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AN INTUITIONISTIC P-FUZZY SOFT HOMOLOGOUS GROUP STRUCTURES

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ABSTRACT

In this paper, we study an intuitionistic P-fuzzy soft set (IPFSS) and intuitionistic P-fuzzy soft normal subgroup (IPFSNS). We also investigate intuitionistic P- fuzzy soft homologous group characterized as normal group which admit a particular type of intuitionistic P- fuzzy soft groups with respect to (min, max) norms.

Keywords: *Intuitionistic P-fuzzy soft set, intuitionistic P-fuzzy soft group, intuitionistic P-fuzzy soft-cut, intuitionistic P- fuzzy soft normal subgroup, intuitionistic P- fuzzy soft centralizer, homologous subgroup.*
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1. INTRODUCTION

The theory of fuzzy groups defined by Rosenfeld [7] is the first application of fuzzy theory in Algebra. Since then a number of works have been done in the area of fuzzy algebra. Gau.W.L and Buehrer.D.J [3] has initiated the study of vague sets as an improvement over the theory of fuzzy sets to interpret and solve real life problems which are in general vague. Soft set theory was introduced by Molodtsov [4] for modeling vagueness and uncertainty and it has received much attention since Maji *et.al* [5] introduced and studied several operations of soft sets. Soft set theory started to progress rapidly in the mean of algebraic structures since Aktas and Cagman [2] defined and studied soft groups. Since then, [1] have studied the soft algebraic structures and soft sets as well. Applying the definition of soft set, Atagun and Sezgin [8] introduced the algebraic soft substructures of rings, fields and modules. Cagman *et.al* [2] studied on soft int-group, which are different from the definitions of soft groups [2]. The new approach is based on the inclusion relation and intersection of sets. It brings the soft set theory, the set theory and the group theory together. On the basic of soft int-groups, Sezgin *et.al* [8] introduced the concept of soft intersection near-rings (soft int-near rings) by using intersection operation of sets and gave the applications of soft int near-rings to the near-ring theory. By introducing soft intersection, union, products and soft characteristic functions, Sezer [9]made a new approach to the classical ring theory via the soft set theory, with the concepts of soft union rings, ideals and bi-ideals. The objective of this paper is to contribute further to the study of intuitionistic P- fuzzy soft group and introducing concepts of intuitionistic P- fuzzy soft normalizer, intuitionistic P- fuzzy soft centralizer and intuitionistic P- fuzzy soft homologous group by imposing fitness condition which we observe can be removed. Also we characterized the intuitionistic P- fuzzy soft normal subgroup and homologous intuitionistic P- fuzzy soft group which admit a particular type of intuitionistic fuzzy soft group.

2. PRELIMINARIES

Definition 2.1: A soft set f_A over U is defined as f_A: E→ P(U) such that $f_A(x) = \emptyset$ if $x \notin$ A. In other words, a soft set f_A over U is a parameterized family of subsets of the universe U. For all ε ∈ A, $f_A(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set f_A.

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A soft set f_A over U can be presented by the set of ordered pairs:

$$f_A = \{(x, f_A(x)) \,/\, x \in E,\ f_A(x) = P(U)\} \tag{1}$$

Clearly, a soft set is not a set. For illustration, Molodtsov [4] consider several examples in [4].

If f_A is a soft set over U, then the image of f_A is defined by $\mathrm{Im}(f_A) = \{f_A(a)/a \in A\}$. The set of all soft sets over U will be denoted by S(U). Some of the operations of soft sets are listed as follows.

Definition 2.2: Let $f_A, f_B \in$ S(U). If $f_A(x) \subseteq f_B(x)$, for all $x \in$ E, then f_A is called a soft subset of f_B and denoted by $f_A \subseteq f_B$. f_A and f_B are called soft equal, denoted by $f_A = f_B$ if and only if $f_A \subseteq f_B$ and $f_B \subseteq f_A$.

