International Research Journal of Pure Algebra-6(12), 2016, 469-475
Available online through www.rjpa.info ISSN 2248–9037

RELATIVE COHERENT MODULES OVER ENDOMORPHISM RINGS

WEI HAN*
Department of Mathematics Teaching, Longqiao,
College of Lanzhou University of Finance and Economics, Lanzhou, China.

RUITONG LI
FuLi West Road Primary School, Lanzhou, China.

(Received On: 16-12-16; Revised & Accepted On: 27-12-16)

ABSTRACT

Let M_R is a right R-module over a ring R with $S = End(M_R)$, n is a nonnegative integer; L is a class of right R-module .we study the coherence of the left S-module $_sM$ relative to L class, many extending known results.

Key Words: *L-n-M-flat module; L-n-Mittag-Leffler module; L—n-coherent module.*

1. INTRODUCTION

Throughout this article, all rings are associative with identity and all modules are unitary. For a ring R, we write Mod-R for the category of all right R-modules. M_R $\left({}_RM\right)$ denotes a right (left) R-module. As usual, E(M) denotes the injective envelope of M. For a module M_R, we denote by S=End $\left(M_R\right)$ the endomorphism ring of M_R and by Add M_R (resp. add M_R) the category consisting of all modules isomorphic to direct summands of (finite) direct sums of copies of M_R. The category consisting of all modules isomorphic to direct summands of direct products of copies of M_R is denoted by Prod $\left(M_R\right)$.

We first recall some known notions and facts which we need in the later sections.

Following T.Y.lam (1998), a flat right R-module have the following equivalent characterization:

Every homomorphism $f: K \to N$ with K finitely presented factors through a finitely generated projective module. The equivalent characterization has been investigated by many authors. τ-flat left R-module was intruded and studied in N.Q. Ding et.al (1993). A left R-module N is τ-flat module if every homomorphism $f: K \to N$ with K τ-finitely presented factors through a finitely generated free module; or equivalently ,if for anyτ-finitely presented left R-module P and homomorphism $f: P \to M$, there is a finitely generated free module F and homomorphism $g: P \to F$, $h: f \to M$ such that $f = hg$ .in Li.Xi. Mao et.al (2007), a left R-module N is τ-M -flat module if every homomorphism $f: K \to N$ with K τ-finitely presented factors through a module of add M_R .

Clarke (1976) called M_R an R-Mittag-Leffler module if the canonical map $M \otimes R^J \to M^J$ is a monomorphism for every set J, or equivalently, if for every finitely generated submodule N of M, the inclusion $N \to M$ factors through a finitely generated right R-module (see Goodearl, 1972, Theorem 1 or Clarke, 1976, Theorem 2.4). The concept of R-Mittag-Leffler modules was called finitely pure-projective modules by Azumaya (see Azumaya, 1987, Note added in proof, p.134). A right R-module N isτ-Mittag-Leffer(see Li.Xi.Mao and N.Q.Ding 2007) if every homomorphism $f: K \to N$ with K τ-finitely presented factors through a finitely presented right R-module .For further concepts and notations about τ-Mittag-Leffer R-module, we refer the reader to Li Xi Mao *et.al* (2007).

Corresponding Author: Wei Han*

Let $\tau =$(F, C) of classes of right R-modules is a cotorsion theory. A right R-module is τ-finitely generated if there is a finitely generated submodule M' of M such that $M / M' \in \tau.$ A right R-module is τ-finitely presented if there exists an exact sequence $0 \to K \to F \to M \to 0,$ where F is finitely generated free and K is τ -finitely generated. Let n is a nonnegative integer. A right R-module is τ -n-finitely presented if there exists an exact sequence $0 \to K_n \to F_{n-1} \to ... \to F_2 \to F_1 \to M \to 0.$ where F_i is finitely generated free and K is τ -finitely generated.

