SECOND GENERALIZED ANDRE PLANE OF ORDER 3⁴

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ABSTRACT

A Generalized Andre plane of order 3^4 has been reported in [19]. Another Generalized Andre plane of order 3^4 is constructed and its translation complement is computed which is found to be of order 6400 and this plane is shown to be different from the plane already reported [19] in view of its orbit structure.

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1. INTRODUCTION

One of the methods of constructing translation planes is through t-spread sets given by Rao and Davis [14]. Bruck and Bose have contributed to the theory of t-spread sets over finite fields for the construction of non Desarguesian translation planes through their papers [1], [2]. In this paper we have given the construction of a 3-spread set which produces a V-W system [1, p.99] and in turn coordinatizes a translation plane of order 3^4. The plane thus constructed is shown to be a generalized Andre plane using the technique given by D.A.Foulser [6]. Collineation groups of the translation plane are determined as they play a vital role in determining the translation complement of the plane. The translation complement is found to be of order 6400 and this plane is shown to have an orbit structure 2, 40, 40 which is different from the plane already reported [19].

2. DESCRIPTION OF THE PLANE π AND IDENTIFYING THE PLANE AS A GENERALIZED ANDRE SYSTEM

It is well known that a translation plane π of finite order can be coordinatized by a V-W system. Conversely given a V-W system (Q, +, ·) a translation plane π(Q) can be associated with Q [8, pp 362]. A V-W system can be constructed from a t-spread set. [1, pp95]. Thus the construction of translation plane of order q^{t+1} reduces to the construction of t-spread set. [3, pp220]

t-spread set: Let t be a positive integer. A set $\mathscr{C}$ of (t+1) by (t+1) matrices over F is a t-spread set over F if it satisfies
   a) $\left| \mathscr{C} \right| = q^{t+1}$, $\mathscr{C}$ contains the zero and identity matrices.
   b)   For all X, Y$\in \mathscr{C}$, $X \neq Y \Rightarrow \det (X-Y) \neq 0$.

Here det A denotes the determinant of the matrix A.

Throughout this paper F, (abcd, efgh, klmn, pqrs) and i.p denote the Galois Field GF(3), the 4x4 matrix
$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ k & l & m & n \\ p & q & r & s \end{pmatrix}$$ and ideal point respectively.

For M, N $\in$ GL(4, 3), T(M, N) = {A$\in$GL(4, 3) $\left|\right.$ A⁻¹MA=N}, Z(M) = T(M,M).

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Let G denote the translation complement of the translation plane π; G_0 (G_{81}) denotes the collineation subgroup of G fixing the i.p 0 (81); $G_{0,81}$ denotes the (autotopism) collineation subgroup of G fixing the i.ps 0,81 and $G_{0,81,1}$ denotes the subgroup of G (conjugation collineation group) fixing the i.ps 0, 81, 1; In general $G_{i, j, k, l, m}$ denotes the collineation subgroup of G fixing the i.ps i, j, k, l, m.

Lemma 2.1: Let $\mathscr{C}$ be a t-Spread set over GF(q), q = p^r, p is prime ≥ 3, r is a natural number with the property,
   -M ∈ $\mathscr{C}$ for all M ∈ $\mathscr{C}$ (2.1)

Let $\pi(\mathscr{C})$ be the translation associated with $\mathscr{C}$. Then [16]
   (a) There exists a collineation which fixes V(∞) and moves V(0) onto V(S), S ∈ $\mathscr{C}$ if and only if M+S ∈ $\mathscr{C}$ for all M ∈ $\mathscr{C}$.
   (b) If there exists a collineation which maps V(S) onto V(∞) and V(∞) onto V(0) where S∈ $\mathscr{C}$, S ≠ 0 then M+S∈ $\mathscr{C}$ for all M ∈ $\mathscr{C}$.
   (c) There exists a collineation which fixes V(0) and moves V(∞) onto V(S), S∈ $\mathscr{C}$, S ≠ 0 if and only if $(M^{-1} + S^{-1})^{-1} ∈ \mathscr{C}$ for all M ∈ $\mathscr{C}$.
   (d) If there exists a collineation which maps V(S) onto V(0) and V(0) onto V(∞) where S∈ $\mathscr{C}$, S ≠ 0 if and only if $(M^{-1} + S^{-1})^{-1} ∈ \mathscr{C}$ for all M ∈ $\mathscr{C}$.

Lemma 2.2: [16] Let $\pi(\mathscr{C})$ be the translation plane associated with a t- spread set $\mathscr{C}$ and has the property that every collineation which fixes V(0) also fixes V(∞).
   (a) If there exists a collineation which fixes V(R) and moves V(S), R, S ∈ $\mathscr{C}$ then no collineation of $\pi(\mathscr{C})$ maps V(0) onto V(R) and V(∞) onto V(S).
   (b) If there exists a collineation which fixes V(R) and moves V(S), R, S ∈ $\mathscr{C}$ then no collineation of $\pi(\mathscr{C})$ maps V(∞) onto V(R) and V(0) onto V(S).

2.3 Construction of translation plane π

The translation plane π under study is constructed through a 3- spread set $\mathscr{C}$ over F. The spread set $\mathscr{C}$ is given by
   $\mathscr{C}$ = {0}∪ A_0 $\mathscr{G}$ ∪A_1 $\mathscr{G}$ ∪A_2 $\mathscr{G}$ ∪A_3 $\mathscr{G}$.
where $\mathscr{G}$ = < X,Y / X,Y∈ GL(4,3), X^5 = I, Y^2 = −I, $Y^{-1}XY$ = X^{-1}> is a metacyclic group of order 20 in GL(4, 3)
where X = (2120, 0212, 2221, 1022), Y = (1100, 1200, 2012, 1222)
and A_0 = (1000, 0100, 0010, 0001)
   A_1 = (0001, 1001, 1122, 1221)
   A_2 = (0010, 0001, 1100, 0110)
   A_3 = (0100, 0010, 0001, 1100)

Let M_0 be the zero matrix and $M_{10j+i} = A_{j_1} M_{11}{}^k M_2{}^{i-1}$ 0 ≤ i ≤ 10, 0 ≤ j ≤ 7, where $j_1 = \left[\dfrac{j}{2}\right]$, k = j - 2$j_1$.

The matrices M_i, 0 ≤ i ≤ 80 are tabulated in Table 1 along with their characteristic polynomials.

The entry [abcd] in table 1 against i indicates that the matrix M_i of C has C.P λ^4 +a λ^3+bλ^2+c λ+d.