Definition 2.3: Let $f_A, f_B \in$ S(U) and let χ be a function from A to B. Then the soft anti-image of f_A under χ denoted by $\chi(f_A)$, is a soft set over U defined by,

$$\chi_{f_A}(b) = \begin{cases} \cap \{ f_A(a)/a \in A, \chi(a) = b\}, if\ \chi^{-1}(b) \neq \emptyset \\ 0 \qquad , \text{ otherwise} \end{cases} \tag{2}$$

for all $b \in$B. And the soft preimage of f_B underχ, denoted by $\chi^{-1}(f_B)$, is a soft set over U defined by
$\chi^{-1}{}_{f_B}(a) = f_B(\chi(a))$, for all $a \in A$.

Note that the concept of level sets in the fuzzy set theory, Cagman *et.al* [2] initiated the concept of lower inclusions soft sets which serves as a bridge between soft sets and crisp sets.

Definition 2.4: Let $\mu_{\tilde{A}}$: U → [0,1] be any function and A be a crisp set in the universe 'U'. Then the ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in U\}$ is called a fuzzy set and $\mu_{\tilde{A}}$ is called a membership function.

Definition 2.5: An intuitionistic P- fuzzy soft set (or in-short IPFSS) in the universe of discourse X is characterized by two membership functions given by

1. a truth membership function $t_A : X \times P \rightarrow [0,1]$
2. a false membership function $f_A : X \times P \rightarrow [0,1]$ such that $t_A(x, p) + f_A(x, p) \leq 1$, for all x∈X and p∈P.

Definition 2.6: The interval $[t_A(x, p), 1 - f_A(x, p)]$ is called an intuitionistic P-fuzzy soft value of X in A and it is denoted by $I_A(x, p)$. (ie) $I_A(x, p) = [t_A(x, p), 1- f_A(x, p)]$.

Definition 2.7: An intuitionistic P-fuzzy soft set 'A' of X with $t_A(x, p) = 0$ and $f_A(x, p) =1$ for all x∈X and p∈P is called zero intuitionistic P-fuzzy soft set of X. A intuitionistic P- fuzzy soft set 'A' of X with $t_A(x, p) = 1$ and $f_A(x, p) = 0$ for all x∈X and p ∈P is called unit intuitionistic P-fuzzy soft set of X.

Definition 2.8: A intuitionistic P-fuzzy soft set 'A' of X with $t_A(x, p) = \alpha$ and $f_A(x, p) =1-\alpha$ for all x∈X and p ∈P is called an α- intuitionistic P- fuzzy soft set of X (α -IPFSS) where α∈[0,1].

Definition 2.9: Let P and G be a set and group respectively. An intuitionistic P-fuzzy soft set 'A' of G is called an intuitionistic P-fuzzy soft subgroup of G if for all x, y in G and p∈P.
(IPFSG1) $I_A(xy, p) \geq T\{I_A(x, p), I_A(y, p)\}$ and (IPFSG2) $I_A(x^{-1}, p) \geq I_A(x, p)$.
(ie) $t_A(xy, p) \geq T\{t_A(x, p), t_A(y, p)\}$, $f_A(xy, p) \leq S\{f_A(x, p), f_A(y, p)\}$ and $t_A(x^{-1}, p) \geq t_A(x, p)$, $f_A(x^{-1}, p) \leq f_A(x, p)$.
Here the element xy stands for x • y.

Definition 2.10: The α-cut A_α of an intuitionistic P-fuzzy soft set 'A' is the (α, α) cut of A and hence given by $A_\alpha = \{x / x \in G,\ t_A(x, p) \geq \alpha\}$.

Definition 2.11: Let 'A' be an intuitionistic P-fuzzy soft subgroup (IPFSG) of G. Then 'A' is called an intuitionistic P-fuzzy soft normal subgroup (IPFSNG) is $I_A(xy, p) = I_A(yx, p)$ for $x, y \in G$, p ∈ P.

Definition 2.12: Let 'A' be a intuitionistic P-fuzzy soft subgroup of G. Then the set $N(A) = \{a \in G / I_A(axa^{-1}, p) = I_A(x, p)\}$, for x∈G and p ∈P is called an intuitionistic P- fuzzy soft normalizer of A.

