International Research Journal of Pure Algebra-7(6), 2017, 758-761
Available online through www.rjpa.info ISSN 2248–9037

ON SOME PROPERTIES OF METRIC F- STRUCTURE SATISFYING $F^5 + F = 0$

LAKHAN SINGH[1], SHAILENDRA KUMAR GAUTAM*[2]

[1]Department of Mathematics, D. J. College, Baraut, Baghpat - (U.P.), India.

[2]Eshan College of Engineering, Mathura - (U.P.), India.

(Received On: 04-04-17; Revised & Accepted On: 05-05-17)

ABSTRACT

In this paper, we have studied various properties of the F- sturcture satisfying $F^5 + F = 0$. *The metric F- structure,* f *induced on each integral manifold of tangent bundle* l^* *have also been discussed*

Key words: *Differentiable manifold, projection operators, tangent bundles and metric.*

1. INTRODUCTION

Let V_n be a differentiable manifold of class C^∞ and F be a C^∞ (1,1) tensor defined on V_n such that

(1.1) $F^5 + F = 0$

we define the the projection operators l and m on V_n by

(1.2) $l = -F^4, \qquad m = I + F^4$

From (1.1) and (1.2), we get

(1.3) $l + m = I, \qquad l^2 = l, \; m^2 = m, \quad lm = ml = 0$

$lF = Fl = F, \quad Fm = mF = 0,$

where I denotes the identify operator.

Theorem 1.1: If $rank((F)) = n$ then

(1.4) $l = I, \; m = 0$

Proof: from the fact

(1.5) $rank((F)) + nulity((F)) = \dim V_n = n$

Thus

(1.6) $nulity((F)) = 0 \Rightarrow \ker((F)) = \{0\}$

Thus

$FX = 0 \Rightarrow X = 0$

Then $FX_1 = FX_2$

$F(X_1 - X_2) = 0$

$X_1 = X_2$ or F is 1-1

Moreover V_n being finite dimensional F is onto also F is invertible operating F^{-1} on

$Fl = F$ and $mF = 0$, we get (1.4)

Corresponding Author: Lakhan Singh[1], Shailendra Kumar Gautam[2]*
[2]Eshan College of Engineering, Mathura - (U.P.), India.

Theorem 1.2: if $rank((F)) = n-1$ then

(1.7) $l = I - A \otimes T, m = A \otimes T$, $AoF = 0, FT = 0$

Proof: from (1.1)

(1.8) $F(F^4 + I) = 0$

(1.9) Let $F^4 + I = A \otimes T$

From (1.8) and (1.9)

(1.10) $FT = 0$

Also from (1.2) and (1.9)

$$l = -F^{-4} = I - A \otimes T$$
$$m = -F^4 + I = A \otimes T$$

From (1.5) and (1.6)

$$F^4 X + X = A \times T$$
$$F^5 X + FX = A(FX)T$$
$$0 = A(FX)T$$

Thus $AoF = 0$

Theorem 1.3: Let the operator m & F satisfying

(1.11) $m^2 = m, Fm = mF = 0, (m + F^2)(m - F^2) = I$

Then we get (1.1)

Proof: from $(m + F^2)(m - F^2) = I$

$$m^2 - mF^2 + F^2 m - F^4 = I$$
$$m - 0 + 0 - F^4 = I$$
$$mF - F^5 = F$$
$$F^5 + F = 0$$

2. **METRIC F-STRUCTURE**

If we define

(2.1) $`F(X, Y) = g(FX, Y)$ is skew- symmetric.

Then

(2.2) $g(FX, Y) = -g(X, FY)$,

Theorem 2.1: the definitions in (2.1) and (2.2), we have

(2.3) $g(F^2 X, F^2 Y) = -g(X, Y) + `m(X, Y)$, where

(2.4) $`m(X, Y) = g(mX, Y) = g(X, mY)$.

Proof: From (1.2), (1.3) and (2.2), (2.4),we have

(2.5) $g(F^2 X, F^2 Y) = g(X, F^4 Y)$

$$= g(X, -lY)$$
$$= -g(X, lY)$$
$$= -g(X, (I - m)Y)$$

$$= -g(X,Y)+g(X,mY)$$
$$= -g(X,Y)+{}'m(X,Y)$$

Theorem 2.2: $\{F,g\}$ is not unique

Proof:

(2.6) let $\mu F' = F\mu,\quad {}'g(X,Y)=g(\mu X,\mu Y)$

Then from (1.1) and (1.2), (1.3), (2.6)

(2.7) $\mu F'^5 = F^5\mu = -F\mu = \mu F'$ or

(2.8) $F'^5 + F' = 0$ .Also

(2.9) $${}'g(F'^2X,F'^2Y) = g(\mu F'^2X,\mu F'^2Y)$$
$$= g(F^2\mu X,F^2\mu Y)$$
$$= g(\mu X,F^4\mu Y)$$
$$= g(\mu X,-l\mu Y)$$
$$= g(\mu X,-(I-m)\mu Y)$$
$$= -g(\mu X,\mu Y)+g(\mu X,m\mu Y)$$
$$= -{}'g(X,Y)+{}'m(X,Y)$$

3. INDUCED STRUCTURE f :

Define

(3.1) $fX' = FX'$ for $X' \in l^*$

Theorem 3.1: lf f satisfying (3.1) and F (1.1) then $\{f^2\}$ is an almost complex structure.

Proof: from (1.2),(1.3) and (3.1)

(3.2) $$f^4lX' = F^4lX'$$
$$= -l^2X'$$
$$= -lX'$$

Thus $\{f^2\}$ as an almost complex structure on l^*

Also

(3.3) $$\mu l' = -\mu F'^4$$
$$= -F^4\mu$$
$$= l\mu$$

(3.4) $$\mu m' = \mu(I+F'^4)$$
$$= \mu + \mu F'^4$$
$$= \mu + F^4\mu$$
$$= (I+F^4)\mu$$
$$= m\mu$$

REFERENCES

1. K. Yano: On a structure defined by a tensor field *f* of the type (1,1) satisfying $f^3+f=0$. Tensor N.S., 14 (1963), 99-109.
2. R. Nivas & S. Yadav: On CR-structures and $F_\lambda(2\nu+3,2)$ - HSU - structure satisfying, $F^{2\nu+3}+\lambda^r F^2=0$ Acta Ciencia Indica, Vol. XXXVII M, No. 4, 645 (2012).
3. Abhisek Singh, Ramesh Kumar Pandey and Sachin Khare On horizontal and complete lifts of (1, 1) tensor fields F satisfying the structure equation $F(2k+S,S)$=0. International Journal of Mathematics and soft computing. Vol. 6, No. 1 (2016), 143-152, ISSN 2249-3328.

Source of Support: Nil, Conflict of interest: None Declared