Table-1

I	M_i	C.P of M_i		I	M_i	C.P of M_i
0	(0000,0000,0000,0000)			41	(0010,0001,1100,0110)	[0121]
1	(1000,0100,0010,0001)	[2021]		42	(1112,2011,1001,1200)	[2211]
2	(1210,0121,1112,2011)	[2121]		43	(0102,2210,0221,1122)	[0121]
3	(2201,1020,0102,2210)	[1111]		44	(1110,0111,1111,1211)	[2101]
4	(1010,0101,1110,0111)	[2121]		45	(2110,0211,1121,1212)	[1101]
5	(2022,2102,2110,0211)	[1111]		46	(0020,0002,2200,0220)	[0111]
6	(2000,0200,0020,0002)	[1011]		47	(2221,1022,2002,2100)	[1221]
7	(2120,0212,2221,1022)	[1111]		48	(0201,1120,0112,2211)	[0111]
8	(1102,2010,0201,1120)	[2121]		49	(2220,0222,2222,2122)	[1101]
9	(2020,0202,2220,0222)	[1111]		50	(1220,0122,2212,2121)	[2101]
10	(1011,1201,1220,0122)	[2121]		51	(2012,1222,2000,0212)	[0001]
11	(1100,1200,2012,1222)	[0201]		52	(1221,1002,2120,2010)	[0001]
12	(1001,1122,1221,1002)	[0201]		53	(2021,2202,1102,0202)	[0001]
13	(0221,1211,2021,2202)	[0201]		54	(0022,0021,2020,1201)	[0001]
14	(1111,1212,0022,0021)	[0201]		55	(0210,1202,1011,0100)	[0001]
15	(1121,0220,0210,1202)	[0201]		56	(1021,2111,1000,0121)	[0001]
16	(2200,2100,1021,2111)	[0201]		57	(2112,2001,1210,1020)	[0001]

17	(2002,2211,2112,2001)	[0201]	58	(1012,1101,2201,0101)	[0001]
18	(0112,2122,1012,1101)	[0201]	59	(0011,0012,1010,2102)	[0001]
19	(2222,2121,0011,0012)	[0201]	60	(0120,2101,2022,0200)	[0001]
20	(2212,0110,0120,2101)	[0201]	61	(0100,0010,0001,1100)	[0022]
21	(0001,1001,1122,1221)	[0102]	62	(0121,1112,2011,1001)	[0022]
22	(2011,0221,1211,2021)	[0102]	63	(1020,0102,2210,0221)	[2112]
23	(2210,1111,1212,0022)	[0102]	64	(0101,1110,0111,1111)	[0202]
24	(0111,1121,0220,0210)	[0102]	65	(2102,2110,0211,1121)	[1122]
25	(0211,2200,2100,1021)	[0102]	66	(0200,0020,0002,2200)	[0012]
26	(0002,2002,2211,2112)	[0102]	67	(0212,2221,1022,2002)	[0012]
27	(1022,0112,2122,1012)	[0102]	68	(2010,0201,1120,0112)	[1122]
28	(1120,2222,2121,0011)	[0102]	69	(0202,2220,0222,2222)	[0202]
29	(0222,2212,0110,0120)	[0102]	70	(1201,1220,0122,2212)	[2112]
30	(0122,1100,1200,2012)	[0102]	71	(1200,2012,1222,2000)	[0202]
31	(1222,2022,2102,2110)	[2002]	72	(1122,1221,1002,2120)	[0202]
32	(1002,2000,0200,0020)	[2002]	73	(1211,2021,2202,1102)	[0202]
33	(2202,2120,0212,2221)	[1222]	74	(1212,0022,0021,2020)	[0202]
34	(0021,1102,2010,0201)	[0102]	75	(0220,0210,1202,1011)	[0202]
35	(1202,2020,0202,2220)	[2212]	76	(2100,1021,2111,1000)	[0202]
36	(2111,1011,1201,1220)	[1002]	77	(2211,2112,2001,1210)	[0202]
37	(2001,1000,0100,0010)	[1002]	78	(2122,1012,1101,2201)	[0202]
38	(1101,1210,0121,1112)	[2212]	79	(2121,0011,0012,1010)	[0202]
39	(0012,2201,1020,0102)	[0102]	80	(0110,0120,2101,2022)	[0202]
40	(2101,1010,0101,1110)	[1222]	81	-----	

2.4. Translation plane π associated with the t-spread set $\mathscr{C}$.

The translation plane π under study is constructed through the 3-spread set $\mathscr{C}$ over F by considering 4-dimensional subspaces V_i, $0 \le i \le 81$ of $V(8, 3)$, the 8-dimensional vector space over F as follows:
Let $V_i = \{(x, y) / y = xM_i, x \in F^4\}$, $0 \le i \le 80$, $V_{81} = \{(0,y) / y \in F^4\}$. The incidence structure whose points are vectors of $V = F^8$ and whose lines are V_i, $0 \le i \le 81$ and their cosets in the additive group of V with inclusion as incidence relation is the translation plane π associated with the 3-spread set $\mathscr{C}$. The matrices of $\mathscr{C}$ are not closed under addition and therefore the translation plane is non Desarguesian and the spread set $\mathscr{C}$ has the property $-M \in \mathscr{C}$ for all $M \in \mathscr{A}$ (2.1)

2.5. Left and Middle nuclei of the t-spread set: If $\mathscr{C}$ is a t-spread set then

$$M_\lambda = \{M \in \mathscr{A} / \mathscr{C}M = \mathscr{C}\}$$
$$M_\mu = \{M \in \mathscr{A} / M\mathscr{C} = \mathscr{C}\}$$

Left nucleus M_λ and middle nucleus M_μ are multiplicative groups of GL(t+1, q) and if $M \in \mathscr{C}$ and $M^2 \notin \mathscr{C}$ Then $M \notin M_\lambda \cup M_\mu$. It can be observed that the left and middle nuclei of the above 3-spread set $\mathscr{C}$ are such that $M_\lambda = M_\mu = \mathscr{G}$

2.6. V-W system associated with the spread set $\mathscr{C}$

Let (Q, +, ·) be a system constructed from the 3-spread set $\mathscr{C}$ where $Q = F^4$, the operation ' +' is the ordinary vector sum. Let e = (1000). For each $y \in Q$ there is a unique matrix $M \in \mathscr{C}$ (denoted by M(y)) such that y = e M. For x, y $\in$ Q, y $\neq$ 0 define y.x= x M(y) and 0.x = 0. The system (Q, +, ·) is a left V-W system coordinatizing the translation plane π. Let N_λ, N_μ be the left and middle nuclei of the V-W system (Q, +, ·)

So $N_\lambda = < (1210),(1100)> = N_\mu$. $N_\lambda \cap N_\mu$ contains a unique cyclic subgroup generated by g of order 10 where g= (1210).

