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OPERATING PROPERTIES OF FRACTAL ALGEBRAIC LANGUAGE SYSTEM

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ABSTRACT

According to the algebraic properties of regular expressions, the regular expression is introduced into the fractal algebraic language system, enhance the functionality of fractal algebraic language system, by defining the algebraic language system of various operations, to realize some fractal graphics attractor of the relationship between the known L system and IFS algebraic language system constructed on the basis of more fractal graph, simulate the more realistic nature of the object.

Keywords: *fractal; iterative function system; L-system; attractor; Fractal Language System operation.*

1. INTRODUCTION

L system and IFS is to produce two different methods of fractal graphics, a lot of research on domestic and foreign scholars in the field of this system, the theory and application aspects have rapid development. L system essential public string rewriting system, have strict to describe the fractal structure of plant morphological structure is very refined. IFS has strict self-similarity, mainly using the affine transformation fractal figure. Fractal language algebra system based on the grammatical forms of L system in the regular expression is introduced into the classification model, the IFS and L system unified symbol, in IFS, L system in construction and extension method for generating fractal figure of algebraic representation. Under the new system by defining addition, direct product, composite products, and similar operations, after the fractal graph and computing is discussed to get the relationship between the fractal graph of the attractor. The algebraic system of various operations in image processing and neural network model has certain significance.

2. BASIC THEORY

2.1 L-System

A D0L system[1-2] is an ordered triple $G=<V,\omega,P>$, where V expressed the alphabet, ω is a non-empty string, referred to as the "axiom", P expressed the set of rewrite rules, using $a\xrightarrow{P}x$ expressed a as P "precursors", x for P "successor". Guiding stipulate, for each $a\in V$ there are at least a non-empty string x, let $a\to x$, if a certain precursor $a\in V$ has not explicitly given x, assuming identical transformation $a\to a$.

2.2 Iterative function system (IFS)

Iterative function system [3-4] is by definition a group of contractive mappings $\{w_n: X\to X\}_{n=1}^{N}$ in complete metric space (X,d) and the corresponding compression factor $\{s_n\}_{n=1}^{N}$. Represented as the IFS: $\{X;w_n\}_{n=1}^{N}$, and the compression attractor is

$$s=\max\{s_n\}_{n=1}^{N}.$$

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Theorem 1: Let $\{X;\omega_i, i=1,2,\cdots,N\}$ is with compression factor s of iteration function system, the transformation $W:H(X)\to H(X)$, set

$$W(B)=\bigcup_{i=1}^{N}\omega_i(B),\forall B\in H(X)$$

is complete space $(H(X),h(d))$ with compression attractor s of compression mapping, now

$$h(W(A),W(B))\le sh(A,B),\forall A,B\in H(X)$$

the only fixed point $P\in H(X)$ satisfies

$$P=W(P)=\bigcup_{i=1}^{N}\omega_i(P)$$

and $P=\lim_{n\to\infty}W^n(B),\forall B\in H(X)$. The set $P\in H(X)$ is called the attractor of IFS. In general, the IFS attractor is fractal.

2.3 Focuses on the definition of system of language and grammar

Language is a quad algebra system $G=(V,\Sigma,P,S)$, The V is a finite set argument; Σ for the finite set of symbols, known as the alphabet; P is to generate rules tables (or production), each rule is composed of rules lvalue and rvalue; S is to string.

The middle of the argument is continuously recursion, get the final result in the middle of the process.

The alphabet is made up of common characters and special characters. Ordinary characters can be said a transformation or a collection of graphics; Special characters, including "*", "+", "|", etc.

Rules are usually expressed in $\alpha\to\beta$, $\alpha\in\Sigma^+$ called l value rules is called matching patterns, $\beta\in\Sigma^*$ is referred to as the rules right value is also called the replacement model. In a group with the same left type $\alpha\to\beta_1,\alpha\to\beta_2,\cdots,\alpha\to\beta_n$ "|" simple record for $\alpha\to\beta_1\,|\,\beta_2\,|\cdots|\,\beta_n$ can be used.
Axiom is a string, general can define one or more of the axiom.

