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# ON GENERALIZED MINIMAL REGULAR SPACES AND GENERALIZED MINIMAL NORMAL SPACES

SUWARNLATHA N. BANASODE

Department of Mathematics,
K. L. E. Society's Raja Lakhamagouda Science Institute,
Belgaum - 590001. Karnataka, India.

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***ABSTRACT***

*In this paper the notions of g-minimal regular spaces and g-minimal normal spaces are introduced and studied in topological spaces. A topological space (X, $\tau$) is said to be generalized minimal regular (briefly g-$m_i$ regular) space if for every g-$m_i$ closed set F of X and each point $x \in F^c$ there exists disjoint open sets U and V of X such that $x \in U$ and $F \subset V$. A topological space (X, $\tau$) is said to be generalized minimal normal (briefly g-$m_i$ normal) space if for any pair of disjoint g-$m_i$ closed sets A and B, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$. Some basic properties of such spaces are obtained.*

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## 1. INTRODUCTION AND PRELIMINARIES

N. Levine [2], in 1970 introduced generalized closed (g-closed) sets in topological spaces as a generalization of closed sets. Since then, many concepts related to g-closed sets were defined and investigated. B. M. Munshi [6] has introduced the notions of g-regular spaces and g-normal spaces in topological spaces. Further T. Noiri and V. Popa [9] studied on g-regular spaces and g-normal spaces. Recently minimal open sets and maximal open sets in topological spaces were introduced and characterized by F.Nakaoka and N. Oda ([7], [8]). In section 2, we obtain new characterizations of g-minimal regular spaces whereas in section 3, g-normal spaces are characterized and studied.. The main purpose of this paper is to obtain several preservation theorems of g-$m_i$ regular spaces and g-$m_i$ normal spaces.

Throughout this paper X, Y and Z represent nonempty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, cl (A), int (A) and $A^c$ denote the closure of A, the interior of A and the complement of A in X, respectively. Let us recall the following definitions, which are useful in the sequel.

**Definition 1.1:** A proper nonempty subset A of a topological space (X, $\tau$) is called
- (i) a minimal open (resp. minimal closed) set [7] if any open (resp. closed) subset of X which is contained in A, is either A or $\phi$.
- (ii) a maximal open (resp. maximal closed) set [8] if any open (resp. closed) set which contains A, is either A or X.

**Definition 1.2:** A subset A of a topological space (X, $\tau$) is called
- (i) a generalized closed[2] (briefly g-closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is an open set in X.
- (i) a generalized minimal closed (briefly g-$m_i$ closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a minimal open set in X.
- (ii) a generalized maximal open (briefly g-$m_a$ open) set iff $A^c$ is a generalized minimal closed set in X.

***Corresponding author: Suwarnlatha N. Banasode**
**Department of Mathematics, K. L. E. Society's Raja Lakhamagouda Science Institute,**
**Belgaum – 590001, Karnataka, India.**

**Definition 1.3:** A topological space (X, $\tau$) is said to be

(i) g-regular [6] if for each g-closed set F of X and each point $x \in F^c$, there exist disjoint open sets U and V of X such that $x \in U$ and $F \subset V$.

(ii) g-normal[6] if for any pair of disjoint g-closed sets A and B, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

**Definition1.4:** A mapping *f*: (X, $\tau$) $\rightarrow$(Y, $\sigma$) is said to be

(i) generalized minimal continuous (briefly g-$m_i$ continuous) map if the inverse image of every minimal closed set in Y is g-minimal closed set in X.

(ii) generalized minimal irresolute (briefly g-$m_i$ irresolute) map if the inverse image of every g- minimal closed set in Y is a g-minimal closed set in X.

**Definition 1.5:** A map *f*: (X, $\tau$) $\rightarrow$ (Y, $\sigma$) is said to be generalized minimal$^*$ closed (briefly g- $m_i^*$ closed) map if the image of every g-minimal closed set in X is a g-minimal closed set in Y.