2.7. V-W system (Q, +, ·) is a generalized Andre system or λ -system:

The quadruples of Q are indexed as follows $Q = \{x_i | x_i = eM_i, M_i \in \mathscr{C}, 0 \le i \le 80\}$ where e = (1000) and $x_2 = (1210) = g$

We observe the following:

$$X_{10j+1}. g = g^{3^{\lambda(x_{10j+1})}}. x_{10j+1} \Leftrightarrow gM(x_{10j+1}) = x_{10j+1} M(g^{3^{\lambda(x_{10j+1})}})$$
$$\Leftrightarrow g A_{j_1} M_{11}{}^k M_2{}^{i-1} = e A_{j_1} M_{11}{}^k M_2{}^{i-1} M_2{}^{3^{\lambda(x_{10j+1})}}$$

$$\Leftrightarrow g\, A_{j_1}\, M_{11}{}^k = e\, A_{j_1}\, M_{11}{}^k M_2{}^{3^{\lambda(x_{10}j+1)}}$$

$$\Leftrightarrow g = e\, (A_{j_1}\, (\, M_{11}{}^k M_2{}^{3^{\lambda(x_{10}j+1)}}\, M_{11}{}^{-k})\, A_{j_1}{}^{-1})$$

$$\Leftrightarrow \lambda(\,x_{10j+1}) = 0 \quad \text{when } j = 0, 3, 4, 6$$
$$= 2 \qquad j = 1, 2.\,5, 7 \tag{I}$$

For $0 \le j \le 7, 1 \le i \le 10$ define $\lambda(\,x_{10j+i}) = \lambda(\,x_{10j+1})$ \hfill II

Now we see $\quad X_{10j+i}\cdot g = g^{3^{\lambda(x_{10}j+i)}}\cdot x_{10j+i} \Leftrightarrow gM(x_{10j+i}) = x_{10j+i}M(g^{3^{\lambda(x_{10}j+1)}})$

$$\Leftrightarrow g\, A_{j_1}\, M_{11}{}^k M_2{}^{i-1} = e\, A_{j_1}\, M_{11}{}^k M_2{}^{i-1} M_2{}^{3^{\lambda(x_{10}j+1)}}$$

$$\Leftrightarrow g\, A_{j_1}\, M_{11}{}^k = e\, A_{j_1}\, M_{11}{}^k M_2{}^{3^{\lambda(x_{10}j+1)}}$$

$$\Leftrightarrow x_{10j+1}\cdot g = g^{3^{\lambda(x_{10}j+1)}}\cdot x_{10j+1}$$

From the above it is clear that the mapping $\lambda: Q^* \rightarrow Z_4$ (integers modulo 4) defined in I and II satisfy the property $x.g = g^{3^{\lambda(x)}}\cdot x$ for all $x \in Q^*$. [By theorem in 13pp 541] V-W system is a λ- system.

3. COLLINEATIONS OF THE TRANSLATION PLANE π

Any non- singular linear transformation on $V=F^8$ induces a collineation of π fixing the point corresponding to the zero vector if and only if the linear transformation permutes the subspaces V_i, $0 \le I \le 81$ among themselves. Equivalently, a non singular linear transformation $T = \begin{bmatrix} B & C \\ D & E \end{bmatrix}$, where B, C, D and E are 4x4 matrices over F, induces a collineation of π fixing the point corresponding to the zero vector if and only if the following conditions (a) and (b) are satisfied.[17, Theorem 1]

(a) If D is non-singular, then $D^{-1}E \in \mathscr{C}$. if D is singular then D is the zero matrix and E is non-singular.

(b) For $M \in \mathscr{A}$ f $(B + MD)$ is non-singular, then $(B+MD)^{-1}(C+ME) \in \mathscr{C}$ if $(B+MD)$ is singular then $(B+MD)$ is the zero matrix and $(C+ME)$ is non- singular.

The group of all collineations leaving the point corresponding to the zero vector of π invariant is called the translation complement of π. Throughout this paper, by a collineation we mean a collineation from the translation complement of π.

3.1. Collineations corresponding to the Left and middle nuclei

The mappings $\alpha = \begin{bmatrix} I & 0 \\ 0 & M_2 \end{bmatrix}$, $\beta = \begin{bmatrix} I & 0 \\ 0 & M_{11} \end{bmatrix}$, $\gamma_1 = \begin{bmatrix} M_2^{-1} & 0 \\ 0 & I \end{bmatrix}$, $\gamma_2 = \begin{bmatrix} M_{11}^{-1} & 0 \\ 0 & I \end{bmatrix}$ are all collineations of π and the actions of the collineations α, β on the set of i.ps. of π are furnished below:

α : (0) (81) (1, 2,…, 10) (11, 12,…, 20) (21, 22,…,30) (31, 32,…,40)
　　(41, 42,…, 50) (51, 52,…, 60) (61, 62,…, 70) (71, 72,…, 80)

β : (0) (81) (1, 11, 6, 16) (2, 20, 7, 15) (3, 19, 8, 14) (4, 18, 9, 13) (5, 17, 10, 12) (21, 31, 26, 36) (22, 40, 27, 35)
　　(23, 39, 28, 34) (24, 38, 29, 33) (25, 37, 30, 32) (41, 51, 46, 56) (42, 60, 47, 55) ((43, 59, 48, 54)
　　(44, 58, 49, 53) (45, 57, 50, 52) (61,71,66,76) (62,80,67,75) (63,79,68,74) (64,78,69,73) (65,77,70,72)

Also $\quad \gamma_i^{-1}\alpha\,\gamma_i = \alpha, \quad \gamma_i^{-1}\beta\,\gamma_i = \beta$, $\quad$ i = 1,2.

The actions of the collineations γ_1, γ_2 on the set of i.ps of π are furnished below.

