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ON THE STRUCTURE EQUATION $F^6 + F^4 + F^2 = 0$

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ABSTRACT

In this paper, we have studied various properties of the F-sturcture satisfying $F^6 + F^4 + F^2 = 0$. *Nijenhuis tensor, metric F-structure and kernel have also been discussed.*

Keywords: *Differnetiable manifold, projection operators, Nijenhuis tensor, metric, kernel, tangent and normal vectrors.*

1. INTRODUCTION

Let M^n be a C^∞ differentiable manifold and F be a C^∞ (1, 1) tensor defined on M^n by

(1.1) $F^6 + F^4 + F^2 = 0$

We define the operators l and m by

(1.2) $l = F^6, \quad m = I - F^6$

From (1.1) and (1.2), we have

(1.3) $l + m = I,\ l^2 = l,\ m^2 = m,\ lm = ml = 0$

$F^2 l = lF^2 = F^2, F^2 m = mF^2 = 0.$

Let

(1.4) $M = \{m - F^6, m - F^5 .... m - F, m + F, m + F^2, ...m + F^6\}$ and

(1.5) $L = \{l - F^6, l - F^5, ... l - F, l + F, l + F^2, ... \ l + F^6\}$

We study properties of some elements of M and L.

Theorem 1.1: We define the (1, 1) tensors by

(1.6) $p = m + F^3, q = m - F^3,$

(1.7) $\alpha = l + F^3, \beta = l - F^3$

(1.8) $\gamma = l + F^2, \delta = l - F^2$

Then we have

(1.9) $pq = m - l, p^2 = q^2 = I, p^2 - p - q + I = 2l$

(1.10) $\alpha^n = 2^{n-1}\alpha, \beta^n = 2^{n-1}\beta$

(1.11) $3\gamma^2 + \delta^2 = 0$

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Proof: We prove only (1.11)

Using (1.1), (1.2), (1.3) and (1.8)

(1.12) $\gamma^2=\left(l+F^2\right)\left(l+F^2\right)$

$$=l^2+lF^2+F^2l+F^4$$

$$=l+F^2+F^2+F^4$$

$$=F^6+F^4+F^2+F^2,$$ Thus

(1.13) $\gamma^2=F^2$

and

(1.14) $\delta^2=\left(l-F^2\right)\left(l-F^2\right)$

$$=l^2-lF^2-F^2l+F^4$$

$$=l-F^2-F^2+F^4$$

$$=F^6+F^4+F^2-3F^2$$

(1.15) $\delta^2=-3F^2,$

From (1.13) and (1.15) we get (1.11)

2. NIJENHUIS TENSOR

The Nijenhuis tensor corresponding to *F*, *l* and *m* be denoted by $\underset{F}{N}, \underset{l}{N}, \underset{m}{N}$ respectively and defined by

(2.1) $\underset{F}{N}\left(X,Y\right)=\left[FX,FY\right]+F^2\left[X,Y\right]-F\left[FX,Y\right]-F\left[X,FY\right]$

(2.2) $\underset{l}{N}\left(X,Y\right)=\left[lX,lY\right]+l^2\left[X,Y\right]-l\left[X,Y\right]-l\left[X,lY\right].$

(2.3) $\underset{m}{N}\left(X,Y\right)=\left[mX,mY\right]+m^2\left[X,Y\right]-m\left[mX,Y\right]-m\left[X,mY\right]$

Theorem 2.1: For the structure *F* satisfying (1.1), we have

(2.4) $\underset{F^2}{N}\left(mX,mY\right)=F^4\left[mX,mY\right]$

(2.5) $F^2\underset{F^2}{N}\left(mX,mY\right)=l\left[mX,mY\right]$

(2.6) $\underset{l}{N}\left(mX,mY\right)=l\left[mX,mY\right]$

(2.7) $\underset{m}{N}\left(lX,lY\right)=m\left[lX,lY\right]$

(2.8) $\underset{l}{N}\left(lX,mY\right)=\underset{m}{N}\left[mX,lY\right]=0$

Proof: Using (1.2) and (1.3) in (2.1), (2.2), (2.3) we get the results.

3. METRIC F-STRUCTURE

Let the Riemannian metric *g* is such that

(3.1) $'F\left(X,Y\right)=g\left(FX,Y\right)$ is symmetric then

(3.2) $g\left(FX,Y\right)=-g\left(X,FY\right)$

(3.3) $'m\left(X,Y\right)=g\left(mX,Y\right)=g\left(X,mY\right)$

Theorem 3.1: For the F-structure satisfying (1.1), we have

(3.4) $g\left(F^3X,F^3Y\right)=g\left(X,Y\right)-{}'m\left(X,Y\right)$

Proof: Using (1.2), (1.3) (3.2) and (3.3) we get the results.

Definition 3.1: Let us define

(3.5) $Ker\ F=\left(X:FX=0\right)$

Theorem 3.2: For the F-structure satisfying (1.1) we have

(3.6) $Ker\ F^2=Ker\ F^4=Ker\ F^6$

Proof: From (1.1), we have $F^8=F^2, F^{10}=F^4, F^{12}=F^6$

Let $X\in Ker\ F^2 \Rightarrow F^2X=0$

$\Rightarrow F^{10}X=0$

$\Rightarrow F^4X=0$

$\Rightarrow X\in Ker\ F^4$

Thus

(3.7) $Ker\ F^2\subseteq Ker\ F^4$

Now let

$X\in Ker\ F^4 \Rightarrow F^4X=0$

$\Rightarrow F^8X=0$

$\Rightarrow F^2X=0$

$\Rightarrow X\in Ker\ F^2$

Thus

(3.8) $Ker\ F^4\subseteq Ker\ F^2$

From (3.7) and (3.8), we get

(3.9) $Ker\ F^2=Ker\ F^4,$

Proceeding similarly we get $Ker\ F^4=Ker\ F^6$ and hence (3.6).

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