The main purpose of this article is to extend the above mentioned fact to a more general setting. In section 2,we introduce the concept of L-finitely generated, L-finitely presented, by expending the concept of $\tau - M$ -flat module and τ -mittag-leffler module, we got the concept of L-n-M-flat module and L-n-Mittag-Leffler module, given some basic properties of the above concepts.

2. DEFINITION AND GENERAL RESULTS

In this section first we define L-n-M-flat module and L-n-Mittag-Leffler module, study the basic properties of them.

Definition 2.1: Let L is a class of module, M_R is a right R-module, n is non-negative integer. M_R is L-finitely generated if M_R have a finitely generated L-dense submodule. Or equivalently, there exist a finitely generated submodule N of M. such that $M / N \in L.$

A right R-module M is L-finitely presented if there exist an exact sequence $0 \to K \to F \to M \to 0,$ where F is finitely generated free and K is L-finitely generated.

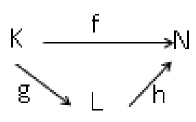
A right R-module M is L-n-presented if there exists a sequence $0 \to K_n \to F_{n-2} \to ... \to F_2 \to F_1 \to M \to 0.$ where $F_i\, F_i$ is finitely generated free and K_n is L-finitely generated.

Remark 2.1:

(1) If M is finitely generated (or finitely presented, n-presented, L-n-presented).
(2) Every L-finitely presented module (L-n-presented module) is finitely generated (n-1-presented).
(3) If L= {0}, then M is L-finitely generated (L-finitely presented L-n-presented) if and only if M is finitely generated (finitely presented, n-presented).
(4) If L=R-MOD, then M is L-finitely presented (L-n-presented) if and only if M is finitely generated (n-1-presented).

Definition 2.2: Let M_R is a right R-module, L is a class of module, n is non-negative integer.

A right R-module N is L-n-M-flat (n-M-flat) if every homomorphism $K \to N$ factor through a module of add M_R, where K is L-n-presented (n-presented). Equivalently there is $L \in addM_R$ and homomorphism g,h, such that the diagram



COMMENTS

A right R-module N is L-n-Mittag-Leffler if every homomorphism $K \to N$ factor through a n-presented module, where K is L-n-presented.

Remark 2.2:

(1) By definitions, the class of L-n-M-flat right R-module is closed under direct summands and finite direct sums. L-n-M-flat R-module are always n-M-flat.
(2) If $N \in addM_F.$ then N is L-n-M-flat. The converse holds if N is L-n-finitely presented.
(3) A L-n- R_R -flat right R-module is L-n-Mittag-Leffler module.
(4) Let L={0}, then every right R-module is L-n-Mittag-Leffler module.

It is clear that L-n-Mittag-Leffler are generalizations of both R-Mittag-Leffler module [9] and τ -M-flat module [12]. The following proposition is also easy to verify.

© 2016, RJPA. All Rights Reserved

Proposition 2.1: Let N be a right R-module. Then:
(1) N is L-n-M-flat if and only if N is both n-M-flat and L-n-Mittag-Leffler for a n-presented M;
(2) N is n-presented if and only if N is both L-n-presented module and L-n-Mittag-Leffler.
(3) Every right R-module is L-n-Mittag-Leffler if and only if every L-n-present module is n-presented.

Proof: by definitions, it is clearly.

In [12] a right R-module epimorphism $f: L \to N$ is called n-pure if for any n-presented P $f_*: Hom_R(P,L) \to Hom_R(P,N)$ is epic. In [12], aright R-module epomorphism $f: L \to N$ is $\tau - Pure$ if for any $\tau -$finitely presented P, $f_*: Hom_R(P,L) \to Hom_R(P,N)$ is epic. For L-n-presented module, we gave the definition of L-n-pure.

Definition 2.3: an R-module epimorphism $f: L \to N$ is called L-n-pure if for any L-n-presented P, $f_*: Hom_R(P,L) \to Hom_R(P,N)$ is epic.

It is clear that an L-n-pure epimorphism is n-pure. But a n-pure epimorphism is not L-n-pure. We have the following proposition.