γ_1: (0) (81) (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) (11, 20, 19, 18, 17, 16, 15, 14, 13, 12)
　　(21, 30, 29, 28, 27, 26, 25, 24, 23, 22) (31, 32, 33, 34, 35, 36, 37, 38, 39, 40)
　　(41, 42, 43, 44, 45, 46, 47, 48, 49, 50) (51, 60, 59, 58, 57, 56, 55, 54, 53, 52)
　　(61, 62, 63, 64, 65, 66, 67, 68, 69, 70) (71, 80, 79, 78, 77, 76, 75, 74, 73, 72)

γ_2 : (0) (81) (1, 11, 6, 16) (2, 12, 7, 17) (3, 13, 8, 18) (4, 14, 9, 19) (5, 15, 10, 20)
　　(21, 32, 26, 37) (22, 33, 27, 38) (23, 34, 28, 39) (24, 35, 29, 40) (25, 36, 30, 31)
　　(41, 59, 46, 54) (42, 60, 47, 55) (43, 51, 48, 56) (44, 52, 49, 57) (45, 53, 50, 58)
　　(61, 80, 66, 75) (62, 71, 67, 76) (63, 72, 68, 77) (64, 73, 69, 78) (65, 74, 70, 79)

Homology groups; [9, pp 385]: From the left nucleus of the plane and the collineations α, β it is clear that <α, β> is the ((∞), [0, 0]) - homology group H_1 of π. From the middle nucleus and the collineations γ_1, γ_2 of π < γ_1, γ_2 > is the ((0), [0])-homology group H_2 of π. Both homology groups are meta cyclic groups of order 20. The collineation group <H_1, H_2> = < α, β, γ_1, γ_2 > divides the set of i.ps of π into six orbits $\mathscr{C}_i$, $1 \leq i \leq 6$ of lengths 1, 1, 20, 20, 20, 20 respectively where $\mathscr{C}_1 = \{0\}$, $\mathscr{C}_2 = \{81\}$, $\mathscr{C}_3 = \{$ i $\mid$ $1 \leq$ i $\leq 2 0\}$, $\mathscr{C}_4 = \{$ i $\mid 21 \leq$ i $\leq 40\}$, $\mathscr{C}_5 = \{i \mid 41 \leq$ i $\leq 60\}$, $\mathscr{C}_6 = \{$ i $\mid 61 \leq$ i $\leq 80\}$

3.2. Conjugacycollineations of the plane

A mapping δ = $\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$, where A $\in$ GL(4, 3) induces a conjugation collineation of π if $A^{-1} \mathscr{C} A = \mathscr{C}$. The set of all conjugation collineations of π forms a group called the conjugation collineation group, and this group fixes the ideal points corresponding to V(0),V(∞), and V(I). Conjugacy collineations of the plane keeps the left and middle nuclei of $\mathscr{C}$ invariant. From Table 1the matrices M_i ,i = 3,5,79 are the only matrices with C.P [1111] and the matrices M_i, $11 \leq i \leq 2 0$ are the only matrices with C.P [0201] and the matrices M_i, i = 41,43 are the only matrices with C.P. [0121]. So every collineation either fixes the i.ps 41 and 43 or flips them while keeping the set of i.ps S = {3, 5, 7.9} and S' = {i $\mid$ $11 \leq$ i $\leq 2 0$} invariant separately. In order to keep the set of i.ps of S and S' invariant under δ the matrix A of δ belong to the following sets:

$K_1 = Z(M_3) \cap Z(M_{41})$ $K_4 = Z(M_3) \cap T(M_{41}, M_{43})$
$K_2 = T(M_3, M_9) \cap Z(M_{41})$ $K_5 = T(M_3, M_9) \cap T(M_{41}, M_{43})$
$K_3 = T(M_3, M_7) \cap Z(M_{41})$ $K_6 = T(M_3, M_7) \cap T(M_{41}, M_{43})$

The sets K_2, K_3, K_4 and K_6 are empty. $K_1 = Z(M_3)$, $K_5 = T(M_3, M_9)$. No conjugacy collineation maps the i.p 3 onto the i.p 7 and every conjugation collineation which fixes the i.p. 3 also fixes the i.p. 41 and every conjugation collineation that flips the i.ps. 3,9 also flips the i.ps 41,43. It follows that every conjugation collineation either fixes the i.p.3 or flips the i.ps. 3, 9 while keeping the set of i.ps. of S' invariant.

Also since $M_\lambda \cap M_\mu = \mathscr{G}$.each matrix of $\mathscr{G}$ induces a conjugacycollineation. Let $\delta_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$ where A = A(1, 1) $\in$ Z (M_3) $\cap$ Z (M_{11}). The collineation $\delta_1$fixes all the i.ps.of the plane π and $G_{0,81,1,3,11} = < \delta_1 > \cong GF^*(3^2)$ and it is of order 8 .

If the mapping δ fixes the i.p.3 and maps the i.p. 11 onto 12, then the matrix A of δ belongs to Z (M_3) $\cap$ T (M_{11},M_{12}) where

$$Z (M_3) \cap T (M_{11,} M_{12}) = \left\{ A(c,d) = \begin{pmatrix} 2c + 2d & 2c & c & d \\ d & 2c & 2c & c \\ c & c + d & 2c & 2c \\ 2c & 0 & c + d & 2c \end{pmatrix} \mid (c,d) \neq (0,0), c, d \in F \right\}$$

If A = A(0,1) = (2001, 1000, 0100, 0010), then $A^{-1}M_{21} A = M_{22}$, $A^{-1}M_{41}A = M_{41}$, $A^{-1}M_{61}A = M_{61}$.

The matrix A induces a conjugation collineation $\delta_2 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$

The action of the conjugation collineation δ_2 on the set of i.ps. of πis furnished below:
δ_2: (0) (81) (1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11, 12, 13, 14, 15, 16, 17, 18, 19, 20)
   (21, 22, 23, 24, 25, 26, 27, 28, 29, 30) (31) (32) (33) (34) (35) (36) (37) (38) (39) (40)
   (41) (42) (43) (44) (45) (46) (47) (48) (49) (50) (51, 52, 53, 54, 55, 56, 57, 58, 59, 60)
   (61) (62) (63) (64) (65) (66) (67) (68) (69) (70) (71, 72, 73, 74, 75, 76, 78, 79, 80)

Since the collineation δ_2 fixes the i.p.3 which is transitive on the set of i.ps. of S', the collineation group $G_{0,81,1,3}$ is transitive on S'. Now
    $G_{0,81,1,3} = \bigcup_{i=0}^{9} G_{0,81,1,3,11}\delta_2^{\ i} = <\delta_1, \delta_2> = <\delta_2>$ since $\delta_2^{10} = \delta_1^{\ 5}$

    $\mid G_{0,81,1,3} \mid = \mid S' \mid . \mid G_{0,81,1,3,11} \mid = 10 \times 8 = 80$

Let $\delta_3 = \gamma_2^{-1} \beta$. The mapping δ_3 is a collineation of π since it is the product of two collineations and flips the i.ps. 3 and 9. The action of δ_3 can be computed from the actions of γ_2 and β and it is given by
δ_3 : (0) (81) (1) (2, 10) (3, 9) (4, 8) (5, 7) (6) (11) (12, 20) (13, 19) (14, 18) (15, 17) (16) (21, 30)
   (22, 29) (23, 28) (24, 27) (25, 26) (31, 32) (33, 40) (34, 39) (35, 38) (36, 37) (41, 43) (42) (44, 50) (45, 49)
   (46, 48) (47) (51, 59) (52, 58) (53, 57) (54, 56) (55) (60) (61, 62) (63, 70) (64, 69) (65, 68) (66, 67) (71, 80)
   (72, 79) (73, 78) (74, 77) (75, 76).

