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COMMON FIXED POINT THEOREMS AND GENERALIZATION OF REICH TYPE MAPPING IN CONE METRIC SPACES

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ABSTRACT

The object of this paper is to establish some common Fixed Point Theorems for weakly compatible pair of self maps in cone metric spaces which is not necessarily normal. It turns out the generalization of results of Huang and Zhang [3], *also we prove the existence of unique common fixed point of sequence of mappings satisfying contractive condition in cone metric space.*

Keywords: *Cone metric space, weakly compatible mapping, common fixed point.*

AMS (1991): *Subject classification: 54H25, 47H10.*

1. INTRODUCTION:

Huang and Zhang [3] initiate the concept of cone metric space as a generalization of metric space by replacing the set of real numbers by an ordered Banach space and gave some fundamental results for a self map satisfying a contractive condition assuming the normality condition in cone metric space. Abbas and Jungck [4] proved some common fixed point theorems in a cone metric space with normality condition. In 2008 Rezapour and Hamlbarani [6] omitted the assumption of normality in cone metric space. The concept of compatibility in cone metric space is introduced by Jain, Jain and Lal Bahadur in [5].

Here we give some common Fixed Point Theorems for four self mappings of a cone metric space and then generalizing the theorem for a sequence of mappings in cone metric spaces without using the normality of cone. It generalizes and extends several known fixed point results in cone metric space.

2. PRELIMINARIES:

Definition 2.1 [3]: *Let E be a real Banach space and P be a subset of E. The set P is called a cone if*

(i) P *is closed, nonempty and* $P \neq \{0\}$, *here* 0 *is the zero vector of* E;
(ii) $a, b \in \Re, a, b \geq 0, x, y \in P \Rightarrow ax + by \in P$;
(iii) $x \in P$ *and* $-x \in P \Rightarrow x = 0$.

Given a cone $P \subset E$, *we define a partial ordering "$\leq$" with respect to* P *by* $x \leq y$ *if and only if* $y - x \in P$. *We write* $x < y$ *to indicate that* $x \leq y$ *but* $x \neq y$. *While* $x << y$ *if and only if* $y - x \in P^0$, *where* P^0 *denotes the interior of* P.

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A cone *P* is called normal if $x, y \in E, 0 \le x \le y \Rightarrow \|x\| \le K\|y\|$ where K>0. The least positive number K satisfies above inequality is called the normal constant of *P*.

Proposition 2.2 [5]: Let P be a cone in a real Banach space E. If for $a \in P$ and $a \le ka$, for some $k \in [0,1)$ then $a = 0$.

Proposition 2.3 [2]: Let P be a cone in a real Banach space E. If for $a \in E$ and $a << c$, for all $c \in P^0$ then $a = 0$.

Proposition 2.4 [5]: Let P be a cone in a real Banach space E. If $a << b$ and $b << c$ then $a << c$; and if $a \le b$ and $b << c$ then $a << c$.

Definition 2.5 [3]: *Let X be a nonempty set, E be a real Banach space. Suppose that the mapping $d : X \times X \to E$ satisfies;*

(i) $0 \le d(x, y)$*, for all* $x, y \in X$ *and* $d(x, y) = 0$ *if and only if* $x = y$*;*
(ii) $d(x, y) = d(y, x)$ *for all* $x, y \in X$*;*
(iii) $d(x, y) \le d(x, z) + d(z, y)$*, for all* $x, y, z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

In the following we always suppose that E is a real Banach space, P is a cone in E with $P^0 \ne \phi$ and "≤" is partial ordering with respect to P.

Definition 2.6 [3]: *Let (X, d) be a cone metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$.*

(i) If for every $c \in E$ with $0 << c$ there is a positive integer n_0 such that for all $n > n_0$, $d(x_n, x) << c$ then the sequence $\{x_n\}$ is said to be converges to x. We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x, (n \to \infty)$.
(ii) If for every $c \in E$ with $0 << c$ there is a positive integer n_0 such that for all $n, m > n_0$, $d(x_n, x_m) << c$ then the sequence $\{x_n\}$ is called a Cauchy sequence in X.

