International Research Journal of Pure Algebra-7(12), 2017, 868-874
Available online through www.rjpa.info ISSN 2248–9037

BETA FUZZY SOFT CONGRUENCE RELATIONS OVER P-FUZZY SOFT SUBGROUP STRUCTURES

M. SUBHA[1], G. SUBBIAH[2*] AND M. NAVANEETHAKRISHNAN[3]

[1]Assistant Professor in Mathematics,
Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.

[2*]Associate Professor in Mathematics,
Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.

[3]Associate Professor in Mathematics,
Kamaraj College, Thoothukudi-628 003, Tamil Nadu, India.

(Received On: 09-12-17; Revised & Accepted On: 16-12-17)

ABSTRACT

In this study, we define some new special fuzzy soft equivalence relations and derive some simple consequences. Then using those soft relations we define suitable P-fuzzy soft subgroupoid and P-fuzzy quotient subgroup of G/H differently.

Keywords: *Group, soft set, P-fuzzy subgroup, fuzzy binary relation, fuzzy left (right) compatible, fuzzy soft normal subgroup, fuzzy soft congruence, fuzzy soft quotient normal subgroup.*

AMS Subject Classification [2000]: *06F35, 03G25, 03E72, 20N25.*

INTRODUCTION

The concept of fuzzy sets was first introduced by Zadeh in 1965 and since then there has been a tremendous interest in the subject due to diverse applications ranging from engineering and computer science to social behavior studies. The concept of fuzzy relation on a set was defined by Zadeh [27] and other authors like Rosenfeld [21], Tamura *et.al* [22] and Yeh and Bang [23] considered it further. The notion of fuzzy congruence on a group was introduced by Kuroki [11] and that the universal algebra was studied by Filep and Maurer [8].The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in 1971. Several mathematicians have followed the Rosenfeld approach in investigating the fuzzy subgroup theory. Fuzzy normal subgroups were studied by Wu [26], Kumar *et.al* [12] and Mukherjee [19]. The concept of fuzzy quotient group was studied by Kuroki [10]. Soft set theory was introduced by Molodtsov [14] for modeling vagueness and uncertainty and it has been received much attention since Maji *et.al* [15]. Sezgin and Atagun [2] introduced and studied operations of soft sets. This theory has started to progress in the mean of algebraic structures, since Aktas and Cagman [6] defined and studied soft groups. Since then, soft substructures of rings, fields and modules [8], union soft substructures of near-rings and near-ring modules [3], normalistic soft groups [2] are defined and studied in detailed. In this study, we define some new special fuzzy equivalence relations and derive some simple consequences. Then using those relations we define suitable P-fuzzy subgroupoid and P- fuzzy quotient subgroup of G / H differently.

2. PRELIMINARIES

In this section, we shall formulate the preliminary definitions and results that are required in more contents of this section are contained in the literature. Let X be a nonempty set and I be the unit interval. A fuzzy binary relation on X is a fuzzy subset A on X×X. By a fuzzy relation, we mean a fuzzy binary relation given by $A: X \times X \rightarrow I$. All fuzzy subsets considered here are assumed to take values in I.

Corresponding Author: G. Subbiah[2]*
[2]Associate Professor in Mathematics,*
Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.

2.1 Definition: A soft set f_A over U is defined as f_A: E→ P(U) such that $f_A(x)$=∅ if $x \notin$ A.

In other words, a soft set f_A over U is a parameterized family of subsets of the universe U. For all $\varepsilon \in$ A, $f_A(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set f_A. A soft set f_A over U can be presented by the set of ordered pair

$$f_A = \{(x, f_A(x)) \;/\; x \in \text{E}, f_A(x) = \text{P(U)}\} \qquad (1)$$

Clearly, a soft set is not a set. For illustration, Molodtsov consider several examples in [14].

If f_A is a soft set over U, then the image of f_A is defined by Im(f_A) = $\{f_A(a)/a \in A\}$. The set of all soft sets over U will be denoted by S(U). Some of the operations of soft sets are listed as follows.