Proposition 2.2: Let L be a class. The following are equivalent for a right R-module N.
(1) N is L-n-Mittag-Leffler;
(2) Every n-pure epimorphism $f: L \to N$ is L-n-pure;
(3) There exists a L-n-pure epimorphism $f: L \to N$ with L is L-n-Mittag-Leffler;
(4) Given a n-pure epimorphism $f: L \to C$ and homeomorphisms $h: N \to C$, $\alpha: P \to N$ with P L-n-presented, there exists a homomorphism $\beta: P \to L$ such that $f\beta = h\alpha.$

Proof:

$(1) \Rightarrow (2):$ Let $f: L \to N$ be a n-pure epimorphism. Assume that P is L-n-presented, and α is any homomorphism. By (1) there exist a n-presented right R-module H, $g: P \to H$ and $h: H \to N$ such that $\alpha = hg$ Since f is n-pure, and H is n-presented, there exists $\beta: H \to L$ such that $f\beta = h.$ So $\alpha = f(\beta g).$ and (2) follows.

$(2) \Rightarrow (3):$ Let f: L → L, for L is L-n-Mittag-Leffler. It is clear that f is n-pure epimorphism. By (2) f is L-n-pure epimorphism, and (3) follows.

$(1) \Rightarrow (3):$ is easy to verify.

$(2) \Rightarrow (4):$ is clear.

$(4) \Rightarrow (2):$ holds by letting $C = N$ and h be the identity map.

Corollary 2.1: Let L be a class of module. The following are equivalent for a right R-module N:
(1) N is L-n- R_R -flat;
(2) Every epimorphism $f: L \to N$ is L-n-pure;
(3) There exists a L-n-pure epimorphism $f: L \to N$ with L is L-n- R_R -flat;
(4) Given a n-pure epimorphism $f: L \to C$ and homeomorphisms $h: N \to C$, $\alpha: P \to N$ with P L-n-presented, there exists a homomorphism $\beta: P \to L$ such that $f\beta = h\alpha.$

In [12], every pure submodule of τ -flat R-module is τ -flat. In [12], if M is pure projective, then pure submodule of τ -M-flat R-module is τ -M-flat. For L-n-M-flat, we consider same question.

Proposition 2.3: Let M be a right R-module. Then:
(1) Every pure submodule of a L-n-M-flat right R-module is L-n-M-flat whenever M_R is pure-projective.
(2) Every pure submodule of a L-n-Mittag-Leffler right R-module is L-n-Mittag-Leffler

Proof: (1) Let N be a pure submodule of a L-n-M-flat right R-module and $f: L \to N$ the inclusion.

For any L-n-presented right R-module P and any homomorphism $f: P \to N$, since L is L-n-M-flat, there are $Q \in addM_R$ and $g: P \to Q$ and $h: Q \to L$ such that $jf = hg$. In [12] there is pure epimorphism $\varphi: H \to L$ with H pure-projective, and we have the pullback diagram of j and φ:

$$\begin{array}{ccccccccc} 0 & \longrightarrow & K & \longrightarrow & H & \longrightarrow & L/N & \longrightarrow & 0 \\ & & \alpha\downarrow & & \varphi\downarrow & & \| & & \\ 0 & \longrightarrow & N & \xrightarrow{i} & L & \xrightarrow{\pi\varphi} & L/N & \longrightarrow & 0 \end{array}$$

Since Q is pure-projective and φ is pure, there exists $I:Q\to H$ such that $h=\varphi I.$ Therefore we have $\pi\varphi Ig=\pi hg=\pi jf=0.$ which implies $Ig(P)\subseteq K.$ Since P is finitely generated, so is $Ig(P)$. Note that j and φ are pure, so λ is pure. On the other hand, since H is pure-projective, we get a homomorphism $k:H\to K$ such that $kIg(p)=Ig(p)$ for all $p\in P$. Let $\beta=\alpha kI,$ then $\beta\in \hom_R(Q,N),$ and for all $p\in P$,
$\beta g(p)=j\alpha k\lg(p)=\varphi\lg(p)=h\ (pg)=jf(p)=f(p).$ .then $f=\beta g.$ Thus N is L-n-M-flat (2) can be proven in a similar way as in the proof of (1).

