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COMMON FIXED POINT THEOREM FOR CYCLIC WEAK ϕ − CONTRACTION IN FUZZY METRIC SPACES

CHITHAPELLI'SRINIVAS[1], M. VIJAYA KUMAR*[2], Dr. PANKAJ TIWRI[3]

[1&2]H. No: 3-10-255, Reddy Colony, Hanamkonda – Warangal, India.

[3]Khjuri Kalan Road, Bhopal (M.P), India.

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ABSTRACT

In this paper we prove some common fixed point theorems for cyclic weak $\varphi-$ contractions in a fuzzy metric space. We offered a generalization of $\varphi-$ contraction in fuzzy metric space. Our results generalize or improve many recent fixed point theorems. some corollaries have been given.

Keywords: *Fixed point theorem, fuzzy metric spaces, contractions.*

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1.1 INTRODUCTION

In 1988, Grabiec [5] defined contraction and contractive mappings on a fuzzy metric space and extended fixed point theorems of Banach and Edelstein in such spaces. Following Grabiec's approach, Mishra *et al.* obtained common fixed point theorems for asymptotically commuting mappings on fuzzy metric spaces. In 1998, Vasuki [17] established a generalization of Grabiec's fuzzy contraction theorem wherein he proved a common fixed point theorem for a sequence of mappings in a fuzzy metric space. Thereafter, Cho [4] extended the concept of compatible mappings of type (alpha) to fuzzy metric spaces and utilize the same to prove fixed point theorems in fuzzy metric spaces. Several years later, Singh and Chauhan [13] introduced the concept of compatible mappings and proved two common fixed point theorems in the fuzzy metric space with the minimum triangular norm. In 2002, Sharma [12] further extended some known results of fixed point theory for compatible mappings in fuzzy metric spaces. At the same time, Gregori and Sapena [26] introduced the notion of fuzzy contractive mapping and proved fixed point theorems in varied classes of complete fuzzy metric spaces in the senses of Veeramani [19], Soon after, Mihet [7] proposed a fuzzy fixed point theorem for (weak) Banach contraction in M-complete fuzzy metric spaces. In this continuation, D Mihet [7] further enriched the fixed point theory for various contraction mappings in fuzzy metric spaces besides introducing variants of some new contraction mappings such as: Edelstein fuzzy contractive mappings, fuzzy ψ-contraction of (ϵ, λ) type etc. In the same spirit, Qiu ***et al.*** [11] also obtained some common fixed point theorems for fuzzy mappings under suitable conditions. In 2010 Pacurar and Rus in [8] , introduced the concept of cyclic ϕ-contraction and utilize the same to prove a fixed point theorem for cyclic$\phi-$contraction in the natural setting of complete metric spaces besides investigating several related problems in respect of fixed points.

The main purpose of this chapter is to introducing sub compatibility and sub sequential continuity in fuzzy metric space and proves some fixed point results related with these new concepts for this first we give some definitions and known results .

2. PRILIMINARIES

Definition 2.1: A binary operation $\star: [0,1] \times [0,1] \to [0,1]$ is continuous $t-$ norm if $\star$ is satisfying the following conditions:

2.1 (i) $\star$ is commutative and associative.
2.1(ii) $\star$ is continuous.
2.1 (iii) $a \star 1 = a$ for all $a \in [0,1]$.
2.1 (iv) $a \star b \le c \star d$ whenever $a \le c$ and $b \le d$,
For $a, b, c, d \in [0,1]$.

*Corresponding Author: M. Vijaya Kumar**

Definition 2.2: A triplet $(X, M,\star)$ is said to be a fuzzy metric space if X is an arbitrary set, $\star$ is a continuous $t-$ norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following condition for all $x, y, z, s, t > 0$,

2.2 (FM − 1) $M(x, y, t) > 0$
2.2 (FM − 2) $M(x, y, t) = 1$ if and only if $x = y$.
2.2 (FM − 3) $M(x, y, t) = M(y, x, t)$
2.2 (FM − 4) $M(x, y, t) \star M(y, z, s) \leq M(x, z, t + s)$
2.2 (FM − 5) $M(x, y, \bullet) : (0,\infty) \to (0,1]$ is continuous.

Then M is called a fuzzy metric on X. the function $M(x, y, t)$ denote the degree of nearness between x and y with respect to t.

Example 2.3: Let (X, d) be a metric space. Define $a * b = \min\{a, b\}$ and

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

For all $x, y \in X$ and all $t > 0$. Then $(X, M,\star)$ is a Fuzzy metric space.

It is called the Fuzzy metric space induced by d.

