(x), \Phi\} = \min\{\mu(x), \Phi\}$ by (1)

And

$\max\{\mu(x^{-1}), \lambda\} = \max\{\mu(ex^{-1}), \lambda\} \leq S\{\max \mu(e), \max \mu(x), \Phi\} = \max\{\mu(x), \Phi\}$ by(2). Again

$\min\{\mu(xy), \lambda\} \leq S\{\min \mu(x), \min \mu(y^{-1}), \Phi\} \leq S\{\min \mu(x), \min \mu(y), \Phi\}$ and

$\max\{\mu(xy), \lambda\} \leq S\{\max \mu(x), \max \mu(y^{-1}), \Phi\} \leq S\{\max(\mu(x), \max \mu(y)), \Phi\}$

Hence μ is a threshold of flexible fuzzy subgroup of G.

3.2 Proposition: If μ is an flexible fuzzy groupoid of a infinite group G, then μ is threshold of flexible fuzzy subgroup of G.

Proof: Let $x \in G$. Since G is finite, x has finite order, say p. then $x^p = e$, where e is the identity of G. Thus $x^{-1} = \mu^{p-1}$. Using the definition of threshold of flexible fuzzy sub groupoid we have

$\min\{\mu(x^{-1}), \lambda\} = \min\{\mu(x^{p-1}), \lambda\} = \min\{\mu(x^{p-2}), \lambda\} \leq S\{\min \mu(x^{p-2}), \mu(x), \Phi\}$.

Again

$\min\{\mu(x^{p-2}), \lambda\} = \min\{\mu(x^{p-3}, x), \lambda\} \leq S\{\min \mu(x^{p-3}), \mu(x), \Phi\}$. Then we have

$\min\{\mu(x^{-1}), \lambda\} \leq S\{\min \mu(x^{p-3}), \min \mu(x), \Phi\}$

So applying the definition of threshold of flexible fuzzy groupoid repeatedly, we have $\min\{\mu(x^{-1}), \lambda\} \leq \min\{\mu(x), \lambda\}$ Similarly we have $\max\{\mu(x^{-1}), \lambda\} \leq \max\{\mu(x), \lambda\}$

Therefore μ is a threshold of flexible fuzzy subgroup.

3.3 Proposition: The Intersection of any two threshold of flexible fuzzy subgroups is also a threshold of flexible fuzzy subgroup

Proof: Let A and B be any two threshold of flexible fuzzy subgroups of G and $x, y \in G$ then

$$\begin{aligned} \min\{(A\cap B)(xy^{-1}), \lambda\} &= S\{\min\{A(xy^{-1}), \min B(xy^{-1})\}, \Phi\} \\ &\leq S\{S\{\{\min A(x), \min A(y)\}, \Phi\}, \{\{\min B(x), \min B(y)\}, \Phi\}\} \\ &= S\{S\{\{\min A(x), \min B(x)\}, \Phi\}, S\{\min A(y), \min B(y)\}, \Phi\}\} \\ &= S\{\min(A\cap B)(x), \min(A\cap B)(y), \Phi\} \qquad (1) \end{aligned}$$

Again

$$\begin{aligned} \max\{(A\cap B)(xy^{-1}), \lambda\} &= S\{\max\{A(xy^{-1}), \max B(xy^{-1})\}, \Phi\} \\ &\leq S\{S\{\{\max A(x), \max A(y)\}, \Phi\}, \{\{\max B(x), \max B(y)\}, \Phi\}\} \\ &= S\{S\{\{\max A(x), \max B(x)\}, \Phi\}, S\{\max A(y), \max B(y)\}, \Phi\}\} \\ &= S\{\max(A\cap B)(x), \max(A\cap B)(y), \Phi\} \qquad (2) \end{aligned}$$

Hence by (1) and (2) and using proposition3.1, we say $A\cap B$ is threshold of flexible fuzzy subgroup of G.

3.4 Proposition: If A is threshold of flexible fuzzy subgroup of a group G having identity e, then $\forall x \in X$

i) $\min\{A(x^{-1}), \lambda\} = \min\{A(x), \lambda\}$ and $\max\{A(x^{-1}), \lambda\} = \max\{A(x), \lambda\}$

ii) $\min\{A(e), \lambda\} \leq \min\{A(x), \lambda\}$ and $\max\{A(e), \lambda\} = \max\{A(x), \lambda\}$.

Proof :

(i) As A is a threshold of flexible fuzzy subgroup of a group G, Then $\min A(mx^{-1}) \leq \min A(x)$

Again $\min\{A(x), \lambda\} = \min\{A((x^{-1})^{-1}), \lambda\} \leq \min\{A(x^{-1}), \lambda\}$

So $\min\{A(x^{-1}), \lambda\} = \min\{A(x), \lambda\}$

Similarly we can prove that $\max\{A(mx^{-1}), \lambda\} = \max\{A(x), \lambda\}$

(ii) $\min\{A(e), \lambda\} = \min\{A(xx^{-1}), \lambda\} \leq S\{\min A(x), \min A(x^{-1}), \Phi\}$
and $\max\{A(e), \lambda\} = \max\{A(xx^{-1}), \lambda\} \leq S\{\max A(x), \max A(x^{-1}), \Phi\}$.

3.5 Proposition: Let $\{\mu_i\}_{i=1}$ be any family of threshold of flexible fuzzy subgroup of G, Then $\mu = \bigcap_{i=1}^{n} \mu_{ii}$ is also flexible fuzzy subgroup of G

Proof: Let $x, y \in N$,

$$\min(\mu(xy),\lambda) = \min(\bigcap_{i=1}^{n} \mu(xy),\lambda) \leq S\{\min_{1\leq i\leq n}\{\min\{(\mu(x)), \min(\mu(y)), \theta\}\}\}$$
$$= S\{\min\{(\bigcap_{i=1}^{n} \mu(x)), (\bigcap_{i=1}^{n} \mu(y)), \theta\}\} = S\{\min\{\mu(x), \mu(y), \theta\}\}.$$

$$\max(\mu(xy),\lambda) = \max\left(\bigcap_{i=1}^{n} \mu(xy),\lambda\right)$$
$$\leq S\{\min_{1\leq i\leq n}\{\max\{(\mu(x)), \max(\mu(y)), \theta\}\}\}$$
$$= S\{\max_{1\leq i\leq n}\{\min\{(\mu(x)), \min(\mu(x)), \theta\}\}\}$$
$$= S\{\max\{(\bigcap_{i=1}^{n} \mu(x)), (\bigcap_{i=1}^{n} \mu(y)), \theta\}\} = S\{\max\{\mu(x), \mu(y), \theta\}\}$$

$$\min\mu(x-1,\lambda) = \min(\bigcap_{i=1}^{n} \mu(x-1),\lambda) \leq \min\{\mu(x), \theta\}$$

$$\max(\mu(x-1),\lambda) = \max(\bigcap_{i=1}^{n} \mu(x-1),\lambda) \leq \max\{\mu(x), \theta\}$$

And hence $\mu = \bigcap_{i=1}^{n} \mu_{ii}$ is also threshold of flexible fuzzy subgroup of G

3.6 Proposition: Let μ and Ω be two threshold of flexible fuzzy subgroups of G_1, G_2 respectively and let Q be a homomorphism from G_1 to G_2. Then

(i) Q(μ) is a threshold of flexible fuzzy subgroup of G_2
(ii) Q(Ω) is threshold of flexible fuzzy subgroup of G_1

Proof: It is trivial

3.1 Remark: If μ is threshold of flexible fuzzy subgroup of G and K is subgroup of G, then the restriction of μ to K(μ/K) is threshold of flexible fuzzy subgroup of K.