Let A, B and M be right R-modules with $S=End(M_R)$. There is a natural homomorphism

$\sigma=\sigma_{A,B}:Hom_R(M,A)\otimes_s Hom_R(B,M)\to Hom_R(B,A)$

defined via $\sigma(f\otimes g)(b)=f(g(b))$ for $f\in Hom_R(M,A), g\in Hom_R(B,M), b\in B.$

It is easy to check that $\sigma_{A,B}$ is an isomorphism if $A\in add(M_R)$ or $B\in add(M_R).$

In [11], N is τ-flat R-module isτ-flat if and only if $\sigma_{p,N}$ is isomorphism for any $\tau-$finitely presented P. For L-n-M-flat, we consider same question.

Proposition 2.4: Let M and A be right R-module, L be a class of modules, the following are equivalent:
(1) A is L-n-M-flat.
(2) $\sigma_{A,B}$ is epimorphism, for any L-n- finitely presented right R-module B.

Proof: $(1)\Rightarrow(2)$: Let B be any L-n-presented right R-module, $f\in Hom_R(B,A).$ By (1), f factors through a right R-module M^n, i.e., there exist $g:B\to M^n$ and $h:M^n\to A$ such that $f=hg.$ Let $\pi_i:M^n\to M$ be the i^th projection, i=1, 2,⋯, n. Put $f_i=h\lambda_i$ and $g_i=\pi_i g.$ It is easy to check that $f=\sigma_{A,B}\left(\sum_{i=1}^n f_i\otimes g_i\right),$ i.e., $\sigma_{A,B}$ is an epimorphism.
$(2)\Rightarrow(1)$: Let B be L-n-presented right R-module and $f\in Hom_R(B,A).$ By (2), there exist $f_i\in Hom_R(B,A)$ and $g_i\in Hom_R(B,M),\ i=1,2,...,n$ such that $f=\sigma_{A,B}\left(\sum_{i=1}^n f_i\otimes g_i\right).$ Define $g:B\to M^n$ via $g(b)=(g_1(b),g_2(b),...g_n(b))$ for $b\in B$ and $h:M^n\to A$ via $h(m_1,m_2,...m_n)=\sum_{i=1}^n f_i(m_i)$ for $m_i\in M.$ Then $f=hg$ and (1) follows.

Proposition 2.5: Let M be a projective right R-module and $0\to A\to B\to C\to 0$ a right R-module exact sequence, L be a class of module. If A and C are L-n-M-flat, then B is L-n-M-flat.

Proof: Let N be a L-n-presented right R-module, $U=Hom_R(N,M).$ Since M is projective right R-module, so we have the following commutative diagram:

$$\begin{array}{ccccc} Hom_R(M,N)\otimes U & \longrightarrow & Hom_R(M,B)\otimes U & \longrightarrow & Hom_R(M,C)\otimes U \\ \sigma_{A,N}\downarrow & & \sigma_{B,N}\downarrow & & \sigma_{C,N}\downarrow \\ Hom_R(N,A)\otimes U & \longrightarrow & Hom_R(N,B)\otimes U & \longrightarrow & Hom_R(N,C)\otimes U \end{array}$$

Since A and C are L-n-M-flat, $\sigma_{A,N},\sigma_{c,N}$ are epimorphism by Proposition 2.4, thus $\sigma_{c,N}$ is epimorphism, and so B is L-n-M-flat.

3. RELATIVE COHERENT MODULE

Definition 3.1: Let M_R be a right R-module, L be a class of module, n is a nonnegative integer, $S = End\left(M_R\right)$. ${}_sM$ is called L-n-coherent if M_R is L-(n+1)-presented and ${}_sHom_R\left(A,M\right)$ is a finitely generated left S-module for any L-(n+1)-presented right R-module A.