According to the differences in production, the language of algebraic system expressed in the form of a regular expression equations (the difference is that a regular expression equation using the equals sign "=" and production with arrow "$\to$"), and according to grammar rules have the following production [5].

production $x\to ax\,|\,x_0$, a is a number of elements sum, where

$$a=\sum_{k=1}^{n}a_k, a_k\in\Sigma^*,\Sigma=\{\omega_1,\omega_2,\cdots,\omega_n\},$$

we write " = "instead of"$\to$" , that is became a regular expression equation. The solution of equation is $x=a^*x_0$. If Σ formed by compression mapping, the corresponding IFS attractor is one of the biggest a^*x_0.

Similarly, production $\boldsymbol{x}\to\boldsymbol{ax}+b\,|\,\boldsymbol{x}_0$, $\boldsymbol{a}=\sum_{k=1}^{n}\boldsymbol{a}_k,\boldsymbol{a}_k\in\Sigma^*,\Sigma=\{\omega_1,\omega_2,\cdots,\omega_n\}$. Using homogeneous coordinates, let $\mathbf{x}=(x\ \ 1)^T$, $\boldsymbol{a}=\begin{pmatrix}a & b\\ 0 & 1\end{pmatrix}$, $\mathbf{x}_0=(x_0\ \ 1)^T$, You can write for production

$\mathbf{x}\to\boldsymbol{a}\,\mathbf{x}\,|\,\mathbf{x}_0$, The solution of equation is $\mathbf{x}=\boldsymbol{a}^*\,\mathbf{x}_0$.

The simple type based on this kind of production should be L-system and the shape of the IFS. $x\to ax\,|\,x_0$, $a=\sum_{k=1}^{n}a_k$,When $n=1$, a for single element, equivalent of L system and transform the IFS; When $n>1$, a is the multiple elements addition, forming regular IFS. When L system can restore production are direction, can use the "+"

will L-system to produce type has nothing to do with all of the argument with type into a sequence of linear systems. Production $\boldsymbol{x} \to \boldsymbol{ax} + b \mid \boldsymbol{x}_0$, if b is a graphic, this production corresponding to the belt coherent set IFS (usually with simulated coherent set of plant, to select the initial set of the same as the coherent set, that is $b = \boldsymbol{x}_0$).

In this paper, language algebra system is only context-free linear system.

3. DEFINITION AND OPERATING PROPERTIES

Definition 1[6]: $L_1 = \{\omega_i, n = 1,2,\cdots,N_1\}$ and $L_2 = \{\xi_j, n = 1,2,\cdots,N_2\}$ is the language of algebraic system on the compression factor for x_0, y_0 compression mapping, A and B defined as

$$L_1 + L_2 = \{\omega_i, \xi_j, i = 1,2,\cdots,N_1,\ j = 1,2,\cdots,N_2\}$$

The compressibility factor is $x = \max\{x_0, y_0\}$.

Definition 2: According to the definition 1, the composite product is defined as

$$L_1 \circ L_2 = \{\omega_i \xi_j, i = 1,2,\cdots,N_1,\ j = 1,2,\cdots,N_2\}$$

The Attractor is $x = x_0 y_0$, In particular, n of L_1 composite product of shorthand for $L_1^{\ n}$, L_1 relative to $L_1^{\ n}$ to write $(L_1^{\ n})^{\frac{1}{n}}$.

Definition 3: According to the definition 1, their direct product is defined as

$$L_1 \otimes L_2 = \{\omega_i \xi_j, \xi_j \omega_i, i = 1,2,\cdots,N_1,\ j = 1,2,\cdots,N_2\}$$

the attractor is $x = x_0 y_0$.