SECTION-4: THRESHOLD OF FLEXIBLE FUZZY NORMAL SUBGROUP WITH FLEXIBLE FUZZY ORDER

4.1 Definition: If μ is of threshold of flexible fuzzy normal subgroup of a group G, then μ is called of threshold of flexible fuzzy normal subgroup of G if for all x, y ∈G

min {μ(xy), λ} = min {μ(yx),Φ} and max{μ(xy), λ} = max {μ(yx), Φ}.

4.2 Definition: Let μ be an threshold of flexible fuzzy subgroup of G. For any x∈G, the smallest positive integer n such that μ(x^n) = μ (e) is called an threshold of flexible fuzzy order of x. If there does not exist such n, then x is said to have an infinite threshold flexible fuzzy order. We shall denote the threshold flexible fuzzy order of x by O(μ (x)).

Example: Let G = {e, a, b, ab} be the Klein four group and let A = {(e, 1/4), (a, 3 /4), (b, 3/4), (ab, 1/4)} be an threshold of flexible fuzzy group, then O (A (ab)) = 1 and O (A (a)) = 2.

4.1 Proposition: The Intersection of any two of threshold of flexible fuzzy normal subgroups of G is also a threshold of flexible fuzzy normal subgroup of G.

Proof: Let A and B be any two of threshold of flexible fuzzy normal subgroups of G. By proposition 3.3, A∩B is an of threshold of flexible fuzzy normal subgroup of G.

Let x, y ∈ G then

min{(A∩B) (xy), λ} = S {min{A(xy), Φ}, min {B(xy), Φ} by definition 4.1
= S {min {A(yx), Φ}, min{B(yx), Φ }
= min{A∩B(yx), Φ}

Similarly max{(A∩B) (xy), λ} = max {(A∩B) (yx), Φ}

This shows that A∩B is of threshold of flexible fuzzy normal subgroup of G.

4.2 Proposition: The Intersection of any arbitrary collection of threshold of flexible fuzzy normal subgroups of a group G is also a threshold of flexible normal fuzzy subgroup of G.

Proof: Let x, y $\in$ G and $\alpha \in$ G

$$\begin{aligned}\min\{A(xy^{-1}), \lambda\} &= \min\{A(\alpha^{-1}xy^{-1}\alpha), \Phi\} \text{ by definition 4.1}\\ &= \min\{A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha), \Phi\}\\ &= \min\{A(\alpha^{-1}x\alpha), A((\alpha^{-1}y\alpha)^{-1}), \Phi\}\\ &\le S\{\min\{A(\alpha^{-1}x\alpha), \Phi\}, \min\{A(\alpha^{-1}y\alpha), \Phi\}\}\\ &= S\{\min\{(A(x), A((y)), \Phi\}\end{aligned}$$

Again

$$\begin{aligned}\max\{A(xy^{-1}), \lambda\} &= \max\{A(\alpha^{-1}xy^{-1}\alpha), \Phi\} \text{ by definition 4.1}\\ &= \max\{A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha), \Phi\}\\ &= \max\{A(\alpha^{-1}x\alpha), A((\alpha^{-1}y\alpha)^{-1}), \Phi\}\\ &\le S\{\max\{A(\alpha^{-1}x\alpha), \Phi\}, \max\{A(\alpha^{-1}y\alpha), \Phi\}\}\\ &= S\{\max\{(A(x), A(y), \Phi\}\end{aligned}$$

Hence by proposition 4.1 A is threshold of flexible fuzzy normal subgroup of G.

4.3 Proposition: Let A be an threshold of flexible fuzzy subgroup of the group G. Then for any integer n and x$\in$G, we have A $(x^n) \le$ A (x).

4.4 Proposition: Let G be a group and let A be an threshold of flexible fuzzy subgroup of the group G; let x $\in$ G be of finite order k; if r $\in$ N and k are relatively prime, then A (x^r) = A (x).

Proof: Since r, k is relatively prime, then by Bizout Theorem, there exists a, b $\in$ Z such that 1 = ar + bk; therefore A (x) = A $(x^{ar + bk})$ = A$((x^{ar}.x^{bk}))$ = A $(x^{ar}) \le A(x^r) \le$ A (x). Then we get A (x) $\le$ A $(x^r) \le$ A(x), therefore A (x^r) = A(x).

4.1 Lemma: Let A be an threshold of flexible fuzzy subgroup of the group G, for x$\in$G. If A (e) = A(x^n) for some n$\in$Z, then O (A(x)) divides n.

Proof: Suppose that O(A(x)) = k, then by Euclidean Algorithm, there exists a, b $\in$ Z. Such that n = ka + b; $0 \le b < k$; thus

$A(x^b) = A(x^{n - ka}) = A(x^n(x^k)^{-a},\lambda) \le S\{\max\{A(x^n),$
$A(x^k)^{-a}, \Phi\}\} \le S\{\max\{A(e), A(x^k), \Phi\}\} = \max\{A(e), A(e), \Phi\} = A(e).$

Thus $A(x^b) \le A(e)$; also A $(x^b) \ge$ A (e), then we get A (x^b) = A(e). Then b = 0; also n =ka which is given O (A(x)) divides n.

4.5 Proposition: Let A be threshold of flexible fuzzy subgroup of the group G, and let O (A(x)) = k, such that x$\in$G. If t $\in$Z with d = (t, k), then O (A (x^t)) = k / d.

Proof: Suppose that O(A(x^t)) = n, we get A$((x^t)^{k/d})$ = A$((x^t)^{ka})$ = A (x^{ka}); for some integer a $\le$ A (x^k) = A(e)

By Lemma 4.1; n/ k /d and d = (t, k) Then there exists b, c $\in$ Z such that bt + ck = d; therefore
A (x^{nd}) = A$(x^{n (bt + ck)})$ = A$((x^{nbt} . x^{nck}), \lambda) \le S\{\max\{A((x^b)^{nt}), A((x^c)^{nk}), \Phi\}\}= S\{\max\{A((x^b)^{nt}), A(((x^c)^n)^k), \Phi\}\}\le S\{\max\{A(x^b), A(x^c)^k, \Phi\}\} \le S\{\max\{A(e), A(e,), \Phi\}\}$ = A (e,) Therefore k /nd by lemma 4.1; this means that k /d /n, consequently n = k /d.

4.6 Proposition: Let A be threshold of flexible fuzzy subgroup of the group G, let O (A(x)) = A; x $\in$ G. If k $\in$ Z such m, n relatively prime, then A(x^k) = A(x,).

Proof: Since m, k relatively prime, (m, k) = 1. Then there exists a, b $\in$ Z such that ma + kb = 1

$$\begin{aligned}A(x, \lambda) &= A(x^{ma + kb}, \lambda) = A((x^m)^a.(x^k)^b, \lambda)\\ &\le S\{\max\{A(x^m)^a, A(x^k)^b, \Phi\}\}\\ &\le S\{\max\{A(x^m), A(x^k), \Phi\}\}\\ &\le S\{\max\{A(e), A(x^k), \Phi\}\}\\ &= S\{A(x^k), \Phi\}\\ &\le A(x)\end{aligned}$$

Therefore A (x^k) = A(x).

4.2 Lemma: Let A be threshold of flexible fuzzy subgroup of the group G. Then O(A(x)) = O(A$(y^{-1}xy)$) for all x,y$\in$ G.

Proof: Let x, y$\in$G, then we have A(x^m) = A$(y^{-1}x^my)$ = A$((y^{-1}xy)^m)$, for all m ε Z. Thus O (A(x)) = O (A$(y^{-1}xy)$).

4.1 Remark: If A is not threshold of flexible fuzzy subgroup of the group G, then above lemma 4.2 is not true. For example Let G be the Dihedral group G = {e, a, b, a^2, ab, ba} and let A = {(e, 1/5), (a, 4 / 5), (b, 1/2), (a^2, 4/5), (ab, 4/5) ,(ba, 4/5)}Since O (A(b)) = 1 $\neq$ 2 = O (A(a^{-1} ba)).