## 2. GENERALIZED MINIMAL REGULAR SPACES

**Definition 2.1:** A topological space (X, $\tau$) is said to be generalized minimal regular (briefly g-$m_i$ regular) space if for every g-$m_i$ closed set F of X and each point $x \in F^c$ there exists disjoint open sets U and V of X such that $x \in U$ and $F \subset V$.

**Theorem 2.2:** Every g-regular space is a g-$m_i$ regular space.

**Proof:** Let X be a g-regular space and F be a g-$m_i$ closed set in X such that for every $x \in X$, $x \in F^c$. Since every g-$m_i$ closed set is a g-closed set in X, F is a g-closed set in X. But X is g-regular space. Therefore for each g-closed set F in X and each point $x \in F^c$, there exists disjoint open sets U and V in X such that $x \in U$ and $F \subset V$. Thus for every g-$m_i$ closed set F in X and each point $x \in F^c$, there exist disjoint open sets U and V in X such that $x \in U$ and $F \subset V$. Hence X is a g-$m_i$ regular space.

**Remark 2.3:** Converse of the Theorem 1.2.2 need not be true.

**Example 2.4:** Let X = {a, b, c, d} with $\tau$ ={$\phi$, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X}.
Open sets: $\phi$, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X.
Closed sets: $\phi$, {a}, {c}, {d}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, d},{a, c, d},{b, c, d}, X.
g-closed sets: $\phi$, {a}, {c}, {d}, {a, c}, {a, d},{b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X.

Here (X, $\tau$) is a g-$m_i$ regular space but not a g-regular space. Since for a g-closed set F = {d}, $F^c$ = {a, b, c} so that for $b \in F^c$ = {a, b, c}, there do not exist disjoint open sets U and V such that $b \in U$ and $F \subset V$.

**Theorem 2.5:** In a topological space (X, $\tau$), the following statements are equivalent.

(i) (X, $\tau$) is a g-$m_i$ regular space.

(ii) For each $x \in X$ and for each g-$m_a$ open set U containing x, there exists an open set V such that $x \in V \subset cl\ (V) \subset U$.

(iii) For each $x \in X$ and for each g-$m_i$ closed set F, such that $x \in F^c$ there exists an open set V such that $x \in V$ and $cl\ (V) \cap F = \phi$.

**Proof:**

**(i)$\Rightarrow$(ii):** Let (X, $\tau$) be a g-$m_i$ regular space and $x \in X$. Let U be any g-$m_a$ open set containing x. Then $U^c$ is a g-$m_i$ closed set such that $x \notin U^c$. Since X is a g-$m_i$ regular space, there exist disjoint open sets V and W of X such that $x \in V$ and $U^c \subset W$.

Now $V \cap W = \phi$ implies $V \subset W^c$ which implies $cl\ (V) \subset cl\ (W^c) = W^c$ which implies that $cl\ (V) \subset W^c$ ……….. (a),

since $W^c$ is a closed set. Again since $U^c \subset W$, $W^c \subset U$…….. (b).

Therefore from (a) and (b), $x \in V \subset cl\ (V) \subset W^c \subset U$. Thus $x \in V \subset cl\ (V) \subset U$.

**(ii)$\Rightarrow$(iii):** For each $x \in X$ let F be any g-$m_i$ closed set in X such that $x \in F^c$. Then $F^c$ is a g-$m_a$ open set containing x. By (ii) there exists an open set V such that $x \in V \subset cl\ (V) \subset F^c$, which implies $x \notin F$. Therefore $cl\ (V) \cap F = \phi$.

**(iii) $\Rightarrow$(i):** Let $x \in X$ and let F be any g-$m_i$ closed set in X such that $x \in F^c$. By (iii) there exists an open set V such that $x \in V$ and $cl\ (V) \cap F = \phi$. Since cl (V) is a closed set, $[cl\ (V)]^c$ is an open set. Now $cl\ (V) \cap F = \phi$ implies that $F \subset [cl\ (V)]^c$. Hence for every g-$m_i$ closed set F in X and for each point $x \in F^c$, there exist disjoint open sets V and $[cl\ (V)]^c$ such that $x \in V$ and $F \subset [cl\ (V)]^c$. Thus (X,$\tau$) is a g-$m_i$ regular space.