Remark 3.1: By [2, lemma3], ${}_sM$ is L-n-coherent if and only if M_R is L-(n+1)-presented and any L-(n+1)-presented right R-module has an $add\left(M_R\right)-$ preenvelope. So it follows that ${}_sM$ is L-n-coherent if and only if M_R is L-(n+1)-presented and any L-(n+1)-presented right R-module has an L-n-M-flat-preenvelope.

For a L-n-coherent module, it is a promotion about n-coherent ring. Then we discusses relationship of these concepts.

Lemma 3.1: Let M and A be right R-module, n is non-negative integer, then:
(1) ${}_sHom_R\left(A,M\right) \in n-$copres ${}_sM$, for any n-presented right R-module A, $n \in N$.
(2) ${}_sHom_R\left(A,M\right) \in n-$copres ${}_sS$, for any $A \in n-presM_R$.

Proof: (1) the conclusion is hold by Angeleri-Hugel (2003,lemma4), when n=1.
Assuming the truth of the result for some n, where $n \le k$. If A_R is (k+1)-presented, then $0 \to K \to R^n \to A \to 0$, where K is (k+1)-presented, so
$0 \to Hom_R\left(A,M\right) \to Hom_R\left(R^n,M\right) \to Hom_R\left(K,M\right) \to 0$. ${}_sMHom_R\left(A,M\right) \in \left(k+1\right)-$ copres ${}_sM$ by $Hom_R\left(R^n,M\right) \cong M^n$ and $Hom_R\left(K,M\right) \in k-copres\ {}_sM$ (2) can be proven in a similar way as in the proof of (1).

Lemma 3.2: S is left coherent ring if and only if any $A \in \left(n+1\right)-presM_R$ has a $addM_R$ preenvelope.

Proof: let $A \in \left(n+1\right)-copres_sS$, then there is an exact sequence $0 \to A \to S^{n_0} \to S^{n_1} \to ... \to S^{n_n} \to V \to 0$ where V is finitely copresented. So V is n-presented. By S is left n-coherent ring, and V is (n+1)-presented, A is finitely generated. i.e. S is left n-coherent ring if and only if any module of $\left(n+1\right)-copres_sS$ is finitely generated.
$Hom_R\left(B,M\right) \in \left(n+1\right)-copres_sS$ by any $B_R \in \left(n+1\right)-presM_R$, so $Hom_R\left(B,M\right)$ is finitely generated. By [2, lemma3] B has an $addM_R$ preenvelope.

Lemma 3.3: If ${}_sM$ is n-coherent module, then every (n+1)-presented module has an $addM_R$ preenvelope. If M_R is (n+1)-presented right R-module and ${}_sM$ is n-presented left S-module, vice versa.

Proof: let A_R is (n+1)-presented, then $Hom_R\left(A,M\right) \in \left(n+1\right)-copres\ {}_sM$ by lemma 2.1, so there is an exact sequence $0 \to Hom_R\left(A,M\right) \to M^{n_1} \to M^{n_2} \to ... \to M^{n_n} \to L \to 0$. for $n_i \in N$. $L \in cogen\ {}_sM$. By assuming that L is n-presented, so L is (n+1)-presented. Hence $Hom_R\left(A.M\right)$ is finitely generated, thus A has an $addM_R$ preenvelope.

Otherwise, if M_R is (n+1)-presented, and every (n+1)-presented module has an $addM_R$ preenvelope, then any $A \in \left(n+1\right)-presM_R$ has an $addM_R$ preenvelope. S is left n-coherent ring by lemma3.2. So ${}_sM$ is n-coherent module by ${}_sM$ is n-presented.

Proposition 3.1: Let M_R is (n+1)-presented right R-module and ${}_sM$ is n-presented left S-module. If ${}_sM$ is L-n-coherent module, then ${}_sM$ is n-coherent module. ${}_sM$ is n-coherent module if and only if S is left n-coherent ring.

Proof: it holds by lemma3.2 and lemma 3.3.

© 2016, RJPA. All Rights Reserved

Remark 2.2: let L={0}, then ${}_sM$ is L-n-coherent if and only if ${}_sM$ is n-coherent and M_R is n-presented.