2.2 Definition: Let $f_A, f_B \in$ S(U). If $f_A(x) \subseteq f_B(x)$, for all $x \in$ E, then f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$. f_A and f_B are called soft equal, denoted by $f_A = f_B$, if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

2.3 Definition: Let $f_A, f_B \in$ S(U) and let χ be a function from A to B. Then the soft anti-image of f_A under χ, denoted by $\chi(f_A)$ is a soft set over U defined by

$$\chi^{-1}(f_B) = \begin{cases} \cap \{ f_A(a)/a \in A, \chi(a) = b\}, if\ \chi^{-1}(b) \neq \emptyset \\ 0 \qquad\qquad , \text{ otherwise} \end{cases} \qquad (2)$$

for all *b*∈B and the soft pre-image of f_B underχ , denoted by ${\chi^{-1}}_{f_B}$, is a soft set over U defined by

$${\chi^{-1}}_{f_B}(\text{a}) = f_B(\chi(a)), \text{ for all } a \in A.$$

Note that the concept of level sets in the fuzzy set theory, Cagman *et.al* [6] initiated the concept of lower inclusions soft sets which serves as a bridge between soft sets and crisp sets.

2.4 Definition: (i) A fuzzy soft relation A on X is said to be reflexive if A(x, x) = 1 for all x∈ X and said to be symmetric if A (x, y) = A(y, x) for all x, y in X (ii) If A_1 andA_2 are two soft relations on X, then their max- product composition denoted by $A_1 \circ A_2$ is defined as $A_1 \circ A_2$ (x, y) = max {A_1(x, z), A_2(z, y)} (iii) If $A_1 = A_2$ = A and A º A ≤ A, then the fuzzy soft relation A is called transitive.

2.5 Definition: A fuzzy binary soft relation A in X is called similarity soft relation if A is reflexive, symmetric and transitive.

Example: Let G = $\{1, \omega, \omega^2\}$ be the group with respect to the usual multiplication, where ω denotes the cube root of unity. Define λ, μ : G → [0,1] by λ(x) = 1 if x =1; 0.6 if x = ω ; 0.5 if x = ω^2 and μ (x) = 0.5 if x =1; 0.4 if x = ω; 0.3 if x = ω^2. It can be found that for every x∈G, $R_{\mu\cap\lambda}$ (x, x) = (μ∩λ)(xx^{-1}) = (μ∩λ)(1) = 0.5. Hence $R_{\mu\cap\lambda}$ is not reflexive and not a similarity soft relation on the group G.

2.6 Definition: Let S be a semi group. A fuzzy binary soft relation A on S is called fuzzy left (right) compatible if and only if A(x, y) ≤ A (tx, ty) for all x, y, t ∈ S and A(x, y) ≤ A(xt, yt) for all x, y, t ∈ S.

2.7 Definition: A fuzzy binary soft relation A on a semi group S is called fuzzy compatible if and only if

min{A(a, b), A(c, d)} ≤ A(ac, bd) for all a, b, c, d ∈ S.

2.8 Definition: Fuzzy compatible similarity soft relation on a semi group S is called fuzzy soft congruence.

2.9 Definition: A mapping μ: X→ [0, 1], where X is an arbitrary non-empty set is called a fuzzy set in X.

2.10 Definition: Let G be any group. A mapping μ: G→ [0, 1] is a fuzzy group if (FG1) μ (xy) ≥ min {μ(x), μ(y)} (FG2) μ(x^{-1}) = μ(x) for all x, y ∈ G.

Example: Let Z be the additive group of all integers. For any integer n, nZ denote the set of all integers multiplies of n. (i,e) n Z = {0, ± n, ±2n, ±3n,……}.
we have Z > 2Z > 4Z > 8Z > 16Z. Define μ : Z→ [0,1] by μ (x) = 1 if x ∈16Z; = 0.7 if x ∈ 8Z -16Z; = 0.5 if x ∈ 4Z-8Z; = 0.2 if x ∈ 2Z- 4Z; = 0 if x ∈ Z-2Z . It can be easily verified that μ is Fuzzy group of Z.