Proposition 1: Set L_1, L_2, L_3 are fractal graphics algebraic language system, About the algebra system addition, direct product, composite product has the following properties:

(1) $L_1 + L_2 = L_2 + L_1$;

(2) $L_1 \otimes L_2 = L_2 \otimes L_1$;

(3) $L_1 \otimes (L_2 + L_3) = (L_1 \otimes L_2) + (L_1 \otimes L_3)$;

(4) $L_1 + L_2 + L_3 = (L_1 + L_2) + L_3 = L_1 + (L_2 + L_3)$;

(5) $L_1 \otimes L_2 \otimes L_3 = (L_1 \otimes L_2) \otimes L_3 = L_1 \otimes (L_2 \otimes L_3)$;

(6) $L_1 \circ L_2 = L_2 \circ L_1$ If and only if $L_1 = L_2$;

(7) $(L_1 + L_2) \circ L_3 = (L_1 \circ L_3) + (L_2 \circ L_3)$;

(8) $L_1 \circ (L_2 + L_3) = (L_1 \circ L_2) + (L_1 \circ L_3)$.

Example 1: A complete metric space $(H(R^2), h(Euclidean))$, there are two algebraic language system $L_1 = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $L_2 = \{\xi_1, \xi_2, \xi_3\}$,

Two different arithmetic of fractal sets fractal structure as shown in figure

$x \to ax \mid x_0$, $\boldsymbol{a} \subset L_1 \circ L_2$, $L_1 = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $L_2 = \{\xi_1, \xi_2, \xi_3\}$,

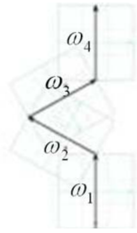
$L_1 \circ L_2 = \{\omega_1\xi_1, \omega_1\xi_2, \omega_1\xi_3, \omega_2\xi_1, \omega_2\xi_2, \omega_2\xi_3, \omega_3\xi_1, \omega_3\xi_2, \omega_3\xi_3, \omega_4\xi_1, \omega_4\xi_2, \omega_4\xi_3\}$,

$L_2 \circ L_1 = \{\xi_1\omega_1, \xi_1\omega_2, \xi_1\omega_3, \xi_1\omega_4, \xi_2\omega_1, \xi_2\omega_2, \xi_2\omega_3, \xi_2\omega_4, \xi_3\omega_1, \xi_3\omega_2, \xi_3\omega_3, \xi_3\omega_4\}$

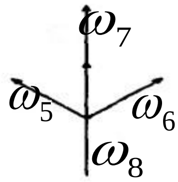
$L_1 \otimes L_2 = \{\omega_1\xi_1, \omega_1\xi_2, \omega_1\xi_3, \omega_2\xi_1, \omega_2\xi_2, \omega_2\xi_3, \omega_3\xi_1, \omega_3\xi_2, \omega_3\xi_3, \omega_4\xi_1, \omega_4\xi_2, \omega_4\xi_3, \xi_1\omega_1, \xi_1\omega_2, \xi_1\omega_3, \xi_1\omega_4,$
$\xi_2\omega_1, \xi_2\omega_2, \xi_2\omega_3, \xi_2\omega_4, \xi_3\omega_1, \xi_3\omega_2, \xi_3\omega_3, \xi_3\omega_4\}$

$L_1 + L_2 = \{\omega_1, \omega_2, \omega_3, \omega_4, \xi_1, \xi_2, \xi_3\}$

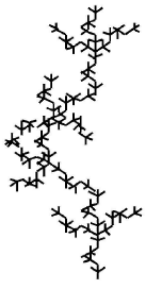
The solution of equation is $\mathbf{x} = \boldsymbol{a}^* \mathbf{x}_0$. If $\boldsymbol{a} \subset L_1 \circ L_2$ fractal figure as shown in figure (c).If $\boldsymbol{a} \subset L_2 \circ L_1$, fractal figure as shown in figure (d), if $\boldsymbol{a} \subset L_1 \otimes L_2$, fractal figure as shown in figure (e), if $\boldsymbol{a} \subset L_1 + L_2$, fractal figure as shown in figure(f).