 

**Theorem 2.6:** If $f$: (X, τ) →(Y, σ) is a bijection, g-$m_i$ irresolute, open map and X is a g-$m_i$ regular space, then Y is a g-$m_i$ regular space.

**Proof:** Let F be any g-$m_i$ closed set in Y and y ∈ $F^c$. Since $f$ is a bijective g-$m_i$ irresolute map, there exists x∈X such that x = $f^{-1}$ (y) which implies $f$(x) = y and $f^{-1}$(F) is a g-$m_i$ closed set in X. Also x ∈$[f^{-1}(F)]^c$. Since X is a g-$m_i$ regular space, by definition for each g-$m_i$ closed set $f^{-1}$(F) in X such that x ∈$[f^{-1}(F)]^c$, there exists disjoint open sets U and V in X such that x∈ U and $f^{-1}$(F) ⊂V. But $f$ is a bijective open map. Therefore $f$(U) and $f$(V) are open sets in Y and $f$(U) ∩ $f$(V) = ϕ. Since U∩V=ϕ, $f$(U∩V) = ϕ which implies that $f$(U) ∩ $f$(V) = ϕ. Now x∈ U implies $f$(x)∈ $f$(U) which implies that y ∈ $f$(U) and $f^{-1}$(F) ⊂V implies F⊂$f$(V). Therefore for each g-$m_i$ closed set F of Y and for each y ∈ $F^c$, there exist disjoint open sets $f$(U) and $f$(V) in Y such that y ∈ $f$(U) and F⊂$f$(V). Thus Y is a g-$m_i$ regular space.

**Theorem 2.7:** If $f$: (X, τ) →(Y, σ) is a continuous, g-$m_i^*$ closed, injection and Y is a g-$m_i$ regular space, and then X is a g-$m_i$ regular space.

**Proof:** Let F be any g-$m_i$ closed set of X and x ∈ $F^c$. Since $f$ is a continuous g-$m_i^*$ closed map, $f$(F) is a g-$m_i$ closed set in Y and $f$(x)∈$[f(F)]^c$. Also since Y is a g-$m_i$ regular space, there exist disjoint open sets U and V such that $f$(x)∈U and $f$(F) ⊂ V which implies that x∈$f^{-1}$(U) and F⊂ $f^{-1}$(V) and $f^{-1}$(U) ∩$f^{-1}$(V) = ϕ. Therefore X is a g-$m_i$ regular space.

## 3. GENERALIZED MINIMAL NORMAL SPACES

**Definition 3.1:** A topological space (X, τ) is said to be generalized minimal normal (briefly g-$m_i$ normal) space if for any pair of disjoint g-$m_i$ closed sets A and B, there exist disjoint open sets U and V such that A⊂ U and B⊂V.

**Theorem 3.2:** Every g-normal space is a g-$m_i$ normal space.

**Proof:** Let (X, τ) be any g-normal space and let A and B be any pair of disjoint g-$m_i$ closed sets in X. Since every g-$m_i$ closed set is a g-closed set, A and B are g-closed set in X. By hypothesis, for any pair of disjoint g-closed sets A and B there exists disjoint open sets U and V such that A⊂ U and B⊂V. Therefore for any pair of disjoint g-$m_i$ closed sets A and B, there exists disjoint open sets U and V such that A⊂ U and B⊂V. Hence (X, τ) is g-$m_i$ normal space.

**Remark 3.3:** Converse of the Theorem 1.3.2 need not be true.

**Example 3.4:** Let X = {a, b, c, d} with τ ={ϕ, {b}, {c}, {a, d}, {b, c}, {a, b, d}, {a, c, d}, X}.
Open sets: ϕ, {b}, {c}, {a, d}, {b, c}, {a, b, d}, {a, c, d}, X..
Closed sets: ϕ, {b}, {c}, {a, d}, {b, c}, {a, b, d}, {a, c, d}, X.
g-closed sets: ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X..
Here (X, τ) is a g-$m_i$ normal space but not a g-normal space. Since
For disjoint g-closed sets {a} and {d} there do not exist disjoint open sets U and V such that {a}⊂ U and {d}⊂V.