2.11 Definition: Let P and G be a set and a group respectively. A mapping μ : G × P→ [0,1] is called P – fuzzy soft set in G . For any P- fuzzy soft set μ in G and t ∈ [0, 1], we define the set U(μ; t) ={ x ∈G / μ(x, p) ≥ t, p∈ P} which is called an upper cut of μ and it can be used to the characterization of μ.

2.12 Definition: A P- fuzzy set A is called a P- fuzzy soft group of G if
(PFSG1) $A(xy, p) \geq \min \{A(x,p), A(y, p)\}$ (PFSG2) $A(x^{-1},p) \geq A(x, p)$ (PFSG3) $A(e, p) = 1$ for all $x, y \in G$ and $p \in P$.

Example: Let G={a, b, c, d} be a set with + operations as follows.
Then (G,+) is a group. Let P= {0,1,2,3,4} and let A be P- fuzzy soft set in R defined by $\mu_A(a,0) = \mu_A(a,1) = \mu_A(a,2) = \mu_A(a,3) = \mu_A(a,4) = 1$,
$\mu_A(b,0) = \mu_A(b,1) = \mu_A(b,2) = \mu_A(b,3) = \mu_A(b,4) = 2/3$,
$\mu_A(c, 0) = \mu_A(c,1) = \mu_A(c,2) = \mu_A(c,3) = \mu_A(c,4) = 1/3$,
$\mu_A(d,0) = \mu_A(d,1) = \mu_A(d,2) = \mu_A(d,3) = \mu_A(d,4) = 1/3$. We can check that A is P- fuzzy soft group of R.

+	A	b	c	d
A	A	b	c	d
B	B	a	d	c
C	C	d	b	a
D	D	c	a	b

2.13 Definition: A P- fuzzy soft subgroup A_H of G is called a P- fuzzy soft normal subgroup of G if
$A_H(xy, p) = A_H(yx, p)$ for all $x, y \in G$ and $p \in P$.

3. β_p – FUZZY RELATION AND FUZZY CONGRUENCE

In this section, we shall define some special fuzzy soft relation and give some its results. We need to define a special soft relation β_p as follows.

3.1 Definition: Let G be a group with identity e and A_H be a P-fuzzy soft subgroup of G. A fuzzy soft relation β_p can be defined on G by $\beta_p(a, b) = \min \{A_H(a, p), A_H(b, p)\}$ if $(a, p) \neq (b, p)$,= $A_H(e, p)$ if $(a, p) = (b, p)$. Now we can show some properties of β_p.

The following propositions are proved based on the definition 3.1.

3.2 Proposition: Let G be a group with identity e and A_H be a P- fuzzy soft subgroup of a group G. Then the soft relation β_p defined on G is P-similarity soft relation on G.

Proof: β_p is reflexive, for each $a \in G$ and $p \in P$, $\beta_p(a, a) = A_H(e, p) = 1$, B_p is symmetric.
$B_p(a, b) = \min \{A_H(a, p), A_H(b, p)\}$
$= \min \{A_H(b, p), A_H(a, p)\}$
$= \beta_p(b, a)$, for $a, b \in G$.
B_p is transitive.
$B_p{}^o\beta_p(a, c) = \max_{b \in G} \{ \beta_p(a, b), \beta_p(b, c)\}$
$= \max_{b \in G} \{\min\{A_H(a, p), A_H(b, p)\}, \min\{A_H(b, p), A_H(c, p)\}\}$
$\leq \{\max_{b \in G} \{\min\{A_H(a, p), A_H(b, p)\}\}, \max_{b \in G} \{\min\{ A_H(b, p), A_H(c, p)\}\}\}$
$\leq \{\max_{b \in G} \{A_H(a, p)\}, \max_{b \in G}\{A_H(c, p)\}\}$
$= A_H(a, p)A_H(c, p) \leq \min \{A_H(a, p), A_H(c, p)\}$
$= \beta_p(a, c)$, for all $a, c \in G$.

