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SOFT UNION IDEAL STRUCTURES OF GAMMA NEAR-RINGS

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ABSTRACT

The structure of soft union Γ-near ring is based on the inclusion relation and union of sets and since this new concept brings the soft set theory, set theory and Γ-near ring theory together, it is very functional by means of improving the soft set theory with respect to Γ-near ring structure. Moreover, we investigate those notions with respect to soft image, soft pre-image and β-inclusion of soft sets. Finally, we give some applications of soft union Γ-near ring to Γ-near ring theory.

Index terms: *Soft set, gamma near- ring SU-action, ideal SU-action, soft image, soft pre-image, β-inclusion*

AMS Subject Classification: *03E70, 58E40.*

SECTION-1 INTRODUCTION

The notion of near ring was first introduced by Dickson and Leonard in 1905 [1].They showed that there do exist "fields with one distributive law" (near fields). It was Zassenhaus who was able to determine all finite near rings. Now a days, near fields are mighty tools in characterizing doubly transitive groups, incidence groups and Frobenius groups. We note that the ideals of near rings play a central role in the structure theory; however, they do not in general coincide with the usual ring ideals of a ring. In 1984, Satyanarayana introduced Γ-near-ring in his doctoral thesis and obtained some basic results [34]. For further see [35, 36]. To solve complicated problems in economics, engineering, environmental science and social science. Methods in classical mathematics are not always successful because of various types of uncertainties presented in these problems. While probability theory, fuzzy set theory [3], rough set theory [4, 5] and other mathematical tools are well known and often useful approaches to describing uncertainty, each of these theories has its inherent difficulties as pointed out in [6, 7]. In 1999, Molodtsov[6] introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting existing methods. Presently, works on soft set theory are progressing rapidly. Maji *et al.* [8] defined several operations on soft sets and made a theoretical study on the theory of soft sets. Since its inception, it has received much attention in the mean of algebraic structures such as groups [9], semi rings [10], rings [11], BCK/BCI-algebras [12, 13, 14], normalistic soft groups [15], BL-algebras [16], BCH-algebras [17] and near-rings [18]. Atagün and Sezgin [19] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Also union soft sub structures of near-rings and near-ring modules are studied in [20].Cagman *et al.* defined two new types of group actions on a soft set, called group SI-action and group SU-action [21], which are based on the inclusion relation and the intersection of sets and union of sets, respectively. Ali *et al.* [23] introduced several operations of soft sets. Sezgin and Atagün[24] studied on soft set operations as well. Soft set relations and functions [25] and soft mappings [26] were proposed and many related concepts were discussed too. Moreover, the theory of soft sets has gone through remarkably rapid strides with a wide-ranging applications especially in soft decision making as in the following studies: [27, 28, 29] and some other fields such as [30, 31, 32, 33].Cagman and Enginoglu [28] redefined the operations of soft sets to develop the soft set theory. By using their definitions, in this paper,

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We define a soft uni Γ-near-ring. The structure of soft uni Γ-near-ring is based on the inclusion relation and intersection of sets and since this new concept brings the soft set theory, set theory and Γ-near-ring theory together, it is very functional by means of improving the soft set theory with respect to Γ-near-ring structure. From this view, it functions as a bridge among soft set theory, set theory and Γ-near-ring theory.

SECTION-2: PRELIMINARIES AND BASIC CONCEPTS

In this section, we recall basic definitions of soft set theory that are useful for subsequent sections. For more detail see the papers [8, 9]

Through out the paper, U refers to an initial universe, E is a set of parameters and P(U) is the power set of U. $\subset$ and $\supset$ stand for proper subset and super set, respectively.

Definition 2.1[9]: For any subset A of E, a soft set λ_A over U is a set, defined by a function λ_A, representing the mapping λ_A: E → P(U). A soft set over U can also be represented by the set of ordered pairs $\lambda_A = \{(x, \lambda_A(x)): x \in E, \lambda_A(x) \in P(U)\}$. Note that the set of all soft sets over U will be denoted by S(U).