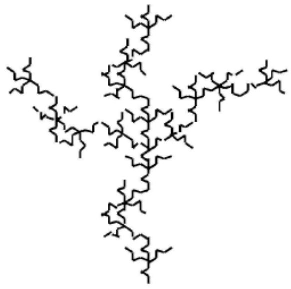
(a)



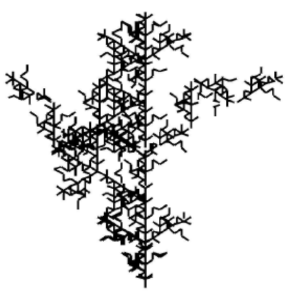
(b)



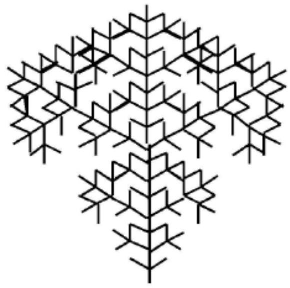
(c)



(d)



(e)

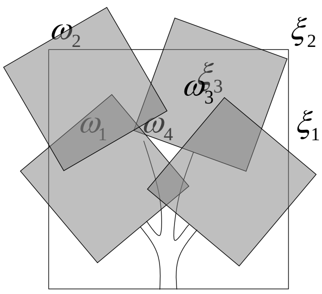


(f)

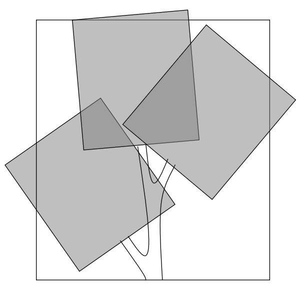
Figure-1: Comparing the fractal sets

Obviously, compound operation generally does not meet the exchange law, and image have obvious differences, but shows the characteristics of the corresponding. L_1 in the former, the overall shape is close to fractal sets L_1. L_2 in the former, the overall shape is close to fractal sets L_2. Fractal sets $L_1 \otimes L_2$ contained in the fractal set $L_1 \circ L_2$ and $L_2 \circ L_1$. $L_1 + L_2$ is three operations in one of the most complex, Fractal sets $L_1, L_2, L_1 \circ L_2, L_2 \circ L_1, L_1 \otimes L_2$ is the subset of fractal sets $L_1 + L_2$.

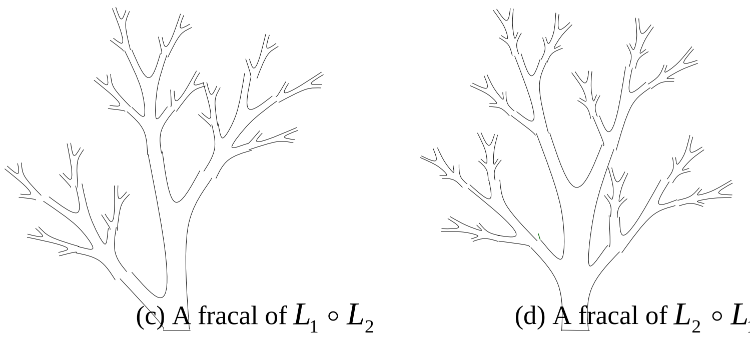
Example 2: L_1 and L_2 are two figure with coherent set of iteration function system, the graph (c) (d) is iterative cubic fractal of $L_1 \circ L_2$ and $L_2 \circ L_1$. We can define a number of iterative function systems with condensed sets, through the iteration can be more realistic simulation of the nature of the trees.



(a) L_1



(b) L_2



(c) A fracal of $L_1 \circ L_2$ (d) A fracal of $L_2 \circ L_1$

Figure-2: Interacting fractal tree

4. CONCLUSION

This paper explores the addition, direct product and complex product operation of fractal algebraic language system, obtains the relationship between the new system and the attractor, constructs more fractal graphs intuitively, and realizes the simulation of the objects in nature. Production for other non-linear type is still under further study, due to limited space, not much in this introduction. The non - linear type has some reference and inspiration for the future simulation of more complex fractal.

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