Therefore β_p is a P- similarity soft relation.

3.3 Corollary: $\beta_p(x^{-1}, y^{-1}) = \beta_p(x, y)$ for all $x, y \in G$, $p \in P$.

Proof: A_H is P- fuzzy soft subgroup of G. It gives that
$B_p(x^{-1}, y^{-1}) = \min\{A_H(x^{-1}, p), A_H(y^{-1}, p)\} = \min\{ A_H(x, p), A_H(y, p)\} = \beta_p(x, y)$.

3.4 Proposition: The fuzzy soft relation β_p defined on G is P- fuzzy compatible.

Proof: By using the definition of P- fuzzy compatible and the definition of β_p

$B_p(ac, bd) = \min\{A_H(ac, p), A_H(bd, p)\}$
$\geq \min\{\min\{A_H(a, p), A_H(c, p)\}, \min\{A_H(b, p), A_H(d, p)\}\}$
$= \min\{\min\{A_H(a, p), A_H(b, p), A_H(c, p), A_H(d, p)\}\}$
$= \min\{\min\{A_H(a, p), A_H(b, p)\}, \min\{A_H(c, p), A_H(d, p)\}\}$
$= \min\{\beta_p(a, b), \beta_p(c, d)\}$. This completes the proof.

3.5 Proposition: The fuzzy soft relation β_p defined on G is a P- fuzzy soft congruence.

Proof: β_p is P- fuzzy compatible is proved in Proposition (3.4). Therefore β_p is a P- fuzzy soft congruence.

3.6 Definition: If P- fuzzy soft set is P-fuzzy soft subgroup of G / H, then it is called P-fuzzy soft quotient subgroup. Similarly, if it is P- fuzzy soft normal subgroup of G / H, then it is called P- fuzzy soft quotient normal subgroup.

By using the P- fuzzy congruence β_p, we define a special function N as follows.

3.7 Definition: Let G be group and A_H be P-fuzzy soft normal subgroup of G. N: G / H × P → [0, 1] can be defined by N (xH, p) = β_p (x, h) for all $h \in H$ and $p \in P$

Now some algebraic properties of N are investigated.

3.8 Proposition: The defined fuzzy set N is a P- fuzzy soft quotient subgroup of G / H.

Proof: We have to show that, N is a P-fuzzy soft subgroup of G / H. A_H is P- fuzzy soft subgroup of G. Using this, for every xH, yH ∈ G / H, we get

$N(xHyH, p) = \beta_p(xy, h) = \min\{A_H(xy, p), A_H(h, p)\}$
$= A_H(xy, p)$
$\geq \min\{A_H(x, p), A_H(y, p)\}$
$= \min\{\min\{A_H(x, p), A_H(h, p)\}, \min\{A_H(y, p), A_H(h, p)\}\}$
$= \min\{\beta_p(x, h), \beta_p(y, h)\}$
$= \min\{N(xH, p), N(yH,p)\}$.

and $N(x^{-1}H, p) = \beta_p(x^{-1}, h)$
$= \min\{A_H(x^{-1}, p), A_H(h, p)\}$
$\geq \min\{A_H(x, p), A_H(h, p)\}$
$= \beta_p(x, h)$
$= N(xH, p)$. Thus N is a P- fuzzy soft quotient subgroup of G / H.

3.9 Proposition: The defined fuzzy soft set N is a P- fuzzy soft quotient normal subgroup of G / H.

Proof: Here we have to prove that, N is a P- fuzzy soft normal subgroup of G / H. Since A_H is P- fuzzy soft normal subgroup of G, it gives that

$N(xHyH, p) = \beta_p(xy, h) = \min\{A_H(xy, p), A_H(h, p)\}$
$\geq \min\{A_H(yx, p), A_H(h, p)\}$
$= \beta_p(yx, p)$
$= N(yHxH, p)$. Hence N is a P- fuzzy soft normal subgroup of G / H.