Definition 2.13: Let 'A' be a intuitionistic P-fuzzy soft subgroup of G. Then $C(A) = \{a \in G / I_A([a, x]_p) = I_A(e, p)\}$, for all x∈G and p∈P is called an intuitionistic P-fuzzy soft centralizer of A where $[a, x]_p = (a^{-1}x^{-1}ax, p)$.

3. CHARACTERIZATIONS OF AN INTUITIONISTIC P-FUZZY SOFT NORMAL SUBGROUP

Proposition 3.1: If 'A' is an intuitionistic P- fuzzy soft normal subgroup of a group G, then
$K = \{x \in G / I_A(x, p) = I_A(e, p)\}$ is a crisp normal subgroup of G.

Proof: 'A' is an intuitionistic P-fuzzy soft normal subgroup of G. Let x, y∈K and p∈P implies $I_A(x, p) = I_A(e, p)$ and $I_A(y, p) = I_A(e, p)$. Consider $I_A(x^{-1}y, p) \geq T\{I_A(x, p), I_A(y, p)\} = T\{I_A(e, p), I_A(e, p)\} = I_A(e, p)$ implies $I_A(x^{-1}y, p) = I_A(e, p)$ implies $x^{-1}y \in H$. Therefore 'K' is a crisp subgroup of G. Let x∈G, y∈K. Consider $I_A(xyx^{-1}, p) = I_A(y, p) = I_A(e, p)$ implies $xyx^{-1} \in K$. Therefore K is a crisp normal subgroup of G.

Proposition 3.2: Let 'A' be an intuitionistic P-fuzzy soft normal subgroup of G. Then α-cut A_α is a crisp normal subgroup of G.

Proof: $A_\alpha = \{x \in G \,/\, t_A(x, p) \geq \alpha\}$. Let x, y $\in A_\alpha$ implies $t_A(x, p) \geq \alpha$ and $t_A(y, p) \geq \alpha$. Consider $t_A(xy^{-1}, p) \geq \min\{t_A(x, p), t_A(y, p)\} \geq \min\{\alpha, \alpha\} = \alpha$ implies $t_A(xy^{-1}, p) \geq \alpha$ implies $xy^{-1} \in A_\alpha$. Therefore A_α is a crisp subgroup of G. Now, for all x∈G, y $\in A_\alpha$, consider $t_A(xyx^{-1}, p) = t_A(y, p) \geq \alpha$ implies $xyx^{-1} \in A_\alpha$. This completes the proof.

Proposition 3.3: If A and B are two intuitionistic P-fuzzy soft normal subgroups of G, then A∩B is also an intuitionistic P-fuzzy soft normal subgroup of G.

Proof: If A and B are two intuitionistic P-fuzzy soft subgroups of G, then A∩B is also intuitionistic P-fuzzy soft subgroup of G.

Now, $t_{A\cap B}(xy, p) = T\{t_A(xy, p), t_B(xy, p) = T\{t_A(yx, p), t_B(yx, p)\} = t_{A\cap B}(yx, p)$.

Also $f_{A\cap B}(xy, p) = S\{f_A(xy, p), f_B(xy, p)\} = S\{f_A(yx, p), f_B(yx, p)\} = f_{A\cap B}(yx, p)$.

Implies that $I_{A\cap B}(xy, p) = I_{A\cap B}(yx, p)$. Thus A∩B is an intuitionistic P-fuzzy soft normal subgroup in G.

Proposition 3.4: Let 'A' be a intuitionistic P-fuzzy soft subgroup of G and B be an intuitionistic P-fuzzy soft normal subgroup of G. Then A∩B is an intuitionistic P-fuzzy soft normal subgroup of the group $K = \{x \in G / I_A(x, p) = I_A(e, p)\}$.

Proof: Since 'A' is a intuitionistic P-fuzzy soft subgroup of G, then $K = \{x \in G \,/\, I_A(x, p) = I_A(e, p)\}$ is a crisp subgroup of G. Also A∩B is an intuitionistic P-fuzzy soft subgroup of G. Now we wish to show that A∩B is an intuitionistic P-fuzzy soft normal subgroup of K. Let x, y∈ K. Then xy ∈ K and yx ