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ON SOME PROPERTIES OF METRIC F- STRUCTURE SATISFYING $F^5 + F = 0$

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ABSTRACT

In this paper, we have studied various properties of the F- sturcture satisfying $F^5 + F = 0$. *The metric F- structure,* f *induced on each integral manifold of tangent bundle* l^* *have also been discussed*

Key words: *Differentiable manifold, projection operators, tangent bundles and metric.*

1. INTRODUCTION

Let V_n be a differentiable manifold of class C^∞ and F be a C^∞ (1,1) tensor defined on V_n such that

(1.1) $\quad F^5 + F = 0$

we define the the projection operators l and m on V_n by

(1.2) $\quad l = -F^4, \qquad m = I + F^4$

From (1.1) and (1.2), we get

(1.3) $\quad l + m = I, \quad l^2 = l, \ m^2 = m, \quad lm = ml = 0$

$\quad lF = Fl = F, \quad Fm = mF = 0,$

where I denotes the identify operator.

Theorem 1.1: If $rank((F)) = n$ then

(1.4) $\quad l = I, \ m = 0$

Proof: from the fact

(1.5) $\quad rank((F)) + nulity((F)) = \dim V_n = n$

Thus

(1.6) $\quad nulity((F)) = 0 \Rightarrow \ \ker((F)) = \{0\}$

Thus

$FX = 0 \Rightarrow X = 0$

Then $\quad FX_1 = FX_2$

$F(X_1 - X_2) = 0$

$X_1 = X_2 \quad$ or F is 1-1

Moreover V_n being finite dimensional F is onto also F is invertible operating F^{-1} on

$Fl = F$ and $mF = 0$, we get (1.4)

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Theorem 1.2: if $rank((F))=n-1$ then

(1.7) $l=I-A\otimes T, m=A\otimes T, AoF=0, FT=0$

Proof: from (1.1)

(1.8) $F(F^4+I)=0$

(1.9) Let $F^4+I=A\otimes T$

From (1.8) and (1.9)

(1.10) $FT=0$

Also from (1.2) and (1.9)

$$l=-F^{-4}=I-A\otimes T$$
$$m=-F^4+I=A\otimes T$$

From (1.5) and (1.6)

$$F^4X+X=A\times T$$
$$F^5X+FX=A(FX)T$$
$$0=A(FX)T$$

Thus $AoF=0$

Theorem 1.3: Let the operator m & F satisfying

(1.11) $m^2=m, Fm=mF=0, (m+F^2)(m-F^2)=I$

Then we get (1.1)

Proof: from $(m+F^2)(m-F^2)=I$

$$m^2-mF^2+F^2m-F^4=I$$
$$m-0+0-F^4=I$$
$$mF-F^5=F$$
$$F^5+F=0$$

2. **METRIC F-STRUCTURE**

If we define

(2.1) $`F(X,Y)=g(FX,Y)$ is skew- symmetric.

Then

(2.2) $g(FX,Y)=-g(X,FY),$

Theorem 2.1: the definitions in (2.1) and (2.2), we have

(2.3) $g(F^2X,F^2Y)=-g(X,Y)+`m(X,Y),$ where

(2.4) $`m(X,Y)=g(mX,Y)=g(X,mY).$

Proof: From (1.2), (1.3) and (2.2), (2.4),we have

(2.5) $g(F^2X,F^2Y)=g(X,F^4Y)$

$$=g(X,-lY)$$
$$=-g(X,lY)$$
$$=-g(X,(I-m)Y)$$

$$= -g(X,Y) + g(X,mY)$$
$$= -g(X,Y) + {}'m(X,Y)$$

Theorem 2.2: $\{F, g\}$ is not unique

Proof:

(2.6) let $\mu F' = F\mu$, ${}'g(X,Y) = g(\mu X, \mu Y)$

Then from (1.1) and (1.2), (1.3), (2.6)

(2.7) $\mu F'^5 = F^5\mu = -F\mu = \mu F'$ or

(2.8) $F'^5 + F' = 0$ .Also

(2.9) ${}'g(F'^2X, F'^2Y) = g(\mu F'^2X, \mu F'^2Y)$

$$= g(F^2\mu X, F^2\mu Y)$$
$$= g(\mu X, F^4\mu Y)$$
$$= g(\mu X, -l\mu Y)$$
$$= g(\mu X, -(I-m)\mu Y)$$
$$= -g(\mu X, \mu Y) + g(\mu X, m\mu Y)$$
$$= -{}'g(X,Y) + {}'m(X,Y)$$

3. INDUCED STRUCTURE f :

Define

(3.1) $fX' = FX'$ for $X' \in l^*$

Theorem 3.1: lf f satisfying (3.1) and F (1.1) then $\{f^2\}$ is an almost complex structure.

Proof: from (1.2),(1.3) and (3.1)

(3.2) $f^4lX' = F^4lX'$

$$= -l^2X'$$
$$= -lX'$$

Thus $\{f^2\}$ as an almost complex structure on l^*

Also

(3.3) $\mu l' = -\mu F'^4$

$$= -F^4\mu$$
$$= l\mu$$

(3.4) $\mu m' = \mu(I + F'^4)$

$$= \mu + \mu F'^4$$
$$= \mu + F^4\mu$$
$$= (I + F^4)\mu$$
$$= m\mu$$

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