3.10 Proposition: If N is a P- fuzzy soft quotient subgroupoid of finite group G / H, then N is P-fuzzy soft subgroup.

Proof: Let xH ∈ G / H. Since G / H is finite, xH has finite order, say r. Then $(xH)^r = x^rH = H$, where H is identity of G/H. Thus $(xH)^{-1} = x^{-1}H = x^{r-1}H$. Now using the definition of P- fuzzy soft subgroupoid repeatedly, it follows that

$N(x^{-1}H, p) = N(x^{r-1}H, p) = \beta_p(x^{r-1}, h)$
$= \min\{A_H(x^{r-2}x, p), A_H(h, p)\}$
$= A_H(x^{r-2}x, p)$
$\geq \min\{A_H(x^{r-2},p), A_H(x,p)\}$
$\geq A_H(x,p)$
$= \min\{A_H(x, p), A_H(h, p)\}$
$= \beta_p(x, h) = N(xH, p)$.

Interchanging xH with $x^{-1}H$, then $N(xH, p) \geq N(x^{-1}H, p)$. Hence N is a P- fuzzy soft quotient subgroup.

3.11 Proposition: Let N be a P- fuzzy soft quotient subgroup of a group G / H and let $xH \in G / H$. Then $N(xHyH, p) = N(yH, p)$, for all $yH \in G / H$ if and only if $N(xH, p) = N(H, p)$.

Proof: Suppose that $N(xHyH, p) = N(yH, p)$ for all $yH \in G/H$. Then by choosing $yH = H$, we obtain $N(xH, p) = N(H, p)$.

Conversely, suppose that $N(xH, p) = N(H, p)$. Since N is a P- fuzzy soft subgroup of G / H and A_H be P- fuzzy soft subgroup of G, it implies that

$$\begin{aligned} N(xHyH, p) &\geq \min\{N(xH, p), N(yH, p)\} \\ &= \min\{N(H, p), N(yH, p)\} \\ &= \min\{\beta_p(e, h), \beta_p(y, h)\} \\ &= \min\{\min\{N_H(h, p)\}, \min\{N_H(y, p), N_H(h, p)\}\} \\ &= \min\{N_H(h, p), N_H(y, p)\} = \beta_p(y, h) = N(yH, p). \end{aligned}$$

Interchanging xHyH with yH, we get $N(yH, p) \geq N(xHyH, p)$.

Therefore $N(xHyH, p) = N(yH, p)$.

3.12 Proposition: Let N and R be two P- fuzzy soft quotient normal subgroups of G / H. Then $N \cap R$ is also P-fuzzy soft quotient normal subgroup of G / H.

Proof: For every $xH, yH \in G / H$ and $p \in P$, the observation is that

$$\begin{aligned} N \cap R(xHyH, p) &= \min\{N(xHyH, p), R(xHyH, p)\} \\ &\geq \min\{\min\{N(xH, p), N(yH, p)\}, \min\{R(xH, p), R(yH, p)\}\} \\ &= \min\{\min\{N(xH, p), R(xH, p)\}, \min\{N(yH, p), R(yH, p)\}\} \\ &= \min\{N \cap R(xH, p), N \cap R(yH, p)\} \text{ and} \end{aligned}$$

$$\begin{aligned} N \cap R(x^{-1}H, p) &= \min\{N(x^{-1}H, p), R(x^{-1}H, p)\} \\ &= \min\{N(xH, p), R(xH, p)\} \\ &\leq N \cap R(xH, p). \end{aligned}$$

Interchanging now xH with $x^{-1}H$, it makes that $N \cap R(xH, p) \leq N \cap R(x^{-1}H, p)$.