A coset decomposition of $G_{0,81,1}$ is given below

$$G_{0,81,1} = G_{0,81,1,3} \cup G_{0,81,1,3}\delta_3 = <\delta_2, \delta_3>$$

$$|G_{0,81,1}| = 2\,|G_{0,81,1,3}| = 2 \times 80 = 160$$

Thus every collineation of π that fixes the i.ps. 0, 81, 1, 3, 11 also fixes all the i.ps. of π.

Therefore the collineation group $G_{0, 81, 1, 3, 11}$ accounts for the $((0, 0), (\infty))$ – homology group H_3 of π and the kernel K of π (or the V–W system Q) is GF(3^2)

3.3. Autotopism Collineation Group $G_{0,81}$ of π

Let $\theta = \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$ where A = (0010, 0001, 1100, 0110), B = $A\,M_{61}^{-1}$ = (0100, 0010, 0001, 1100)

The mapping θ maps the matrix M $\in \mathscr{C}$ onto the matrix $M_{61}A^{-1}M^{-1}A$ and induces a collineation of π if θ permutes the subspaces V_i, $1\leq i \leq 80$ among themselves. It may be seen that
θ: $M_2{\rightarrow}M_{10}$, $M_{11}{\rightarrow}M_{74}$, $M_{21}{\rightarrow} M_{54}$, $M_{41}{\rightarrow} M_{37}$, $M_{61}{\rightarrow} M_1$

Further θ maps the subspaces corresponding to the i.ps. 0, 81, 1 onto 81, 0, 61 respectively and $V_{21}\theta = V_{54}$, $V_{41}\theta = V_{37}$, $V_{61}\theta = V_1$ Also $\theta^{-1}\alpha\theta = \gamma_1^{-1}$, $\theta^{-1}\beta\theta = \gamma_2^{-1}\gamma_1^{-9}$. From these relations we get the following:

$V_{1+i}\theta = V_{61+k_1}$ where $k_1 \equiv 9i\ (mod\,10)$

$V_{11+i}\theta = V_{71+k_2}$ where $k_2 \equiv 3 + i\ (mod\,10)$

$V_{21+i}\theta = V_{51+k_2}$

$V_{31+i}\theta = V_{41+k_3}$ where $k_3 \equiv 6 + 9i\ (mod\,10)$

$V_{41+i}\theta = V_{31+k_3}$

$V_{51+i}\theta = V_{21+k_4}$ where $k_4 \equiv 4 + i\ (mod\,10)$

$V_{61+i}\theta = V_{1+k_1}$ and $V_{71+i}\theta = V_{71+k_2}$

From the above it is clear that θ permutes the subspaces of the spread $\mathscr{S}$ among themselves. Thus θ is a collineation of π whose action on the set of i.ps. of π is as follows:

θ :   (0, 81) (1, 61) (2, 70) (3, 69) (4, 68) (5, 67) (6, 66) (7, 65) (8, 64) (9, 63) (10, 62)

      (11, 74, 18, 71, 15, 78, 12, 75, 19, 72, 16, 79, 13, 76, 20, 73, 17, 80, 14, 77)

      (21, 54, 28, 51, 25, 58, 22, 55, 29, 52, 26, 59, 23, 56, 30, 53, 27, 60, 24, 57)

      (31, 47) (32, 46) (33, 45) (34, 44) (35, 43) (36, 42) (37, 41) (38, 50) (39, 49) (40, 48)

Lemma 3.4: (a) No collineation of π flips the i.ps. 0, 81 and fixes the i.p.1.

Proof: A collineation with the required action on the set of i.ps. of π is a mapping μ of the form $\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$, A $\in$ GL(4,3) is such that for each non zero matrix M $\in \mathscr{C}$ there is a matrix N $\in \mathscr{C}$ with the property $A^{-1} M^{-1} A = N$. i.e., the spread $(\mathscr{C})^{-1}$ is isomorphic to $\mathscr{C}^{*}$.

If M $\in \mathscr{C}$, M $\neq$ 0 has C.P. [abcd] then d $\neq$ 0 and the C.P. of M^{-1} is [$d^{-1}c$, $d^{-1}b$, $d^{-1}a$, d^{-1}].

It may be seen that $\mathscr{C}^{*}$ and $(\mathscr{C})^{-1}$ have the same C.P. structure. Further a close examination of the matrices of $\mathscr{C}$ reveals that M_{42}, M_{47} are the only matrices with C.P.[2211], [1221] respectively; M_{31}, M_{32} are the only two matrices in $\mathscr{C}$ with C.P. [2002] and M_{66}, M_{67} are the only two matrices in $\mathscr{C}$ with C.P.[0012]. If μ is a collineation of π flipping the i.ps. 0, 81 and fixing the i.p.1 then μ must map the i.p. 42 onto the i.p. 47 while mapping the set of i.ps. {31, 32} onto the set of i.ps. {66, 67}, besides mapping the set of i.ps. S = {3, 5, 7, 9} among themselves. The matrix A of μ belongs to the following sets:

    $E_1 = Z(M_3) \cap T(M_{42}^{-1}, M_{47})$

    $E_2 = T(M_3, M_9) \cap T(M_{42}^{-1}, M_{47})$

    $E_3 = T(M_3, M_7) \cap T(M_{42}^{-1}, M_{47})$

    $E_4 = T(M_3, M_5) \cap T(M_{42}^{-1}, M_{47})$

A simple computation shows that the sets E_1 and E_2 are empty. Now concentrating on the action of μ on the i.p. 31 the mapping μ either maps the i.p. 31 onto the i.p. 66 or onto the i.p. 67. Under these circumstances the matrix A of μ belongs to the following sets:

    $E_3 \cap T(M_{31}^{-1}, M_{66})$, $E_3 \cap T(M_{31}^{-1}, M_{67})$, $E_4 \cap T(M_{31}^{-1}, M_{66})$, $E_4 \cap T(M_{31}^{-1}, M_{67})$

On computing we find all the above sets are empty. This shows that no collineation of π is of the form $\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$. That is no collineation of π flips the i.ps. 0, 81 and fixes the i.p. 1. Hence the lemma.