Definition 2.2[9]: Let λ, μ ∈S(U). Then

(i) If $\lambda(e) = \emptyset$ for all e ∈ E, λ is said to be a null soft set, denoted by $\emptyset$.
(ii) If λ(e) = U for all e ∈ E, λ is said to be an absolute soft set, denoted by U.
(iii) λ is a soft subset of μ, denoted $\lambda \subseteq \mu$, if $\lambda(e) \subseteq \mu(e)$ for all e∈ E.
(iv) Soft union of λ and μ, denoted by $\lambda \cup \mu$, is a soft set over U and defined by $\lambda \cup \mu$: E → P(U) such that $(\lambda \cup \mu)(e) = \lambda(e) \cup \mu(e)$ for all e ∈ E.
(v) $\lambda = \mu$ if $\lambda \subseteq \mu$ and $\lambda \supseteq \mu$.
(vi) Soft intersection of λ and μ, denoted by $\lambda \cap \mu$, is a soft set over U and defined by $\lambda \cap \mu$: E → P(U) such that $(\lambda \cap \mu)(e) = \lambda(e) \cap \mu(e)$ for all e ∈ E.
(vii) Soft complement of λ is denoted by λ^C and defined by λ^C : E → P(U) such that $\lambda^C(e) = \cup/\lambda(e)$ for all e ∈ E.

Definition 2.3: Let E be a parameter set, S ⊂ E and λ: S → E be an injection function. Then S∪ λ(s) is called extended parameter set of S and denoted by ξ_S.

If S = E, then extended parameter set of S will be denoted by ξ.

Definition 2.4: Let R be a Γ-near ring and f_R be a soft set over U. Then, f_R is said to be soft union Γ-near ring over U, if it satisfies the following conditions hold:

(i) $f_R(x+y) \subseteq f_R(x) \cup f_R(y)$.
(ii) $f_R(-x) = f_R(x)$.
(iii) $f_R(x\,\alpha\,y) \subseteq f_R(x) \cup f_R(y)$, for all x, y ∈ R and $\alpha \in \Gamma$.

Example 1: Let R = {0, 1, 2, 3} and $\Gamma = \{\alpha, \beta\}$ be non-empty sets. The binary operations defined as

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

α	0	1	2	3
0	0	0	0	0
1	0	1	0	2
2	0	0	0	0
3	0	2	0	2

β	0	1	2	3
0	0	0	0	0
1	0	1	0	0
2	0	0	2	0
3	0	0	0	2

Clearly, (R, +, Γ) is a Γ-near ring.

Assume that R is the set of parameters and $U = \left\{\begin{bmatrix} x & x \\ x & 0 \end{bmatrix} / x \in Z_4\right\}$, 2×2 matrices with Z_4 terms, in the universal set. We construct a soft set f_R over U by

$f_R(0) = \left\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right\}$,

$f_R(1) = \left\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 4 & 0 \end{bmatrix}\right\}$,

$f_R(2) = \left\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 3 & 0 \end{bmatrix}\right\}$,

$f_R(3) = \left\{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 4 & 0 \end{bmatrix}\right\}$. Then, one can easily show that the soft set f_R is a soft union Γ-near ring over U.

Example 2: In example-1, assume that R={0, 1, 2, 3} is again the set of parameters and U= S_3, Symmetric group, is the universal set. We defined a soft set f_R by,

$f_R(0)$ = {(1 2), (2 3)}
$f_R(1)$ = {(1 2), (1 3), (1 2 3)}
$f_R(2)$ = {(1 2), (2 3), (1 2 3)}
$f_R(3)$ = {(1 2), (1 3), (1 2 3)}

f_R is not a soft union Γ-near ring, because

$f_R(1+1) = f_R(0)$ = {(1 2), (2 3)} ⊈ { (1 2), (1 3), (1 2 3)}.

Note-1: If f_R is a soft union Γ-near ring over U, then $f_R(0) \subseteq f_R(y)$, for all y ∈ R

SECTION-3: SOME IMPORTANT THEOREMS

Theorem 3.1: Let R be a Γ-near ring and f_R a soft set over U. Then, f_R is a soft union Γ-near ring if and only if $f_R(x - y) \subseteq f_R(x) \cup f_R(y)$ and $f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$, for all x, y ∈ R and $\alpha \in \Gamma$.

Proof: Assume that f_R is a soft union Γ-near ring over U. Then, by definition of a soft union Γ-near ring, we have

$f_R(x - y) \subseteq f_R(x) \cup f_R(-y) = f_R(x) \cup f_R(y)$ and
$f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$, for all x, y ∈ R and $\alpha \in \Gamma$.

Conversely, assume that $f_R(x - y) \subseteq f_R(x) \cup f_R(y)$ and $f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$, for all x, y ∈ R and $\alpha \in \Gamma$. If we choose x = 0, then $f_R(0 - y) = f_R(-y) \subseteq f_R(0) \cup f_R(y) = f_R(y)$.