We note that, $M(x, y, t)$ can be realized as the measure of nearness between x and y with respect to t. It is known that $M(x, y,\cdot)$ is non decreasing for all $x, y \in X$.

Let $M(x, y,\star)$ be a fuzzy metric space for $t > 0$, the open ball

$$B(x, r, t) = \{y \in X: M(x, y, t) > 1 - r\}.$$

Now, the collection $\{B(x, r, t): x \in X, 0 < r < 1, t > 0\}$ is a neighborhood system for a topology τ on X induced by the fuzzy metric M. This topology is Housdroff and first countable.

Definition 2.4: A sequence $\{x_n\}$ in a fuzzy metric space $(X, M,\star)$ is said to be a converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.

Definition 2.5: A sequence $\{x_n\}$ in a fuzzy metric space $(X, M,\star)$ is said to be a G- Cauchy sequence converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_m, x_n, t) > 1 - \varepsilon$ for all $m, n \geq n_0$.

A fuzzy metric space $(X, M,\star)$ is said to be complete if every G- Cauchy sequence in it converges to a point in it.

3. MAIN THEOREM

Fixed Point Theorem for Cyclic Contraction In Fuzzy Metric Spaces

Theorem 3.1: Let $(X, M,\star)$ be a fuzzy metric spaces, $A_1, A_2, \ldots, A_m$ be closed subsets of X and $Y = \cup_{i=1}^{m} A_i$ be $G-$ complete. Suppose that $\phi: [0,\infty) \to [0,\infty)$ is a continuous non decreasing function with $\phi(r) > 0$ for each $r \in (0, +\infty)$ and $\phi(0) = 0$. If $f: Y \to Y$ satisfying,

$$\left(\frac{1}{M(fx, fy, \varphi(t))} - 1\right) \leq \left(\frac{1}{M(x, y, \varphi(t))} - 1\right) - \phi\left(\frac{1}{M(x, y, \varphi(t))} - 1\right) \qquad 3.1(i)$$

then f has a unique fixed point $z \in \cap_{i=1}^{m} A_i$.

Proof: Let $x_0 \in Y = \cup_{i=1}^{m} A_i$ and set $x_n = fx_{n-1}$ $(n \geq 1)$. Clearly, we get $M(x_n, x_{n+1}, t) = M(fx_{n-1}, fx_n, t)$ for any $t > 0$. Beside for any $n \geq 0$, there exists $i_n \in \{1,2, \ldots, m\}$ such that $x_n \in A_{i_n}$ and $x_{n+1} \in A_{i_{n+1}}$. Then by 3.1(i), (for $t > 0$) we have

$$\begin{aligned}\left(\frac{1}{M(x_n, x_{n+1}, \varphi(t))} - 1\right) &= \left(\frac{1}{M(fx_{n-1}, fx_n, \varphi(t))} - 1\right) \qquad 4.2.1(ii)\\ &\leq \left(\frac{1}{M(x_{n-1}, x_n, \varphi(t))} - 1\right) - \phi\left(\frac{1}{M(x_{n-1}, x_n, \varphi(t))} - 1\right)\\ &\leq \left(\frac{1}{M(x_{n-1}, x_n, \varphi(t))} - 1\right)\end{aligned}$$

Which implies that

$$M(x_n, x_{n+1}, \varphi(t)) \geq M(x_{n-1}, x_{n,} \varphi(t))$$

and hence

$$M(x_n, x_{n+1}, \varphi(t)) \geq M(x_{n-1}, x_n, \varphi(t))$$

For all $n \geq 1$ and so $\{M(x_{n-1}, x_n, \varphi(t))\}$ is non decreasing sequence of positive real numbers in $(0,1]$.

Let $S(t) = \lim_{n\to+\infty} M(x_{n-1}, x_n, \varphi(t))$. Now we show that $S(t) = 1$ for all $t > 0$.

If not, there exists some $t > 0$ such that $S(t) < 1$. Then, on making $n \to +\infty$, we obtain

$$\left(\frac{1}{S(\varphi(t))} - 1\right) \leq \left(\frac{1}{S(\varphi(t))} - 1\right) - \phi\left(\frac{1}{S(\varphi(t))} - 1\right)$$

Which is a contradiction. Therefore $M^2(x_n, x_{n+1}, \varphi(t)) \to 1$ as $n \to +\infty$. Now for each positive integer p we have

$$M(x_n, x_{n+p}, \varphi(t)) \geq M\left(x_n, x_{n+1}, \frac{\varphi(t)}{p}\right) \star M\left(x_{n+1}, x_{n+2}, \frac{\varphi(t)}{p}\right) \ldots\ldots \star M\left(x_{n+p-1}, x_{n+p}, \frac{\varphi(t)}{p}\right)$$