We now concentrate on the existence or otherwise of the collineations of π which fix the i.p. 0, 81 and move the i.p. 1 using the following lemma

Lemma 3.5:
    (a) No collineation of π fixes the i.ps. 0, 81 and maps the i.p.1 onto the i.p.21
    (b) No collineation of π flips the i.ps. 0, 81 and maps the i.p.1 onto the i.p.21
    (c) No collineation of π fixes the i.ps. 0, 81 and maps the i.p.1 onto the i.p. k where k ∈ {41, 61}

Proof: If η is a collineation fixing the i.ps. 0, 81 and maps the i.p.1 onto the i.p.k then η is of the form $\begin{pmatrix} B & 0 \\ 0 & A \end{pmatrix}$, A, B ∈ GL(4,3), B = AM_k^{-1} such that for each matrix M ∈ $\mathscr{C}$ there is a matrix N ∈ $\mathscr{C}$ satisfying $N = MkA^{-1}MA$ i.e., $A^{-1}MA = M_k^{-1}N$. From this it follows that if η is a collineation then $\mathscr{C}$ and $M_k^{-1}\mathscr{C}$ are conjugate.

Let k = 21 and $\mathscr{C} = M_{21}^{-1}\mathscr{C} = \{N_i \mid N_i = M_{21}^{-1}M_i, M \in \mathscr{C}, 1 \le i \le 80\}$. Some spot checks show that there are 13 matrices N_i, $1 \le i \le 10$, i= 14, 19, 41 of $\mathscr{C}$ with C.P. [0202], where as the spread set $\mathscr{C}$ has only 12 matrices M_i $71 \le i \le 80$ and i =64, 69. Thus the spread set $\mathscr{C}$ and $\mathscr{C}^i$ cannot be conjugate. From this the truth of the part (a) of the lemma follows.

If μ is a collineation which flips the i.ps.0, 81 and maps the i.p. 1 onto the i.p. 21 then μ is of the form $\begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}$, A,B ∈ GL(4,3), B = AM_{21}^{-1} such that for each nonzero matrix M ∈ $\mathscr{C}$ there is a matrix N ∈ $\mathscr{C}$ satisfying $M_{21} A^{-1} M^{-1} A = N$ i.e., $A^{-1} M^{-1} A = M_{21}^{-1}N$. From this it follows that $\left(\mathscr{C} \right)^{-1}$ is conjugate to $M_{21}^{-1} \mathscr{C}$. It is already observed (by the proof of lemma 1) that $\mathscr{C}$ and $\left(\mathscr{C} \right)^{-1}$ have the same C.P. structure. It now follows that $\left(\mathscr{C} \right)^{-1}$ and $M_{21}^{-1} \mathscr{C}$ cannot be conjugate on comparing the number of matrices with C.P.[0201] in both the sets as in the proof of part(a). Thus no collineation of π flips the i.ps. 0, 81 and maps the i.p. 1 onto the i.p. 21.

Suppose η_2 is a collineation fixing the i.p. 0, 81 and mapping the i.p. 1 onto i.p. 41. We have a collineation $\tau = \theta^3\beta^{-1}$ which flips the i.p. 0, 81 and maps the i.p. 21 onto the i.p.41. Now $\mu\tau^{-1}$ is a collineation flipping the i.ps. 0, 81 and mapping the i.p. 1 onto the i.p. 21 – a contradiction to part (b) of the lemma. This shows that no collineation of π fixes the i.ps. 0, 81 and maps the i.p. 1 onto the i.p. 41. This proves the lemma.

If η_3 is a collineation fixing the i.ps. 0, 81 and mapping the i.p. 1 onto the i.p. 61 then $\theta\mu^{-1}$ is a collineation which flips the i.ps. 0, 81 and fixes the i.p. 1 – a contradiction to lemma 1.from this the other part of part (a) of the lemma follows. Hence the lemma. We now compute the autotopism group $G_{0,81}$ of π.

We compute the autotopism group $G_{0,81}$ of π

Theorem 3.6: The autotopism group $G_{0,81}$ of π is given by $G_{0,81} = <\alpha, \beta, \delta_2, \delta_3>$ It is of order 3200 and divides the set of i.ps. into six orbits $\mathscr{O}_i$, $1 \le i \le 6$ of lengths 1, 1, 20, 20, 20, 20.

Proof: The collineation group $H_1 = <\alpha, \beta>$ is a subgroup of $G_{0,81}$ and it is transitive on $\mathscr{O}_i$ -{ i | $1 \le i \le 2 0$}

In view of lemma 3.5 no collineation of π moves the i.p. 1 onto the i.p. k, k ∈ {21, 41, 61} while fixing the i.ps. 0, 81. From this it follows that no collineation of $G_{0,81}$ maps the i.p.1 onto an i.p. k where k ∈ $\cup_{i=1}^6 O_i'$.
Thus $G_{0,81}$ is transitive on $\mathscr{O}_i$ whose length is 20. A coset decomposition of $G_{0,81}$ is given by
      $G_{0,81} = \cup_{i=1}^{20} G_{0,81,1} \xi_i$ where $\xi_i \in H_1$
          $= < G_{0,81,1}, H_1> = <\alpha, \beta, \delta_2, \delta_3>$

  and $|G_{0,81}| = |O'_3||G_{0,81,1}| = 20 \times 160 = 3200$

Hence the theorem

The following lemma gives the order of the collineation group G'

Lemma 3.7:
   (a) $G_0 = G_{81} = G_{0,81}$
   (b) The group G' of all collineations which fix the i.ps. 0, 81 or flip them is given by $G' = <G_{0,81}, \theta>$

It is of order 6400 and it divides the set of i.ps. into three orbits $\mathscr{C}_i$, $1 \leq i \leq 3$ of lengths 2, 2, 40 where $\mathscr{C}_1 = \mathscr{C}_1' \cup \mathscr{C}_2'$, $\mathscr{C}_2 = \mathscr{C}_3 \cup \mathscr{C}_6'$, $\mathscr{C}_3 = \mathscr{C}_4' \cup \mathscr{C}_5'$

Proof: The first part of the lemma follows by a result of Foulser [9, pp.390] since π is a λ- plane with proper kern. Thus no collineation of π fixes the i.p.0 (81) and moves the i.p. 81(0). Therefore every collineation of π that fixes the i.p. 0 also fixes the i.p. 81. The first part of lemma now follows.

The collineation group $< G_{0,81}, \theta >$ divides the set of i.ps. of π into three orbits $\mathscr{C}_i$, $1 \leq i \leq 3$ of lengths 2,40,40 where $\mathscr{C}_1 = \mathscr{C}_1' \cup \mathscr{C}_2'$, $\mathscr{C}_2 = \mathscr{C}_3 \cup \mathscr{C}_6'$, $\mathscr{C}_3 = \mathscr{C}_4' \cup \mathscr{C}_5'$. Suppose$\mu$ is a coolineation flipping the i.ps 0, 81 and moving the i.p. 1 onto the i.p. k of $\mathscr{C}_3$.