Now,

$f_R(y) = f_R(-(-y)) \subseteq f_R(-y)$. Thus $f_R(y) = f_R(-y)$ for all y ∈ R. Also, by assumption, we have
$f_R(x + y) = f_R(x - (-y)) \subseteq f_R(x) \cup f_R(-y) = f_R(x) \cup f_R(y)$. Thus f_R is a soft union Γ-near ring over U.

Note-2: Let f_R is a soft union Γ-near ring over U.

(i) If $f_R(x - y) = 0$ for all x, y ∈ R, then $f_R(x) = f_R(y)$.
(ii) If $f_R(x - y) = f_R(0)$ for all x, y ∈ R, then $f_R(x) = f_R(y)$.

It is known that if (R, +, Γ) is a Γ-near ring, then (R, +) is a group but not necessarily abelian. That is, for any x, y ∈ R, x + y needs not be equal to y + x. However, we have the following.

Theorem 3.2: Let $f_R$be a soft union Γ-near ring over U and x ∈ R. Then
$f_R(x) = f_R(y) \Leftrightarrow f_R(x + y) = f_R(y + x)$. for all y∈R

Proof: Straight forward.

Theorem 3.3: Let R be a Γ-near field and f_R a soft set over U. If $f_R(0) \subseteq f_R(1_R) = f_R(x)$ for all 0≠ x ∈ R, then f_R is a soft union Γ-near ring over U.

Proof: Suppose that $f_R(0) \subseteq f_R(1_R) = f_R(x)$ for all 0≠ x ∈ R. In order to prove that f_R is a soft union Γ-near ring over U, it is enough to prove that $f_R(x - y) \subseteq f_R(x) \cup f_R(y)$ and $f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$.

Let x, y ∈ R and $\alpha \in \Gamma$. Then, we have the following cases:

Case-1: Suppose that x ≠ 0 and y = 0 or x = 0 and y ≠ 0. Since R is a Γ-near field, so it follows that $x\, \alpha\, y = 0$ and

$f_R(x\, \alpha\, y) = f_R(0)$. Since $f_R(0) \subseteq f_R(x)$ for all x ∈ R,

So, $f_R(x\, \alpha\, y) = f_R(0) \subseteq f_R(x)$ and $f_R(x\, \alpha\, y) = f_R(0) \subseteq f_R(y)$. This imply $f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$.

Case-2: Suppose that x ≠ 0 and y≠ 0. It follows that $x\, \alpha\, y \neq 0$. Then

$f_R(x\, \alpha\, y) = f_R(1_R) = f_R(x)$ and $f_R(x\, \alpha\, y) = f_R(1_R) = f_R(y)$. This imply
$f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$.

Case-3: Suppose that x = 0 and y = 0, then clearly $f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$. Hence

$f_R(x\, \alpha\, y) \subseteq f_R(x) \cup f_R(y)$ for all x, y ∈ R and $\alpha \in \Gamma$.

Theorem 3.4: If f_R and f_S are soft union Γ-near rings over U_1 and U_2, then $f_R \times f_S$ is also soft union Γ-near ring over $U_1 \times U_2$.

Proof: Let (x_1, y_1), $(x_2, y_2) \in R\times S$. Then

$$\begin{aligned} \hbar_{R\times S}\{(x_1,y_1) - (x_2,y_2)\} &= \hbar_{R\times S}(x_1 - x_2,\, y_1 - y_2) \\ &= \hbar_R(x_1 - x_2) \times \hbar_s(y_1 - y_2) \\ &\subseteq (\hbar_R(x_1)\cup \hbar_R(x_2)) \times (\hbar_s(y_1)\cup \hbar_s(y_2)) \\ &= (\hbar_R(x_1)\times \hbar_s(y_1)) \cup(\hbar_R(x_2)\times \hbar_s(y_2)) \\ &= \hbar_{R\times S}(x_1,y_1) \cup \hbar_{R\times S}(x_2,y_2). \end{aligned}$$

Let (x_1, y_1), $(x_2, y_2) \in R\times S$ and $(\alpha_1, \alpha_2) \in \Gamma_1 \times \Gamma_2$. Then

$$\begin{aligned} \hbar_{R\times S}\{(x_1,y_1)(\alpha_1,\alpha_2)(x_2,y_2)\} &= \hbar_{R\times S}(x_1\alpha_1x_2,\, y_1\alpha_2y_2) \\ &= \hbar_R(x_1\alpha_1x_2) \times \hbar_s(y_1\alpha_2y_2) \\ &\subseteq (\hbar_R(x_1)\cup \hbar_R(x_2)) \times (\hbar_s(y_1)\cup \hbar_s(y_2)) \\ &= (\hbar_R(x_1)\times \hbar_s(y_1)) \cup(\hbar_R(x_2)\times \hbar_s(y_2)) \\ &= \hbar_{R\times S}(x_1,y_1) \cup \hbar_{R\times S}(x_2,y_2). \end{aligned}$$

Hence $f_R \times f_S$ is soft union Γ-near ring over $U_1 \times U_2$.