For L-n-coherent, it can be describe by L-n-flat and L-n-Mittag-Leffler, so we have the following theorem.

Theorem 3.3: Let M_R be (n+1)-presented. $S = End\left(M_R\right).$ Then the following are equivalent:

(1) ${}_sM$ is L-n-coherent;
(2) Every right R-module has a L-n-flat preenvelope;
(3) All direct products of copies of M_R are L-n-M-flat;
(4) All direct products of L-n-M-flat right R-module are τ-n-M-flat;
(5) ${}_sM$ is n-coherent and all direct products of copies of M_R are L-n-Mittag-Leffler
(6) ${}_sM$ is n-coherent and all direct products of N_i with $N_i \in AddM_R$ are L-n-Mittag-Leffler
(7) The right R-module $Hom_R\left(P,M\right)$ is L-n-M-flat for any projective left S-module P.

Proof: $\left(2\right)\Rightarrow\left(1\right),\left(4\right)\Rightarrow\left(3\right),$ and $\left(6\right)\Rightarrow\left(5\right)$ are trivial.

$\left(3\right)\Rightarrow\left(1\right)$: Let A be a L-n-presented right R-module. For every index set I, we have the following commutative diagram:

$$\begin{array}{ccc} Hom_R\left(M,M^I\right)\otimes_s Hom_R\left(A,M\right) & & \\ \varphi\downarrow & \searrow^{\sigma_{M^I,A}} & \\ \left(Hom_R\left(A,M\right)\right)^I & \xrightarrow{\theta} & Hom_R\left(A,M\right) \end{array}$$

Where θ is an isomorphism, and φ is a canonical homomorphism. By Proposition 2.4, $\sigma_{M^I,A}$ is epic since M^I is L-n-M-flat. Thus φ is epic, and hence $Hom_R\left(A,M\right)$ is a finitely generated left S-module by [13, lemma 13.1, P41].

$\left(1\right)\Rightarrow\left(4\right)$: let M_i be a family of L-n-M-flat right R-modules and N any τ-finitely presented right R-module. For any homomorphism $f_i : F_i \to M_i,$ since M_i is L-n-M-flat, there exist $F_i \in addM_R$ and homomorphism $g_i : N \to F_i,\ h_i : F_i \to M_i,$ such that $f_i : h_i g_i.$ Since N has an $addM_R$-preenvelope $f : N \to F$ by (1), there is $k_i : F \to F_i,$ such that $g_i : k_i f \to M_i$ Hence $f_i = \left(h_i k_i\right) f.$ It follows that the sequence $Hom_R\left(F,M_i\right) \to Hom_R\left(N,M_i\right) \to 0$ is exact. Thus we get the exact sequence $\left(Hom_R\left(F,M_i\right)\right)^I \to \left(Hom_R\left(N,M_i\right)\right)^I \to 0$

Note that $\left(Hom_R\left(F,M_i\right)\right)^I \cong Hom_R\left(F,M_i^I\right)$ and $\left(Hom_R\left(N,M_i\right)\right)^I \cong Hom_R\left(N,M_i^I\right).$, thus every homomorphism from N to M_i^I factors through F. So (4) follows.

$\left(1\right)\Rightarrow\left(5\right)$: let N be any right R-module. By [lemma5.3.12] there is a cardinal number $\aleph_\alpha$ such that for any R-homomorphism $f : N \to L$ with L L-n-M-flat. There is a pure submodule Q of L such that $Card\left(Q\right) \le \aleph_\alpha$ and $f\left(N\right) \subseteq Q$. Q is L-n-m-flat by Proposition 2.3, and so N has a L-n-M-flat preenvelope by (5) and [14, Proposition 6.2.1].

$\left(1\right)\Rightarrow\left(5\right)$: ${}_sM$ is coherent by Proposition 3.1.By the preceding proof, thus all the products of copies of M_R are L-n-M-flat, and hence L-n-Mittag-Leffler by Proposition 1.1 since M_R is (n+1)-presented.