(X, d) *is called a complete cone metric space, if every Cauchy sequence in X is convergent in X.*

Lemma 2.7 [5]: *Let (X, d) be a cone metric space and P be a cone in a real Banach space E and $k_1, k_2, k > 0$. If $x_n \to x, y_n \to y$ in X and $ka \le k_1 d(x_n, x) + k_2 d(y_n, y)$, then $a = 0$.*

Definition 2.8 [2]: *Let A and S be self maps of a set X. If $w = Ax = Sx$, for some $x \in X$, then w is called a coincidence point of A and S.*

Definition 2.9 [1]: *Let X be any se a pair of self maps (A, S) in X is said to be weakly compatible if $u \in X, Au = Su$ imply $SAu = ASu$.*

3. MAIN RESULTS:

Theorem 3.1: Let (X, d) be a complete cone metric space (not necessarily normal). Suppose that A, A_1, A_2, f and g are self maps of X, such that, $AA_1(X) \subseteq g(X)$ and $A_2(X) \subseteq Af(X)$ and satisfying,

$$d(AA_1x, A_2y) \le ad(Afx, gy) + bd(Afx, AA_1x) + cd(gy, A_2y) + e[d(Afx, A_2y) + d(gy, AA_1x)] \tag{3.1.1}$$

for all $x, y \in X$ and $a, b, c, e \in [0,1)$ with $a+b+c+2e<1$. Suppose that the pairs $\{f, A_1\}$ and $\{g, A_2\}$ are weakly compatible, and A commutes with A_1 and f, then the mapping $A, A_1, A_2, f, g, AA_1, AA_2$ and Af have unique common fixed point.

Proof: Let $x_0 \in X$ is arbitrary and we define a sequence $\{y_n\}$ as follows

$AA_1x_{2n} = gx_{2n+1} = y_{2n}$ $Afx_{2n+2} = A_2x_{2n+1} = y_{2n+1}$ now taking $x = x_{2n}$, $y = x_{2n+1}$ in (3.1.1) we get

$$d(AA_1x_{2n}, A_2x_{2n+1}) \le ad(Afx_{2n}, gx_{2n+1}) + bd(Afx_{2n}, AA_1x_{2n}) + cd(gx_{2n+1}, A_2x_{2n+1}) \\ +e[d(Afx_{2n}, A_2x_{2n+1}) + d(gx_{2n+1}, AA_1x_{2n})]$$

$$\Rightarrow d(y_{2n}, y_{2n+1}) \le ad(y_{2n-1}, y_{2n}) + bd(y_{2n-1}, y_{2n}) + cd(y_{2n}, y_{2n+1}) + e[d(y_{2n-1}, y_{2n+1}) + d(y_{2n}, y_{2n})]$$

writing $d_n = d(y_n, y_{n+1})$, we get

$$d_{2n} \le ad_{2n-1} + bd_{2n-1} + cd_{2n} + e[d_{2n-1} + d_{2n}]$$

i.e. $d_{2n} \le \dfrac{a+b+e}{1-c-e} d_{2n-1} = \lambda d_{2n-1}$

where $\lambda = \dfrac{a+b+e}{1-c-e}$.

So, $d_{2n} = \lambda d_{2n-1}$ again taking $x = x_{2n+2}$, $y = x_{2n+1}$ in (3.1.1) we get

$$d(AA_1x_{2n+2}, A_2x_{2n+1}) \le ad(Afx_{2n+2}, gx_{2n+1}) + bd(Afx_{2n+2}, AA_1x_{2n+2}) + cd(gx_{2n+1}, A_2x_{2n+1}) \\ +e[d(Afx_{2n+2}, A_2x_{2n+1}) + d(gx_{2n+1}, AA_1x_{2n+2})]$$

$$d(y_{2n+2}, y_{2n+1}) \le ad(y_{2n+1}, y_{2n}) + bd(y_{2n+1}, y_{2n+2}) + cd(y_{2n}, y_{2n+1}) + ed(y_{2n}, y_{2n+2})$$

or $\quad d_{2n+1} \le ad_{2n} + bd_{2n+1} + cd_{2n} + ed_{2n} + ed_{2n+1}$

i.e. $\Rightarrow d_{2n+1} \le \dfrac{a+c+e}{1-b-e} d_{2n} = \mu d_{2n}$, $\mu = \dfrac{a+c+e}{1-b-e} < 1$

Hence, $d_{2n+1} \le \mu d_{2n}$

Thus $\quad d_{2n} \le \lambda d_{2n-1} \le \lambda\mu d_{2n-2} \le \lambda^2 \mu d_{2n-3} \le \lambda^2 \mu^2 d_{2n-4} .... \le \lambda^n \mu^n d_0$

And $d_{2n+1} \le \mu d_{2n} \le \mu\lambda d_{2n-1} \le \mu^2 \lambda d_{2n-2} \le ... \le \lambda^n \mu^{n+1} d_0$.