Theorem 3.5: If f_R and f_S are soft union Γ-near rings over U, then $f_R \cap f_S$ is also soft union Γ-near ring over U.

Proof: Now, let x, y ∈ R then

$$\begin{aligned} (f_R \cup f_S)\ (x - y) &= f_R(x- y) \cup f_S(x- y) \\ &\subseteq (f_R(x) \cup f_R(y)) \cup (f_S(x) \cup f_S(y)) \\ &= (f_R(x) \cup f_S(y) \cup (f_R(x)) \cup f_S(y)) \\ &= f_{R\cup S}(x) \cup f_{R\cup S}(y) \end{aligned}$$

Now, let x, y ∈ R and $\alpha \in \Gamma$ then

$$\begin{aligned} (f_R \cup f_S)\ (x\,\alpha\, y) &= f_R(x\,\alpha\, y) \cup f_S(x\,\alpha\, y) \\ &\subseteq (f_R(x) \cup f_R(y)) \cup (f_S(x) \cup f_S(y)) \\ &= (f_R(x) \cup f_S(x)) \cup ((f_R(y) \cup f_S(y)) \\ &= f_{R\cup S}(x) \cup f_{R\cup S}(y). \end{aligned}$$

Hence $f_R \cup f_S$ is a soft union Γ-near ring over U.

4. SOFT UNION IDEALS IN GAMMA NEAR -RINGS

Definition 4.1: Let R be a Γ-near ring and f_R be a soft union Γ-near ring over U. Then, $f_R$is said to be a soft union Γ-ideal of R over U, if the following conditions hold:

(i) $f_R(x + y - x) \subseteq f_R(x) \cup f_R(y)$.
(ii) $f_R(x\,\alpha\, y)\ ) \subseteq f_R(x)$.
(iii) $f_R(x\,\alpha\,(y + z) - x\,\alpha\, y) \subseteq f_R(z)$, for all x, y, z ∈ R and $\alpha \in \Gamma$.

If f_R is soft union Γ-near ring over U and the conditions (i) and (ii) hold, then f_R is called a soft union right Γ-ideal of R over U and if conditions (i) and (iii) hold, then f_R is called a soft union left Γ-ideal of R over U

Example 1: Let R = {0, 1, 2, 3} and Γ = $\{\alpha, \beta\}$ be non-empty sets. The binary operations defined as

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

α	0	1	2	3
0	0	0	0	0
1	1	0	1	1
2	0	1	2	3
3	0	0	3	2

β	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	2	0	3
3	0	0	0	2

Clearly, (R, +, Γ) is a Γ-near ring.

Assume that R is the set of parameters and
U=$D_3 = \{ (x, y): x^3 = y^3 = (xy)^2 = e, xy = yx^2\} = \{e, x, x^2, y, yx, y^2x\}$ dihedral group, the universal set.

We define a soft set f_R over U by
$f_R(0) = f_R(3) = D_3$
$f_R(2) = f_R(1) = \{e, x\}$.
Then, clearly $f_R$is a soft union left Γ-ideal and right Γ-ideal of R over U.

Theorem 4.2: Let R be a Γ-near field and f_R a soft union Γ-ideal of R over U. Then,
$f_R(0) \subseteq f_R(1_R) = f_R(x)$ for all $0 \neq x \in R$.

Proof: suppose that f_R is a soft union Γ-ideal of R over U, then f_R is a soft union Γ-near-ring of R over U. since $f_R(0) \subseteq f_R(x)$, so in particular $f_R(0) \subseteq f_R(1_R)$.

Now, let $0 \neq x \in R$. Then

$f_R(x) = f_R(1_R \cdot x) \subseteq f_R(1_R) = f_R(x \cdot x^{-1}) \subseteq f_R(x)$

Imply that $f_R(x) = f_R(1_R)$ for all $0 \neq x \in R$.

For a near –ring R, the zero-symmetric part of R denoted by R_0 is defined by $R_0=\{r \in R / r0 = 0\}$.

It is a zero-symmetric near-ring and $I_i \nabla R$, then RI ⊇ R. Hence, we have an analog for this case.

Theorem 4.3: Let R=R_0 and f_R *be* a soft set of R over U. Then $f_R(x\,\alpha\,(y + z) - x\,\alpha\,y) \subseteq f_R(z)$ implies that $f_R(xz) \subseteq f_R(z)$ for all $x, y, z \in R$.