$\left(5\right)\Rightarrow\left(1\right)$: We shall show that any L-n-presented right R-module has an $addM_R$-preenvelope . Let N_R be L-n-presented. Then the product map $f : N \to M^J$ induced by all maps in $J = Hom_R\left(N,M\right)$ is a $prod\left(M\right)$-preenvelope. Thus, by (5), there exist a n-presented right R-module L and homomorphism $g : N \to L,\ h : L \to M^J$

such that $f = hg$. Note that L has an $addM_R$ -preenvelope $k : L \to M^R$ since $_sM$ is coherent. It is easy to verity that $kg : N \to M^n$ is an $addM_R$ -preenvelope of N.

$(5) \Rightarrow (6)$: Let $\{N_i\}_{i\in I} \subseteq addM_R$ with I an index set. Then N_i is a direct summand of $M^{(J_i)}$ for some index set J_i. Since $M^{(J_i)}$ is a pure submodule of M^{J_i} by [7, lemma 1(1)], N_i is pure in M^{J_i}. Thus $\prod_{i\in I} N_i$ is a pure submodule of $\prod_{i\in I} M^{J_i}$ by [7, lemma1 (2)]. So the result follows from Proposition 2.3(2).

$(7) \Rightarrow (3)$: is obvious by choosing P to be S^I for any index set I.

By specializing Theorem 3.3 to the case L={0}, we have the following corollary.

Corollary 3.4: Let M_R be n-presented. $S = End(M_R)$. Then the following are equivalent:

(1) $_sM$ is n-coherent;
(2) Every right R-module has an n-M-flat-preenvelope.
(3) All direct products of copies of M_R are n-M-flat.
(4) All direct products of n-M-flat right R-module are n-M-flat.
(5) The right R-module
(6) $Hom_s(P, M)$ is n-M-flat for any projective left S-module P.

REFERENCES

1. F.W. Anderson, K.R. Fuller, Rings and Categories of Modules. [M]New York: Springer-Verlag, (1974).
2. L. Angeleri-Hgeul, Endocoherent modules [J], Pacific J. Math. (2003)212(1):1-11
3. G. Azumaya, Finite splitness and finite projectivity. [J], J. Algebra. (1987)106:114-134.
4. G. Azumaya, Some characterizations of regular modules. [J], Publ. Mat. (1998)34:241-248.
5. V. Camillo, Coherence for polynomial rings. [J], J. Algebra. (1990)132:72-76.
6. S.U. Chase, Direct products of modules. [J], Trans. Amer. Math. Soc. (1960)97,457-473.
7. T.J. Cheatham, D.R. Stone, Flat and projective character modules. [J], Proc. Amer. Math. Soc. (1981) 81(2): 175-177.
8. J. Chen, N. Ding, On n-coherent rings. [J], Comm. Algebra. (1996)24:3211-3216.
9. T.G. Clarke, On $\mathcal{N}_{-1}$-Projective Modules. [J], Ph.D. thesis. Kent State University.(1973)
10. D. Zhou, On n-coherent rings and (n,d)-rings.[J], Comm. Algebra. (2004)32:2425-2441.
11. N.Q. Ding, J.L. Chen, Relative coherence and preenvelopes. [J], Manuscripta Math.(1993)81:243-262
12. L.X. Mao, N.Q. Ding, Relative flatness, mittag-lefffler modules, and endocohence. [J], Comm. Algebra. (2006)34:3281-3299.
13. B. Stenstrom, Rings of Quotients. [M], Berlin-Heidelberg-New York: Springer-Verlag, (1975)
14. E.E. Enochs, O.M.G. Jenda, Relative Homological Algebra. [M], Berlin- New York: Walter de Geuyter, (2000)

Source of Support: Nil, Conflict of interest: None Declared

[Copy right © 2016, RJPA. All Rights Reserved. This is an Open Access article distributed under the terms of the International Research Journal of Pure Algebra (IRJPA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]