**Theorem 3.5:** In a topological space (X, τ), the following statements are equivalent.
(i) (X, τ) is a g-$m_i$ normal space.
(ii) For each g-$m_i$ closed set A and each g-$m_a$ open set U such that A⊂ U, there exists an open set V such that A⊂ V ⊂ cl (V) ⊂U.
(iii) For every pair of disjoint g-$m_i$ closed sets A and B of X, there exists an open set V such that A⊂V and cl (V) ∩ B = ϕ.

**Proof:**
**(i) ⇒ (ii):** Let (X, τ) be any g-$m_i$ normal space. Let A be a g-$m_i$ closed set A and U be g-$m_a$ open set such that A⊂ U. Then $U^c$ is a g-$m_i$ closed set in X. Now A and $U^c$ are disjoint g-$m_i$ closed sets. By (i), there exist disjoint open sets V and W such that A ⊂ V and $U^c$ ⊂ W. Now V ∩W = Φ implies V ⊂ $W^c$ which implies cl (V) ⊂ cl ($W^c$) = $W^c$ which implies cl (V) ⊂ $W^c$, since $W^c$ is a closed set. Again, since $U^c$ ⊂ W, $W^c$ ⊂ U. Therefore A⊂ V⊂ cl (V) ⊂ U.

**(ii) ⇒(iii):** Let A and B be any pair of disjoint g-$m_i$ closed sets so that A ∩ B = ϕ then A⊂$B^c$. Since A is a g-$m_i$ closed set and $B^c$ is a g-$m_a$ open set such that A⊂$B^c$, by (ii) there exists an open set V such that A⊂ V⊂ cl (V) ⊂ $B^c$ which implies cl (V) ∩ B = ϕ.

**(iii) ⇒(i):** Let A and B be any pair of disjoint g-$m_i$ closed sets in X.

By (iii), there exists an open set V such that A⊂V and cl (V) ∩ B = ϕ which implies A⊂V and B ⊂ $[cl (V)]^c$. Therefore for any pair of disjoint g-$m_i$ closed sets in X, there exists disjoint open sets V and $[cl (V)]^c$ such that A ⊂ V and B ⊂ $[cl (V)]^c$. Therefore (X, τ) is a g-$m_i$ normal space.

 

**Theorem 3.6:** If $f$: (X, $\tau$) $\rightarrow$ (Y, $\sigma$) is bijection, g-$m_i$ irresolute, open map and X is a g-$m_i$ normal space, then Y is a g-$m_i$ normal space.

**Proof:** Let A and B be any pair of disjoint g-$m_i$ closed sets in Y. Since $f$ is a g-$m_i$ irresolute map, $f^{-1}$(A) and $f^{-1}$(B) are g-$m_i$ closed sets in X and hence $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \phi$. But X is a g-$m_i$ normal space, so there exists disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since $f$ is open, bijective map, $A \subset f(U)$, $B \subset f(V)$ and $f(U) \cap f(V) = \phi$. Also $f$ (U) and $f$ (V) are open in Y. This shows that Y is a g-$m_i$ normal space.

**Theorem 3.7:** If $f$: (X, $\tau$) $\rightarrow$ (Y, $\sigma$) is continuous, g-$m_i^*$ closed, injection and Y is a g-$m_i$ normal space, and then X is a g-$m_i$ normal space.

**Proof:** Let A and B be any pair of disjoint g-$m_i$ closed sets in X. Since $f$ is a g-$m_i^*$ closed map, $f$(A) and $f$(B) are g-$m_i$ closed sets in Y and $f(A) \cap f(B) = \phi$. But Y is a g-$m_i$ normal space. So there exist disjoint open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Thus we obtain $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Since $f$ is a continuous map, $f^{-1}$(U) and $f^{-1}$(V) are open sets in X. This shows that X is a g-$m_i$ normal space.

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