Theorem 4.4: If $f_R$and f_S are soft union Γ-ideals over U, then $f_R f_S$ is also soft union Γ-ideal over U.

Proof: Let f_R and f_S are soft union Γ-ideals over U. Let $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in R\times S$ and $(\sigma_1, \sigma_2) \in \Gamma_1 \times \Gamma_2$. Then

$$(f_R f_S)\big((x_1, y_1) + (x_2, y_2) - (x_1, y_1)\big) = (f_R f_S)\big(x_1 + x_2 - x_1, y_{1+}y_2 - y_1\big)$$
$$= f_R(x_1 + x_2 - x_1) \cup f_S\,(y_{1+}y_2 - y_1)$$
$$\subseteq f_R(x_2) \cup f_S\,(y_2) = f_R f_S(x_2, y_2)$$

$$(f_R f_S)\big((x_1, y_1)(x_2, y_2)\big) = (f_R f_S)\,(x_1x_2, y_1y_2)$$
$$= f_R(x_1x_2) f_S\,(y_1y_2)$$
$$\subseteq f_R(x_1) \cup f_S\,(y_1) = (f_R f_S)(x_1, y_1) \text{ and}$$

$$(f_R f_S)\big((x_1, y_1)(\sigma_1, \sigma_2)((x_2, y_2) + (x_3, y_3)\big) - (x_1, y_1)(\sigma_1, \sigma_2)(x_2, y_2))$$
$$= (f_R f_S)\,(x_1\,\sigma_1(x_2 + x_3) - x_1\,\sigma_1x_2, y_1\sigma_2(y_2 + y_3) - y_1\sigma_2y_2)$$
$$= f_R(x_1\sigma_1\,(x_2 + x_3) - x_1\,\sigma_1x_2) \cup f_S\,(y_1\sigma_2(y_2 + y_3) - y_1\sigma_2y_2)$$
$$\subseteq f_R(x_3) \cup f_S\,(y_3) = (f_R f_S)(x_3, y_3)$$

Hence, $f_R f_S$ is a soft union Γ-idel of R×S over U.

Theorem 4.5: If $f_R$and f_S are soft union Γ-ideals over U_1 and U_2, then $f_R \times f_S$ is also soft union Γ-ideal over $U_1 \times U_2$.

Proof: similar to previous theorem.

Theorem 4.6: If $f_R$is a soft union Γ-ideal of Γ near-ring R over U, then R_f={x∈ R: $f_R(x) = f_R(0)$} is a Γ-ideal of R over U

Proof: It is oblivious that $0 \in R_f \subseteq R$. We need to prove that

(i) $x - y \in R_f$ (ii) $n + x - n \in R_f$ (iii) $x\alpha n \in R_f$ and $n\alpha(i+x) - n\alpha i \in R_f$

for all $x, y \in R_f$, $n, i \in R$ and $\alpha \in \Gamma$.

If $x, y \in R_f$, then $f_R(x) = f_R(y) = f_R(0)$. So by Theorem 4.2, it follows that $f_R(0) \subseteq f_R(x - y)$, $f_R(0) \subseteq f_R(n + x - n)$, and $\lambda_R(0) \subseteq f_R(\,n\,\alpha(i + x) - n\,\alpha i)$

for all $x, y \in R_f$, $n, i \in R$ and $\alpha \in \Gamma$

Since $f_R$is a soft union Γ-ideal of R over U, so

(i) $f_R(x - y) \subseteq f_R(x) \cup f_R(y) = f_R(0)$

(ii) $f_R(n + x - n) \subseteq f_R(x) = f_R(0)$

(iii) $f_R(x\alpha\, n) \subseteq f_R(x) = f_R(0)$ and $f_R(n\alpha\,(i + x) - n\alpha i) \subseteq f_R(x) = f_R(0)$

This implies that (i)$f_R(x - y) = f_R(0)$,(ii)$f_R\,(n + x - n) = f_R(0)$, (iii)$f_R(x\alpha\, n) = f_R(0)$ and $f_R(n\alpha\,(i + x) - n\alpha i) = f_R(0)$ forall $x,y \in R_f$, $n, i \in R$ and $\alpha \in \Gamma$

Thus, $R_f$is a Γ-ideal of R over U.

Theorem 4.7: Let f_R be a soft set over U and $\boldsymbol{\mathcal{B}}$ be a subset of U such that $\emptyset \neq \boldsymbol{\mathcal{B}} \supseteq f_R(0)$.