4.7 Proposition: Let G be a finite group and let A, B be threshold of flexible fuzzy subgroups. If $\lambda \in G$ and A(e) = B(e). Then O (A (x))/ O(B(x)) for all $x \in G$ such that O(B (x)) is finite.

Proof:-Suppose that O (B(x)) = k, then A (e) = B(e) = B(x^n) $\leq$ A(x^n), since A(e)$\leq$ A (x^{n}) and A(e) = A (x^n). Thus, O(A(x)/n (by lemma 4.2).

4.3 Lemma: Let A be flexible fuzzy subgroup of the group G and let x, y ε G, such that (O(A(x)), O(A(y)))=1 [that is, O(A(x)), O(A(y)) relatively prime] and xy = yx. If A (e) = A (xy) then A (x) and A(y) =A(e).

Proof: Suppose that O (A(x)) = k and O (A(y)) = m, then A(e) = A(xy) $\geq$ A ($(xy)^n$) = A ((x^n y^n)), since A((x^n y^n))= A(e) but A (x^n) = A((x^n y^n y^{-n})) $\leq$ S{max{(A (x^n y^n)), A(y^{-n})}} = S{max {A(x^n y^n), A(y^n)}}=S{max {A (e), A (e)} = A(e) Therefore A(x^n) = A(y^n) = A(e), then k / n by lemma 4.1. Since k, m relatively prime [(k, m) = 1] thus k = 1 this means that A (x) = A (e); the same proof for A (y) = A (e).

4.8 Proposition: Let A be threshold of flexible fuzzy subgroup of a cyclic group G and let x, y be two generators of G, then O(A (x)) = O (A(y)).

Proof: Suppose that G is a finite cyclic group with O(G) = m, since G is generated by x and y. Then we get O(x) = O(y) = m, on the other hand y = x n; n $\in$ Z and we must have (m, n) = 1; thus
O (A (x)) = O (A (x^{n})) = O (A (y)) = n. Note if G is an infinite group, then y = x^{-1}

4.4 Lemma: If A is an threshold of flexible fuzzy subgroup of G, then max {A (e), λ} $\leq$ S {max (A(x), Φ}, for every $x \in G$.

Proof: max {A(e), λ} = max {A (x. x^{-1}), λ} $\leq$ S {max A (x), max A (x^{-1}), Φ}= S{(max A (x), max A (x)), Φ} = A (x).

4.9 Proposition: Let G be a group and A be threshold of flexible fuzzy subgroup of G. Then for any x, y $\in$G such that max A (x) $\neq$ max A (y), we have max {A (xy), λ} = S {(max A(x), max A(y)), Φ}.

Proof: Assume that max {A (y), λ} > max {A (x), λ}, then max {A (y), λ} = max {A ((x^{-1}xy), λ)} $\leq$ S {max {A (x^{-1}) , max A (xy), Φ}= S {max A (x), max A (xy)), Φ} = max {A (xy), Φ}.Also A is threshold flexible fuzzy subgroup, then max {A (xy), λ}$\leq$ S{max A (x), max A (y), Φ} = max {A (y), Φ}. Thus max {A (y), λ}$\leq$ max {A (xy), λ}$\leq$ max {A(y), λ} this implies that max {A (xy), λ} = S{max A (x), max A (y), Φ}.

CONCLUSION

We have studied in this paper the definition of the threshold of flexible fuzzy subgroup over an arbitrary group. Some proposition, lemma and examples given for it and this proposition and lemma are generalization for some properties in group theory. The concept of flexible fuzzy subset is introduced and there after we defined threshold of flexible fuzzy subgroup and a few of its properties are discussed. One can obtain the similar idea by means of vague set, rough set with some suitable threshold.

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