If k $\in \mathscr{C}_4'$ then by transitivity of $G_{0,81}$ on $\mathscr{C}_4'$ there exists a collineation $\tau \in G_{0,81}$ mapping the i.p. k onto the i.p. 21. Then the collineation $\mu \tau$ flips the i.ps. 0 and 81 and maps the i.p. 1 onto i.p. 21 – a contradiction.

If k$\in \mathscr{C}_5'$ then by transitivity of $G_{0,81}$ on $\mathscr{C}_5'$ there exists a collineation $\tau \in G_{0,81}$ mapping the i.p. k onto the i.p. k onto the i.p. 41. Then the collineation $\mu\tau\beta\theta^{-3} \epsilon G_{0,81}$ mapping the i.p. 1 onto i.p. 21 – a contradiction.

From the above discussion we conclude that no collineation of π maps i.p. of $\mathscr{C}_2$ onto $\mathscr{C}_3$, while flipping the i.ps. 0, 81.

Thus any flipping collineation belongs to $G_{0,81} \theta$. Therefore the group G' of all collineations that fix the i.p. 0, 81 or flips them is given by
   $G' = G_{0,81} \cup G_{0,81}\theta$
    $= <G_{0,81}\theta >$

Since G' is transitive on $\mathscr{C}_1$ whose length is two,
   $|G'| = |\mathscr{C}_1| \, |G_0|$
      $= |\mathscr{C}_1| \, |G_{81}| = |\mathscr{C}_1| \, |G_{0,81}| = 2 \times 3200 = 6400$
Hence the theorem.

4. THE TRANSLATION COMPLEMENT OF π

This section is devoted to determine the translation complement of the translation plane π. It is shown that there are no collineations of π other than G' i.e., the translation complement G is G'. To the end it is shown that the translation planes π_9 and π are not isomorphic even though both the planes are generalized andre planes with translation complements of the same order by comparing their kernels and the action of the translation complements on the sets of i.ps. of the planes.

Lemma 4.1: No collineation of π
   (i) Maps the i.p.1(21) onto the i.p. 81 and the i.p. 81 onto the i.p.(0)
   (ii) Maps the i.p.1(21) onto the i.p 0 and the i.p. 0 onto the i.p.81
   (iii) Maps the i.p. k onto the i.p. 81(0) and the i.p. 81(0) onto the i.p.0(81) where k$\neq$ 0,81.

Proof: The first two parts of the lemma follows from lemma 2.1, the property (2.1) and the following observations:
   $M_1 + M_{21} = (1001, 1101, 1102, 1222) \notin \mathscr{C}_{10}$
   $(M_1^{-1} + M_{21}^{-1})^{-1} = (0210, 1000, 1201, 0121) \notin \mathscr{C}_{10}$

Assume that μ is a collineation of π mapping the i.p. k onto the i.p. 81 and the i.p. 81 onto the i.p. 0, k $\neq$ 0, 81. If k$\in \mathscr{C}_2$ then there exists a collineation τ flipping the i.ps. 0, 81 and mapping the i.p. 1 onto the i.p. k. Hence $\tau\mu\tau^{-1}$ is a collineation of π which maps the i.p. 1 onto the i.p. 0 and the i.p. 0 onto the i.p. 81 – a contradiction to a part of this lemma. If k $\in \mathscr{C}_3$ then there exists a collineation $\tau \in G_{0,81}\theta$ mapping the i.p. 21 onto the i.p. k. now $\tau\mu\tau^{-1}$ is a collineation of π mapping the i.p. 21 onto the i.p. 0 and the i.p. 0 onto the i.p. 81 – a contradiction to a part of this lemma. The truth of the last part of the lemma now follows.

Lemma4.2: None of the following 3- Spread sets of π:
   $\Gamma_{i,j,k}$ i = 1, j $\in$ {6, 42, 47}, k $\neq$ i, k $\neq$ j is conjugate to $\mathscr{C}$ where
   $\Gamma_{i,j,k} = \{N_l = [(M_l - M_i)^{-1} - (M_j - M_i)^{-1}] \, [(M_k - M_i)^{-1}]^{-1} \mid M_l \in \mathscr{C}_{10}, 1 \leq l \leq 80\}$

Proof: we have already observed that $\mathscr{C}$ has a property (2.1). if $\mathscr{C}$ and $\Gamma_{i,j,k}$ are conjugate then $\Gamma_{i,j,k}$ must also have the property (2.1) i.e., for every $N \in \Gamma_{i,j,k}$ we must have $-N \in \Gamma_{i,j,k}$. It may be seen that $\Gamma_{i,j,k}$ has the property (2.1) if and only if $\Gamma_{i,j}$ has the property (2.1), where

$\Gamma_{i,j} = \{N_l = (M_l - M_i)^{-1} - (M_j - M_i)^{-1} \mid M_l \in \mathscr{C} \{1 \leq l \leq 80\}$. Thus it is enough to consider $\Gamma_{i,j}$ to discuss about the property (2.1).

If $i = 1$, $j = 6$ then $N_{41} = (1101, 1210, 0121, 1112) \in \Gamma_{1,6}$. Assume that $-N_{41} \in \Gamma_{1,6}$. Now we look for the matrix M in $\mathscr{C}$ for which $-N_{41} \in \Gamma_{1,6}$ .A simple computation shows M = (1012, 0120, 1002, 0110) and we notice M $\in \mathscr{C}$ This shows that $\Gamma_{1,6}$ does not possess the property (2.1) and there by $\Gamma_{1,6,k}$, $k \neq 1,6$ does not posses the property (2.1) and hence $\Gamma_{1,6,k}$ is not conjugate to $\mathscr{C}$.

If $i = 1$, $j = 42$ the $N_{21} = (1120, 2100, 2112, 0210) \in \Gamma_{1,42} \Rightarrow -N_{21} \in \Gamma_{1,42} \Rightarrow$ M= (0101, 0120, 1002, 0110) $\in \mathscr{C}$ a contradiction. Similarly we conclude $\Gamma_{1,42,k}$ is not conjugate to $\mathscr{C}$.

If $i = 1$, $j = 47$ then $N_{21} = (2120, 2200, 2122, 0211) \in \Gamma_{1,47}$. $-N_{21} \in \Gamma_{1,47} \Rightarrow$ M = (1121, 0001, 0221, 1100) $\in \mathscr{C}$ a contradiction. Thus $\Gamma_{1,47,k}$ is not conjugate to $\mathscr{C}$.