If f_R is a soft union Γ-ideal of R over U, then $f_R^{\subseteq \boldsymbol{\mathcal{B}}}$={x ∈ R/ $f_R(x) \subseteq \boldsymbol{\mathcal{B}}$}is a Γ-ideal of R over U.

Proof: Since $f_R(0) \subseteq \mathcal{B}$, so $0 \in f_R^{\subseteq \mathcal{B}}$ and $\emptyset \neq f_R^{\subseteq \mathcal{B}} \supseteq$ R. Take x, y $\in f_R^{\subseteq \mathcal{B}}$, n, i $\in$ R and $\alpha \in \Gamma$, which implies that f_R(x) $\subseteq \mathcal{B}$ and f_R(y) $\subseteq \mathcal{B}$. Now we need to prove that

(i) x− y $\in f_R^{\subseteq \mathcal{B}}$ (ii) n+x−n $\in f_R^{\subseteq \mathcal{B}}$(iii) xαn $\in f_R^{\subseteq \mathcal{B}}$ and nα(i+x) − nαi $\in f_R^{\subseteq \mathcal{B}}$

for all x, y $\in f_R^{\subseteq \mathcal{B}}$, n, i$\in R$ and $\alpha \in \Gamma$. Since f_R is a soft union Γ-ideal of R over U, so it follows that

(i) $f_R(\mathrm{x} - \mathrm{y}) \subseteq f_R(\mathrm{x}) \cup f_R(\mathrm{y}) \subseteq \mathcal{B} \cup \mathcal{B} = \mathcal{B}$

(ii) $f_R(\mathrm{n} + \mathrm{x} - \mathrm{n}) \subseteq f_R(\mathrm{x}) \subseteq \mathcal{B}$

(iii) $f_R(\mathrm{x}\alpha\, \mathrm{n}) \subseteq f_R(x) \subseteq \mathcal{B}$ and

(iv) $f_R(\mathrm{n}\alpha\ (\mathrm{i} + \mathrm{x}) - \mathrm{n}\alpha\mathrm{i}) \subseteq f_R(\mathrm{x}) \subseteq \mathcal{B}$

Thus, this completes the proof.

Theorem 4.8: Let f_R and f_S are soft sets over U and φ be a Γ-near ring isomorphism from R to S.

(i) If f_R is a soft union Γ-ideal of R over U, then $\varphi(f_R)$ is a soft union Γ-ideal of S over U.

(ii) If f_S is a soft union Γ-ideal of S over U, then $\varphi^{-1}(f_S)$ is a soft union Γ-ideal of R over U.

Proof: (i) let $\mathrm{x}_1, \mathrm{x}_2 \in$ S. since φ is surjective, there exists $\mathrm{y}_1, \mathrm{y}_2 \in$ R such that $\varphi(y_1)= \mathrm{x}_1, \varphi(y_2)= \mathrm{x}_2$. We have

$$\begin{aligned}
\varphi(f_R)(\mathrm{x}_1 - \mathrm{x}_2) &= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, \varphi(y) = \mathrm{x}_1 - \mathrm{x}_2\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, y = \varphi^{-1}(\mathrm{x}_1 - \mathrm{x}_2)\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, y = \varphi^{-1}(\varphi(\mathrm{y}_1 - \mathrm{y}_2)) = \mathrm{y}_1 - \mathrm{y}_2\} \\
&= \cup \{f_R(\mathrm{y}_1 - \mathrm{y}_2) / \mathrm{y}_\mathrm{i} \in R,\ \varphi(\mathrm{y}_\mathrm{i}) = \mathrm{x}_\mathrm{i}, \mathrm{i}=1,2\} \\
&\subseteq \cup \{f_R(\mathrm{y}_1) \cup f_R(\mathrm{y}_2) / \ \mathrm{y}_\mathrm{i} \in R,\ \varphi(\mathrm{y}_\mathrm{i}) = \mathrm{x}_\mathrm{i}, \mathrm{i}=1,2\} \\
&= \cup \{f_R(\mathrm{y}_1) / \ \mathrm{y}_1 \in R, \varphi(\mathrm{y}_1) = \mathrm{x}_1\} \cup \{ f_R(\mathrm{y}_2) / \mathrm{y}_2 \in R, \varphi(\mathrm{y}_2) = \mathrm{x}_2\} \\
&= \varphi(f_R)(\mathrm{x}_1) \cup \varphi(f_R)(\mathrm{x}_2)
\end{aligned}$$

Thus $\varphi(f_R)(\mathrm{x}_1 - \mathrm{x}_2) \subseteq \varphi(f_R)(\mathrm{x}_1) \cup \varphi(f_R)(\mathrm{x}_2)$. Similarly, we can prove that

$\varphi(f_R)(\mathrm{x}_1\alpha_1\mathrm{x}_2) \subseteq \varphi(f_R)(\mathrm{x}_1) \cup \varphi(f_R)(\mathrm{x}_2)$.