Hence $d_{2n} + d_{2n+1} \le \lambda^n \mu^n (1+\mu) d_0$ (3.1.2)

And, $d_{2n+1} + d_{2n+2} \le \lambda^n \mu^{n+1} (1+\lambda) d_0$ (3.1.3)

Now if n is even and $m > n$ are natural no's then (3.1.2) gives

$$d(y_n, y_m) \le d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + ... + d(y_{m-1}, y_m)$$

$$\leq (d_n + d_{n+1}) + (d_{n+2} + d_{n+3}) + \ldots$$
$$\leq \lambda^{n/2}\mu^{n/2}(1+\mu)d_0 + \lambda^{n/2+1}\mu^{n/2+1}(1+\mu)d_0 + \ldots$$
$$\leq (\lambda\mu)^{n/2}(1+\mu)d_0[1+\lambda\mu+\ldots]$$

$$\Rightarrow \quad d(y_n, y_m) \leq \frac{(\lambda\mu)^{n/2}(1+\mu)d_0}{1-\lambda\mu}, \text{ where } d_0 = d(y_0, y_1)$$

Now since $\lambda\mu < 1$ hence right hand side of above inequality converges to 0.

Hence for any $c \in P^0$, We can choose $n_0 \in \mathbb{N}$ such that for all $n > n_0$,

$\frac{(\lambda\mu)^{1/2}(1+\mu)}{1-\lambda\mu} d_0 \ll c$. Hence using proposition 2.4 we get $d(y_n, y_m) << c$ for all $n > n_0, m > n$.

Now if n is odd then again using (3.1.3)

$$d(y_n, y_m) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots + d(y_{m-1}, y_m)$$
$$\leq (d_n + d_{n+1}) + (d_{n+2} + d_{n+3}) + \ldots$$
$$\leq \lambda^{(n-1)/2}\mu^{(n+1)/2}(1+\lambda)d_0 + \lambda^{n+1/2}\mu^{n+3/2}(1+\lambda)d_0 + \ldots$$
$$\leq (\lambda\mu)^{(n-1)/2}\mu(1+\lambda)d_0[1+\lambda\mu+\ldots]$$

$$\Rightarrow \quad d(y_n, y_m) \leq \frac{(\lambda\mu)^{(n-1)/2}\mu(1+\lambda)}{1-\lambda\mu} d_0, \text{ where } d_0 = d(y_0, y_1)$$

Now since $\lambda\mu < 1$ hence right hand side of above inequality converges to 0.

Hence for any $c \in P^0$, We can choose $n_1 \in \mathbb{N}$ such that for all $n > n_1$,

$d(y_n, y_m) << c$ for all $n > n_1$, $m > n$, for some $n_1 \in \mathbb{N}$, hence choosing $n_2 = \max\{n_0, n_1\}$ we get $d(y_n, y_m) << c$ for all $n > n_2, m > n$.

So, $\{y_n\}$ is Cauchy sequence in X, and by completeness of X it must be convergent to u (say) in X i.e. $y_n \to u$ as $n \to \infty$ hence

$$\lim_{n\to\infty} AA_1x_{2n} = \lim_{n\to\infty} gx_{2n+1} = \lim_{n\to\infty} Afx_{2n+2} = \lim_{n\to\infty} A_2x_{2n+1} = u$$

Now since $A_2(X) \subseteq f(X)$ hence there is $z \in X$ such that $Afz = u$

$$d(AA_1z, u) \leq d(AA_1z, A_2x_{2n+1}) + d(A_2x_{2n+1}, u)$$

$$\leq ad(Afz, gx_{2n+1}) + bd(Afz, AA_1z) + cd(gx_{2n+1}, A_2x_{2n+1}) \\ + e[d(Afz, A_2x_{2n+1}) + d(gx_{2n+1}, AA_1z)] + d(A_2x_{2n+1}, u)$$

$$\leq ad(u, y_{2n}) + bd(u, AA_1z) + cd(y_{2n}, u) + cd(y_{2n+1}, u) \\ + e[d(y_{2n+1}, u) + d(y_{2n}, u) + d(u, AA_1z)] + d(y_{2n+1}, u)$$

So $(1-b-e)d(AA_1 z,u)\le(a+c+e)d(u,y_{2n})+(c+e+1)d(u,y_{2n+1})$

Since $(1-b-e)>0$ hence by lemma 2.7 we have $d(AA_1 z,u)=0$ So $AA_1 z=u=Afz$.