Hence $N \cap R$ is a P-fuzzy soft subgroup of G / H.

$$\begin{aligned} N \cap R(xHyH, p) &= \min\{N(xHyH, p), R(xHyH, p)\} \\ &= \min\{N(yHxH, p), R(yHxH, p)\} \\ &\leq N \cap R(yHxH, p). \end{aligned}$$

Hence $N \cap R$ is a P- fuzzy soft quotient normal subgroup of G / H.

By using, P-fuzzy soft quotient normal subgroup of N, we define P- fuzzy soft relation μ_N is as follows.

3.13 Definition: For all $(xH, yH) \in G / H \times G / H$, the P-fuzzy soft relation μ_N on G / H is defined by

$$\mu_N(xH, yH) = N(xHy^{-1}H, p) \text{ where } p \in P.$$

3.14 Proposition: The P-fuzzy soft relation μ_N is a P-fuzzy congruence on G / H.

Proof: Let xH, yH be any two elements of G / H. Then μ_N is fuzzy reflexive, since $\mu_N(xH, xH) = N(xHx^{-1}H, p) = N(H, p) = 1$. μ_N is a fuzzy symmetric, since

$$\begin{aligned} \mu_N(xH, yH) &= N(xHy^{-1}H, p) \\ &= N((yx^{-1})^{-1}H, p) \\ &= N(yx^{-1}H, p) \\ &= N(yHx^{-1}H, p) \\ &= \mu_N(yH, xH). \end{aligned}$$

Let xH, yH be any two elements of G / H and N_H be P- fuzzy soft normal subgroup of G.
Then N_H is P-fuzzy transitive, since

$$\begin{aligned} \mu_N{}^o\mu_N(xH, yH) &= \max\{\mu_N(xH, zH), \mu_N(zH, yH)\} \\ &= \max\{N(xHz^{-1}H, p), N(zHy^{-1}H, p)\} \\ &= \max\{N(xz^{-1}H, p), N(zy^{-1}H, p)\} \\ &= \max\{\beta_p(xz^{-1}, h), \beta_p(zy^{-1}, h)\} \\ &= \max\{\min\{N_H(xz^{-1}, p), N_H(h, p)\}, \min\{N_H(zy^{-1}, p), N_H(h, p)\}\} \\ &\leq \max\{\min\{\min\{N_H(xz^{-1}, p), N_H(zy^{-1}, p), N_H(h, p)\}\}\} \\ &\leq \max\{\min\{N_H(xy^{-1}, p), N_H(h, p)\}\} \\ &= \min\{N_H(xy^{-1}, p), N_H(h, p)\} \end{aligned}$$

$= \beta_p(xy^{-1}, h) = N(xHy^{-1}H, p)$
$= \mu_N(xH, yH)$.

μ_N is P- fuzzy compatible, since

$\min\{\mu_N(xH, yH), \mu_N(zH, wH)\} = \min\{N(xHy^{-1}H, p), N(zHw^{-1}H, p)\}$
$= \min\{N(xy^{-1}H, p), N(zw^{-1}H, p)\}$
$= \min\{\beta_p(xy^{-1}, h), \beta_p(zw^{-1}, h)\}$
$= \min\{\min\{N_H(xy^{-1}, p), N_H(h, p)\}, \min\{N_H(zw^{-1}, p), N_H(h, p)\}\}$
$= \min\{N_H(xy^{-1}, p), N_H(zw^{-1}, p)\}$
$= \min\{N_H(y^{-1}x, p), N_H(zw^{-1}, p)\}$.

Since N_H is P- fuzzy soft normal subgroup of G

$\leq N_H(y^{-1}xzw^{-1}, p) = N_H(xzw^{-1}y^{-1}, p)$.

Since N_H is P- fuzzy normal subgroup of G

$= \min\{N_H(xz(yw)^{-1}, p), N_H(h, p)\}$
$= \beta_p(xz(yw)^{-1}, h)$
$= N(xzH(yw)^{-1}H, p)$
$= \mu_N(xzH, ywH)$,

So it is P- fuzzy congruence on G / H. This completes the proof.