Now we prove that

$$\begin{aligned}
\varphi(f_R)(\mathrm{x}_1 + \mathrm{x}_2 - \mathrm{x}_1) &= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, \varphi(y) = (\mathrm{x}_1 + \mathrm{x}_2 - \mathrm{x}_1)\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, y = \varphi^{-1}(\mathrm{x}_1 + \mathrm{x}_2 - \mathrm{x}_1)\} \\
&= \cup \{f_R(\mathrm{y}) / \ \mathrm{y} \in R, y = \varphi^{-1}\big(\varphi(\mathrm{y}_1 + \mathrm{y}_2 - \mathrm{y}_1)\big) = (\mathrm{y}_1 + \mathrm{y}_2 - \mathrm{y}_1)\} \\
&= \cup \{f_R(\mathrm{y}_1 + \mathrm{y}_2 - \mathrm{y}_1) / \ \mathrm{y}_\mathrm{i} \in R, (\mathrm{y}_\mathrm{i}) = \mathrm{x}_\mathrm{i}, \mathrm{i}=1, 2\} \\
&\subseteq \cup \{f_R(\mathrm{y}_2) / \ \mathrm{y}_2 \in R, \varphi(\mathrm{y}_2) = \mathrm{x}_2 \} \\
&= \varphi(f_R)\ (\mathrm{x}_2)
\end{aligned}$$

Thus $\varphi(f_R)(\mathrm{x}_1 + \mathrm{x}_2 - \mathrm{x}_1) \subseteq \varphi(f_R)(\mathrm{x}_2)$.

Now, let $\mathrm{x}_1, \mathrm{x}_2 \in$ S, $\mathrm{y}_1, \mathrm{y}_2 \in$ R and $\alpha \in \Gamma$, then we have

$$\begin{aligned}
\emptyset(f_R)(\mathrm{x}_1\alpha_1\mathrm{x}_2) &= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, \varphi(y) = \mathrm{x}_1\alpha_1\mathrm{x}_2\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, y = \varphi^{-1}(\mathrm{x}_1\alpha_1\mathrm{x}_2)\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, y = \varphi^{-1}\big(\varphi(\mathrm{y}_1\alpha\mathrm{y}_2)\big) = \mathrm{y}_1\alpha\mathrm{y}_2\} \\
&= \cup \{f_R(\mathrm{y}_1\alpha\mathrm{y}_2) / \ \mathrm{y}_\mathrm{i} \in R, (\mathrm{y}_\mathrm{i}) = \mathrm{x}_\mathrm{i}, \mathrm{i}=1,2\} \\
&\subseteq \cup \{f_R(\mathrm{y}_2) / \ \mathrm{y}_2 \in R,\ \varphi(\mathrm{y}_2) = \mathrm{x}_2\} \\
&= \varphi(f_R)\ (\mathrm{x}_1) \cup \varphi(f_R)\ (\mathrm{x}_2)
\end{aligned}$$

Now let $\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3 \in$ S, $\mathrm{y}_1, \mathrm{y}_2, \mathrm{y}_3 \in$ R and $\alpha_1 \in \Gamma$. Then we have

$$\begin{aligned}
\varphi(f_R)\big(\mathrm{x}_1\alpha_1(\mathrm{x}_2{+}\mathrm{x}_3) - (\mathrm{x}_1\alpha_1\mathrm{x}_2)\big) &= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R, \varphi(y) = \mathrm{x}_1\alpha_1(\mathrm{x}_2{+}\mathrm{x}_3) - \mathrm{x}_1\alpha_1\mathrm{x}_2\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R,\ y = \varphi^{-1}(\mathrm{x}_1\alpha_1(\mathrm{x}_2{+}\mathrm{x}_3) - \mathrm{x}_1\alpha_1\mathrm{x}_2)\} \\
&= \cup \{f_R(\mathrm{y}) / \mathrm{y} \in R,\ y = \varphi^{-1}(\varphi(\mathrm{y}_1\alpha(\mathrm{y}_2 + \mathrm{y}_3) - \mathrm{y}_1\alpha\mathrm{y}_2)) \\
&= (\mathrm{y}_1\alpha(\mathrm{y}_2 + \mathrm{y}_3) - \mathrm{y}_1\alpha\mathrm{y}_2 \\
&= \cup \{f_R(\mathrm{y}_1\alpha(\mathrm{y}_2 + \mathrm{y}_3) - \mathrm{y}_1\alpha\mathrm{y}_2);\ \mathrm{y}_\mathrm{i} \in R, (\mathrm{y}_\mathrm{i}) = \mathrm{x}_\mathrm{i}, \mathrm{i}=1, 2, 3\} \\
&\subseteq \cup \{f_R(\mathrm{y}_3) / \ \mathrm{y}_3 \in R,\ \varphi(\mathrm{y}_3) = \mathrm{x}_3 \} \\
&= \varphi(f_R)\ (\mathrm{x}_3).
\end{aligned}$$