Again since $AA_1(X)\subseteq g(X)$ hence there is $v\in X$ such that $gv=u$ and

$$\begin{aligned}d(u,A_2v)&=d(AA_1z,A_2v)\\&\le ad(Afz,gv)+bd(Afz,AA_1z)+cd(gv,A_2v)+e[d(Afz,A_2v)+d(gv,AA_1z)]\\&\le ad(u,u)+bd(u,u)+cd(u,A_2v)+e[d(u,A_2v)+d(u,u)]\end{aligned}$$

Hence $d(u,A_2v)\le(c+e)d(u,A_2v)$

Since $c+e<1$ hence by proposition 2.2 we have $d(u,A_2v)=0$, i.e. $A_2v=u=gv=AA_1z=Afz$.

Now since pair $\{f,A_1\}$ is weakly compatible and A is commuting with f and A_1 so $\{Af,AA_1\}$ also weakly Compatible and $AA_1z=Afz$ so $AA_1Afz=AfAA_1z$ i.e. $AA_1u=Afu$

Thus we have $A_2v=gv=AA_1z=Afz=u$ and $AA_1u=Afu$

$$\begin{aligned}d(AA_1u,u)&=d(AA_1u,A_2v)\\&\le ad(Afu,gv)+bd(Afu,AA_1u)+cd(gv,A_2v)+e[d(Afu,A_2v)+d(gv,AA_1u)]\\&=ad(AA_1u,u)+bd(AA_1u,AA_1u)+cd(u,u)+e[d(AA_1u,u)+d(u,AA_1u)]\\&=(a+2e)d(AA_1u,u)\end{aligned}$$

hence $d(AA_1u,u)\le(a+2e)d(AA_1u,u)$

since $a+2e<1$, hence $d(AA_1u,u)=0$ so $AA_1u=u=Afu$ Again as pair $\{A_2,g\}$ is weakly compatible and

$A_2v=gv$ hence $A_2gv=gA_2v \Rightarrow A_2u=gu$

Thus we have $A_2z=gz=AA_1u=Afu=u$ and $A_2u=gu$ (3.1.5)

Now

$$\begin{aligned}d(u,A_2u)&\le d(AA_1u,A_2u)\\&=ad(Afu,gu)+bd(Afu,AA_1u)+cd(gu,A_2u)+e[d(Afu,A_2u)+d(gu,AA_1u)]\\&\le ad(u,A_2u)+bd(u,u)+cd(A_2u,A_2u)+e[d(u,A_2u)+d(A_2u,u)]\\&=(a+2e)d(u,A_2u)\end{aligned}$$

Since $a+2e<1$, hence we get $d(u,A_2u)=0$ *i.e.* $A_2u=u=gu$

Thus we have $A_2u=gu=AA_1u=Afu=u$ (3.1.6)

Again using (3.1.1) with $x=Au,\ y=u$ and (3.1.6) and fact that A commutes with f and A_1, we get,

$$d(AA_1Au,A_2u)\le ad(AfAu,gu)+bd(AfAu,AAA_1u)+cd(gu,A_2u)+e[d(AfAu,A_2u)+d(gu,AAA_1u)]$$

$\Rightarrow d(Au,u) \le ad(Au,u) + bd(Au,Au) + cd(u,u) + e[d(Au,u) + d(u,Au)]$

$\Rightarrow \ d(Au,u) \le (a+2e)d(Au,u)$ and $a+2e<1$ hence we have $d(Au,u)=0$ i.e. $Au=u$.

Therefore, $Au = u = A_2u = gu = AA_1u = Afu$.

Finally since A commutes with A_1 and f we have $A_1u = A_1Au = AA_1u = u$ similarly $fu = u$.
Thus $Au = A_1u = A_2u = fu = gu = AA_1u = Afu$ hence u is common fixed point of A, A_1, A_2, f, g, AA_1 and Af.