CONCLUSION

W.M.Wu [26] and A.Rosenfeld [21] introduced the concept of fuzzy soft normal subgroup and fuzzy groups. We investigate the concept of special fuzzy soft relations of P- fuzzy soft group and derive some simple consequences.

REFERENCES

1. Antony, J.M and Sherwood, H., Fuzzy groups redefined, J. Math. Anal. Appl. 69, 124-130, 1979.
2. Atagun AO,Sezgin A, Soft substructures of near ring modules (submitted)
3. Atagun AO, Soft sub near rings, soft ideals and soft N-subgroups of near rings (submitted)
4. Ashhan Sezgin, Akm Osman Atagun, Naim Cagman, "Union soft substructure of near rings and N-groups" Neural Compt & Applic (2012), 21(supppl): S133-S143.
5. U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Computers and Mathematics with Applications, 59 (2010) 3458-3463.
6. H. Aktas and N. Cagman, Soft sets and soft group, Information Science 177 (2007), pp. 2726–2735.
7. D. Chen, E.C.C. Tsang, D.S. Yeung and X. Wang, The parameterization reduction of soft set and its applications, Computers & Mathematics with Applications 49 (2005), pp. 757–763.
8. L. Filep and I. Maurer, Fuzzy congruence's and compatible fuzzy partitions, Fuzzy sets and systems 29(1989), 357-361.
9. F.Feng,Y.B.Jun and X.Z.Zhao, Soft semi-rings, Computers Math. Appl.56 (2008), 2621-2628.
10. Kuroki, N., Fuzzy congruencies and fuzzy normal subgroups, Inform. Sci. 60, 247 – 259 (1992).
11. N.Kuroki, Fuzzy congruence and Fuzzy normal subgroups, Inform.Sci, 60 (1992), 247-361.
12. I. J. Kumar, P. K. Saxena and Pratibha Yadava, Fuzzy normal subgroups and fuzzy quotients, Fuzzy Sets and Systems, 46 (1992), 121-132.
13. G.J. Klir and B. Yuan, Fuzzy sets and fuzzy logic, Theory and Applications, Prentice-Hall Inc, New Jersey (1995)
14. D. Molodtsov, Soft set theory–First results, Computers & Mathematics with Applications 37 (4/5) (1999), pp. 19–31.
15. P.K. Maji and A.R. Roy, An application of Soft set in decision making problem, Computers & Mathematics with Applications 44 (2002), pp. 1077–1083.
16. P.K. Maji, R. Biswas and A.R. Roy, Soft set theory, Computers & Mathematics with Applications 45 (2003), pp. 555–562.
17. P.K. Maji, R. Biswas and A.R. Roy, Fuzzy soft sets, Journal of Fuzzy Mathematics 9 (3) (2001), pp. 589–602.
18. Naim cagman, Sezgin A,Atagun AO, α-inclusions and their applications to group theory (submitted).
19. N.P. Mukherjee and P. Battacharya, Fuzzy groups, some group theoretic analogs, Inform.sci 39 (1986), 247 – 268.
20. D. S. Malik and J. Mordesen, A note on fuzzy relations and fuzzy groups, Inform.Sci.56 (1991), 193-198.
21. A.Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971), 512-517.
22. S. Tamura, S. Higuchi and K. Tanaka, Pattern classification based on fuzzy relations, IEEE Trans. System Man Cybernet, SMC 1(1971), 61-66.
23. R. T. Yeh and S. Y. Bang, Fuzzy relations, Fuzzy graphs and their Applications, Academic press New York, 1975.

24. Wanging Wu, Normal fuzzy subgroups, Fuzzy Math I(1981), 21-30.
25. Wanging Wu, Fuzzy congruence's and normal fuzzy groups, Fuzzy Math. 3(1988). 9-20.
26. W.M.Wu,Normal fuzzy subgroups, Fuzzy Math,1,(1981),21-23.
27. L. A. Zadeh, Fuzzy sets, Inform.control.8 (1965), 338-353.

Source of Support: Nil, Conflict of interest: None Declared