It follows that $\lim_{n\to+\infty} M(x_{n-1}, x_n, \varphi(t)) \geq 1 \star 1 \star \ldots\ldots \star 1 = 1$

So that $\{x_n\}$ is a G-Cauchy sequence. As Y is G-complete, then there exists $y \in Y$ such that $\lim_{n\to+\infty} x_n = y$. On the other hand, by it follows that the iterative sequence $\{x_n\}$ has a infinite numbers of terms in A_i for each $i = 1,2, \ldots\ldots m$. Since Y is G-complete, for each A_i, $i = 1,2, \ldots\ldots m$ one can exists a subsequence of $\{x_n\}$ that converges to y. By virtue of the fact that each $A_i, i = 1,2, \ldots, m$ is closed, we conclude that $y \in \cap_{i=1}^m A_i \neq \phi$. Obviously, $\cap_{i=1}^m A_i$ is closed and $G-$ complete. Now we consider the restriction of f on $\cap_{i=1}^m A_i$, i.e. $f|\cap_{i=1}^m A_i : \cap_{i=1}^m A_i \to \cap_{i=1}^m A_i$ thus $f|\cap_{i=1}^m A_i$ has a unique fixed point in $\cap_{i=1}^m A_i$, say z, which is obtained by iteration form the starting point $x_0 \in Y$. To this end, we have show that $x_n \to z$ as $n \to \infty$, then, by 3.1(i), we have

$$\left(\frac{1}{M(x_n, z, \varphi(t))} - 1\right) \leq \left(\frac{1}{M(x_{n-1}, z, \varphi(t))} - 1\right) - \phi\left(\frac{1}{M(x_{n-1}, z\varphi(t))} - 1\right)$$

Now letting $n \to +\infty$, we get

$$\left(\frac{1}{M(y, z, \varphi(t))} - 1\right) \leq \left(\frac{1}{M(y, z, \varphi(t))} - 1\right) - \phi\left(\frac{1}{M(y, z, \varphi(t))} - 1\right)$$

Which is contradiction if $M(y, z, \varphi(t)) < 1$, and so, we conclude that $z = y$. Obviously z is the unique fixed point of f.

Corollary 3.2: Let $(X, M, \star)$ be a fuzzy metric spaces, $A_1, A_2, \ldots, A_m$ be closed subsets of X and $Y = \cup_{i=1}^m A_i$ be $G-$ complete. Suppose that $\phi: [0, \infty) \to [0, \infty)$ is a continuous non decreasing function with $\phi(r) > 0$ for each $r \in (0, +\infty)$ and $\phi(0) = 0$. If $f: Y \to Y$ satisfying,

$$\left(\frac{1}{M(fx, fy, \varphi(t))} - 1\right) \leq \left(\frac{1}{M(x, y, \varphi(t))} - 1\right) - \phi\left(\frac{1}{M(x, y, \varphi(t))} - 1\right) \qquad 3.2(i)$$

and there exists a sequence $\{y_n\}$ in Y such that $M(y_n, fy_n, t) \to 1$ as $n \to +\infty$ for any $t > 0$, then $y_n \to z$ as $n \to +\infty$, provides that the fuzzy metric M is triangular, and z is the unique fixed point of f in $\cap_{i=1}^m A_i$.

Corollary 3.3: Let $(X, M, \star)$ be a fuzzy metric spaces, $A_1, A_2, \ldots, A_m$ be closed subsets of X and $Y = \cup_{i=1}^m A_i$ be $G-$ complete. Suppose that $\phi: [0, \infty) \to [0, \infty)$ is a continuous non decreasing function with $\phi(r) > 0$ for each $r \in (0, +\infty)$ and $\phi(0) = 0$. If $f: Y \to Y$ satisfying,

$$\left(\frac{1}{M(fx, fy, \varphi(t))} - 1\right) \leq \left(\frac{1}{M(x, y, \varphi(t))} - 1\right) - \phi\left(\frac{1}{M(x, y, \varphi(t))} - 1\right) \qquad 3.1(i)$$

and there exists a sequence $\{y_n\}$ in Y such that $M(y_{n+1}, fy_n, \varphi(t)) \to 1$ as $n \to +\infty$ for any $t > 0$, then there exists $x \in Y$ such that $M(y_n, f^n x, \varphi(t)) \to 1$ as $n \to +\infty$ provides that the fuzzy metric M is triangular.

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