UNIQUENESS:

Let w is another common fixed point then by (3.1.1). We have

$$\begin{aligned} d(u,w) &= d(AA_1u, A_2w) \\ &\le ad(Afu, gw) + bd(Afu, AA_1u) + cd(gw, A_2w) + e[d(Afu, A_2w) + d(gw, AA_1u)] \\ &= ad(u,w) + bd(u,u) + cd(w,w) + e[d(u,w) + d(w,u)] \end{aligned}$$

$\Rightarrow d(u,w) \le (a+2e)d(u,w)$

and since $a+2e<1$, hence $d(u,w)=0$ i.e. $u=w$, which proves uniqueness.

Taking $A=I$ and $b=c$ in theorem 3.1 we get the following corollary.

Corollary 3.2: Let (X,d) be a complete cone metric space suppose mapping A_1, A_2, f and g are four self maps of X such that $A_2(X) \subseteq f(X)$ and $A_1(X) \subseteq g(X)$ and satisfying

$d(A_1x, A_2y) \le ad(fx, gy) + b[d(fx, A_1x) + d(gy, A_2y)] + e[d(fx, A_2y) + d(gy, A_1x)]$, for all $x, y \in X$ where $a,b,e \in [0,1)$ and $a+2b+2e<1$, suppose that the pairs $\{f, A_1\}$ and $\{g, A_2\}$ are weakly compatible, then f, g, A_1 and A_2 have a unique common fixed point.

Note that we have proved above results without using the normality of cone.

If we take $A=I, f=g$ and $b=c$ in Theorem 3.1, we get the following corollary.

Corollary 3.2: Let (X,d) be a complete cone metric space. Suppose mappings f, A_1 and A_2 are three self maps of X such that $A_1(X) \subseteq f(X)$ and $A_2(X) \subseteq f(X)$ and satisfying

$$d(A_1x, A_2y) \le ad(fx, fy) + b[d(fx, A_1x) + d(fy, A_2y)] + e[d(fx, A_2y) + d(fy, A_1x)]$$

for all $x, y \in X$ where $a,b,e \in [0,1)$ and $a+2b+2e<1$.

Suppose that the pairs $\{f, A_1\}$ and $\{f, A_2\}$ are weakly compatible then f, A_1 and A_2 have a unique common fixed point.

If we take $A=I, f=g=I$ in Theorem 3.1 we get the following corollary.

Corollary 3.3: Let (X,d) be a complete metric space suppose mapping A_1 and A_2 are two self maps of X and satisfying

$$d(A_1x, A_2y) \le ad(x,y) + bd(x, A_1x) + cd(y, A_2y) + e[d(x, A_2y) + d(y, A_1x)]$$

for all $x, y \in X$ where $a,b,c,e \in [0,1)$ and $a+b+c+2e<1$ then A_1 and A_2 have unique common fixed point.

Now we give fixed point results for sequence of self maps.

Theorem 3.4: Let (X,d) be an complete cone metric space suppose that A, f, g are self maps of X, and $\{A_n\}$ be a sequence of self maps on *X*, such that $AA_1(X) \subseteq g(X)$ and $A_2(X) \subseteq Af(X)$ and satisfying,

$$d(AA_ix, A_{i+1}y) \le a_i d(Afx, gy) + b_i d(Afx, AA_ix) + c_i d(gy, A_{i+1}y) + e_i[d(Afx, A_{i+1}y) + d(gy, AA_ix)] \quad (3.4.1)$$

for all $x, y \in X$ where $a_i, b_i, c_i, e_i \in [0,1)$ and $a_i + b_i + c_i + 2e_i < 1$.
Suppose that pairs $\{A_1, f\}$ and $\{A_2, g\}$ are weakly compatible and A commutes with A_1 and f then A, f, g, Af and all the mappings of sequence $\{A_n\}$ and $\{AA_n\}$ have a unique common fixed point.

Proof: If $i=1$ then by theorem 3.1 $A, A_1, A_2, f, g, AA_1, AA_2$ and A, f have a unique common fixed point (Say u).