Hence $\varphi(f_R)$ is a soft union Γ-ideal of S over U.

(ii) Let $\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3 \in$ S and $\alpha_1 \in \Gamma$ and $\mathcal{B}_1 \in \Gamma_1$.

$$\begin{aligned}
\text{Then, } (\varphi^{-1}(f_S))(\mathrm{x}_1\alpha_1\mathrm{x}_2) &= f_S(\varphi(\mathrm{x}_1\alpha_1\mathrm{x}_2)) = f_S(\varphi(\mathrm{x}_1)\mathcal{B}\ \varphi(\mathrm{x}_2)) \\
&\subseteq f_S(\varphi(\mathrm{x}_1)) \cup f_S(\varphi(\mathrm{x}_2)) \\
&= (\varphi^{-1}(f_S))(\mathrm{x}_1) \cup (\varphi^{-1}(f_S))(\mathrm{x}_2)
\end{aligned}$$

Similarly, $(\varphi^{-1}(f_S))(x_1 - x_2) \subseteq (\varphi^{-1}(f_S))(x_1) \cup (\varphi^{-1}(f_S))(x_2)$.

Also,
$$\begin{aligned}(\varphi^{-1}(f_S))(x_1 + x_2 - x_1) &= f_S(\varphi(x_1 + x_2 - x_1)) \\ &= f_S(\varphi(x_1) + \varphi(x_2) - \varphi(x_1)) \\ &\subseteq f_S(\varphi(x_2)) = (\varphi^{-1}(f_S))(x_2).\end{aligned}$$

Now, let $x_1, x_2 \in S$ and $\alpha_1 \in \Gamma$ and $\boldsymbol{\mathcal{B}_1} \in \Gamma_1$.
$$(\varphi^{-1}(f_S))(x_1\alpha_1 x_2) = f_S(\varphi(x_1\alpha_1 x_2)) = f_S(\varphi(x_1)\boldsymbol{\mathcal{B}_1}\varphi(x_2)) \subseteq f_S(\varphi(x_1)) = (\varphi^{-1}(f_S))(x_1).$$

Finally, Let $x_1, x_2, x_3 \in S$ and $\alpha_1 \in \Gamma$ and $\boldsymbol{\mathcal{B}_1} \in \Gamma_1$.

Then,
$$\begin{aligned}(\varphi^{-1}(f_S))(x_1\alpha_1(x_2 + x_3) - (x_1\alpha_1 x_2)) &= f_S(\varphi(x_1\alpha_1(x_2 + x_3) - (x_1\alpha_1 x_2)) \\ &= f_S(\varphi(x_1)\boldsymbol{\mathcal{B}_1}\varphi(x_2)) + \varphi(x_3) - \varphi(x_1)\boldsymbol{\mathcal{B}_1}\varphi(x_2)) \\ &\subseteq f_S(\varphi(x_3)) = (\varphi^{-1}(f_S))(x_3).\end{aligned}$$

Hence $\varphi^{-1}(f_S)$ is a soft union Γ-ideal of R over U.

CONCLUSION

Fuzzy set theory, rough set theory and soft set theory are all mathematical tools for dealing with uncertainties. This paper is devoted to discussion of combination of soft set theory, set theory and Γ-near-ring. By using soft sets and union operation of sets, we have defined a new concept, called soft union Γ-near-ring. This new notion brings the soft set theory, set theory and Γ-near-ring theory together and therefore is very functional for obtaining results by means of Γ-near-ring structure. Based on the definition, we have introduced the concepts of soft union sub Γ-near-rings and soft union Γ-ideals of a Γ-near-ring with illustrative examples. We have then investigated these notions with respect to soft image, soft pre-image and β-inclusion of soft sets. Finally, we give some applications of soft union Γ-near-rings to Γ-near-ring theory.

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