If $i = 42$. $j = 1$ (6) (47) then $N_{21} = (1001, 1101, 1102, 1222) \in \Gamma_{42,1}$.
$N_{21} = (2222, 2020, 0121, 0021) \in \Gamma_{42,6}$, $N_{21} = (2001, 1201, 1112, 1210) \in \Gamma_{42,47}$ respectively.
  $-N_{21} \in \Gamma_{41,1,} \Rightarrow$ M = (0102, 0122, 1000, 0110) $\in \mathscr{C}_{10}$ - a contradiction.
  $-N_{21} \in \Gamma_{41,6} \Rightarrow$ M= (2012, 1020, 2220, 0200) $\in \mathscr{C}_{10}$ - a contradiction.
  $-N_{21} \in \Gamma_{42,47} \Rightarrow$ M= (0222, 0010, 2011, 0221) $\in \mathscr{C}_{10}$ - a contradiction.

From the above, the spread sets, $\Gamma_{42,1,k}$, $\Gamma_{42,6,k}$, $\Gamma_{42,47,k}$ are not conjugate to $\mathscr{C}_{10}$ as these spread sets do not have the property (**2.1**). Hence the lemma.

Corollary 4.3:
  (a) No collineation of π maps the i.p. 81 onto the i.p. 1 and the i.p. 0 onto the i.p. k where $k \in \{6, 42, 47\}$.
  (b) No collineation of π maps the i.p. 81 onto the i.p. 42 and the i.p. 0 onto the i.p.k, $k \in \{1, 6, 47\}$.

Proof: By a result of Maduram on the Matrix representative sets associated with translation planes there exists a collineation mapping i.ps. 0, 81, 1 onto the i.ps. i, j, k respectively if and only if the matrix representative sets with these fundamental subspaces are conjugate i.e., $\mathscr{C}$ and $\Gamma_{i,j,k}$ are conjugate. The truth of the corollaries follow from the above lemma. Hence the result.

Lemma 4.4:
  (a) No collineation of π moves the i.p. 81 onto the i.p.1
  (b) No collineation of π moves the i.p. 81 onto the i.p.42
  (c) No collineation of π moves the i.ps. 0, 81 outside the orbit $\mathscr{O}_r$.

Proof: By the actions of δ_2, δ_3 we have already noticed that they fix i.ps. 1, 6, 42, 47 besides the i.ps. 0, 81 and moves all the remaining i.ps. By lemma 2.2 the i.p. 81 may be mapped onto one of the i.ps. i of the set S" = {1, 6, 42, 47} and the i.p. 0 onto one of the i.ps. j of S", j ≠ i. This means, if there is a collineation mapping the i.p. 1, then that collineation must map the i.p. 0 onto the i.p. k, k ∈ {6, 42, 47}. Further if there is a collineation which maps the i.p. 81 onto the i.p. 42 then this collineation must map the i.p. 0 onto the i.p. k, k ∈ {1, 6, 47}. By the corollary 7 no collineation of π maps the i.p. 81 onto the i.p. 1 and the i.p. 0 onto the i.p. k, k {6, 42, 47}.From this it may be concluded that no collineation of π maps the i.p. 81 onto the i.p. 1. Also no collineation of π maps the i.p. 81 onto the i.p. 42 and the i.p. 0 onto the i.p. k, k ∈ {1, 6, 47}. From this it may be concluded that no collineation of π maps the i.p. 81 onto the i.p. 42. This proves parts (a) and (b) of the lemma.

If μ is a collineation mapping the i.p. 81 onto the i.p. 6 then the i.p. 0 must be mapped onto one of the i.ps. of the set {1, 42, 47}. Now $\alpha^{-5}\mu\alpha^5$ is a collineation which moves the i.p. 81 onto the i.p. 1 – a contradiction.

If μ is a collineation of π mapping the i.p. 81 onto the i.p. 47 then the i.p. 0 must be mapped onto one of the i.ps. of the set {1, 6. 42}. Now the collineation $\alpha^{-5}\mu\alpha^5$ maps the i.p. 81 onto the i.p. 42- a contradiction.

By part (a) of lemma **3.7** no collineation of π fixes one of the i.ps. of $\mathscr{C}_1$ and moves the other i.e., every collineation of π fixes one of the i.ps. of $\mathscr{C}_1$ also fixes the other. By part (iii) of lemma 4.1 no collineation of π maps the i.p. k onto one of the i.ps. of $\mathscr{C}_1$ via the other i.p. of $\mathscr{C}_1$, $k \neq 0.81$. Assume that the i.p. 0 and 81 move to the i.ps. outside $\mathscr{C}_1$ .Suppose that μ is a collineation mapping the i.p. 81 onto the i.p. k. If k $\in \mathscr{C}_2$

Then there is a collineation $\tau \in G_{0,81}\,\theta$ mapping the i.p. k onto the i.p. 1. Now the collineation $\mu\,\tau$ maps the i.p. 81 onto the i.p. 1- a contradiction to part(a) of the lemma. If $k \in \mathscr{C}_3$ then there is a collineation. $\tau \in G_{0,81}\,\theta$ mapping the i.p. k onto the i.p. 42. Now the collineation $\mu\,\tau$ maps the i.p. 81 onto the i.p. 42 – a contradiction to part (b) of the lemma. From this the truth of the third part of the lemma follows

Theorem 4.5: The translation complement G of the translation plane π is given by G' = < G $_{0,81}$, θ> and it is of order 6400. G divides the set of i.ps. of π into 3 orbits $\mathscr{C}_i$, $1 \le i \le 3$ of lengths 2, 40, 40 where $\mathscr{C}_1$= { 0,81}, $\mathscr{C}_2$={i│ $1 \le i \le 2$ 0 , $61 \le i \le 8$ 0}, $\mathscr{C}_3$ -{i│$21 \le i \le 6$ 0}

Proof: In view of lemma 3.7 and lemma 4.4(c), every collineation of π either fixes both the i.ps. 0, 81 or flips them. From this it follows that G = G'. The rest of the theorem is evident

Theorem 4.6: The translation planes π is not isomorphic to the plane already reported. [19].

Proof: Both the translation planes π and the plane reported in [19] are generalized Andre planes with the order of the translation complement 6400. These two planes are distinct in view of the orbit structure of the set of i.ps. under the translation complement 40, 40 and 2, 80. Moreover these two planes are not isomorphic since the kernel of the plane reported is trivial and the kernel of π is isomorphic to GF(3^2). Hence the result.

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