Now for $i=2$, from (3.4.1) we have,

$$d(AA_2x, A_3y) \le a_2 d(Afx, gy) + b_2 d(Afx, AA_2x) + c_2 d(gy, A_3y) + e_2[d(Afx, A_3y) + d(gy, AA_2x)] \quad (3.4.2)$$

for all $x, y \in X$ where $a_2, b_2, c_2, e_2 \in [0,1)$ and $a_2 + b_2 + c_2 + 2e_2 < 1$.Using above equation with $x = y = u$, we have

$$d(AA_2u, A_3u) \le a_2 d(Afu, gu) + b_2 d(Afu, AA_2u) + c_2 d(gu, A_3u) + e_2[d(Afu, A_3u) + d(gu, AA_2u)]$$

$$\Rightarrow d(u, A_3u) \le a_2 d(u,u) + b_2 d(u,u) + c_2 d(u, A_3u) + e_2[d(u, A_3u) + d(u,u)]$$
$$\Rightarrow d(u, A_3u) \le (c_2 + e_2) d(u, A_3u)$$

and $c_2 + e_2 < 1$ hence we have $d(u, A_3u) = 0$ i.e. $A_3u = u$ and so $A_3u = u$, also $AA_3u = Au = u$, hence u is common fixed point of $A, A_1, A_2, A_3, f, g, AA_1, AA_2, AA_3, Af$ and if it is not unique let w is another common fixed point then using (3.4.2) with $x = u$, $y = w$ we get,

$$\Rightarrow d(AA_2u, A_3w) \le a_2 d(Afu, gw) + b_2 d(Afu, AA_2u) + c_2 d(gw, A_3w) + e_2[d(Afu, A_3w) + d(gw, AA_2u)]$$

$$\Rightarrow d(u,w) \le ad(u,w) + 2e_2 d(u,w) = (a+2e)d(u,w)$$

Since $a + 2e < 1$ hence we get $d(u,w) = 0$ i.e. $u = w$.Hence fixed point is unique.

By induction on i we get

$$Au = A_1u = A_2u = A_3u = \ldots = A_nu = \ldots = AA_1u = AA_2u == AA_3u = \ldots = AA_nu = \ldots = fu = gu$$
$$= Afu = u$$

Therefore u is unique common fixed point of A, f, g and all the mappings of the sequence $\{A_n\}$ and $\{AA_n\}$.

If we take $A = I$ in above theorem we get the following theorem.

Theorem 3.5: Let (X,d) be a complete cone metric space suppose that f,g are self maps of X and $\{A_n\}$ be a sequence of self maps of X such that $A_1(X)\subseteq g(X)$ and $A_2(X)\subseteq f(X)$ and satisfying

$$d(A_i x, A_{i+1}y)\le a_i d(fx,gy)+b_i d(fx,A_i x)+c_i d(gy,A_{i+1}y)+e_i[d(fx,A_{i+1}y)+d(gy,A_i x)]$$

for $x,y\in X$ where $a_i,b_i,c_i,e_i\in[0,1)$ and $a_i+b_i+c_i+2e_i<1$. Suppose that pairs $\{A_1,f\}$ and $\{A_2,g\}$ are weakly compatible then f,g and all the maps of sequence $\{A_n\}$ have unique common fixed point.

If we take $A=f=g=I$ in theorem 3.4 we get the following corollary.

Corollary 3.6: Let (X,d) be an complete cone metric space suppose that $\{A_n\}$ be a sequence of self maps on *X*, satisfying, $d(A_i x,A_{i+1}y)\le a_i d(x,y)+b_i d(x,A_i x)+c_i d(y,A_{i+1}y)+e_i[d(x,A_{i+1}y)+d(y,A_i x)]$ for all $x,y\in X$ where $a_i,b_i,c_i,e_i\in[0,1)$ and $a_i+b_i+c_i+2e_i<1$. Then all the mappings of sequence $\{A_n\}$ have a unique common fixed point.

If we take $A_i=A$ for all $i\in\mathbb{N}$ and $a_i=a,b_i=b,c_i=c$ for all $i\in\mathbb{N}$, in above corollary we get the following corollary which generalize the result of Reich [7], in cone metric spaces.

Corollary 3.7: Let (X,d) be an complete cone metric space suppose that $A:X\to X$ be a self map of *X*, satisfying, $d(Ax,Ay)\le ad(x,y)+bd(x,Ax)+cd(y,Ay)$ for all $x,y\in X$ where $a,b,c\in[0,1)$ and $a+b+c<1$. Then A has a unique fixed point in *X*.

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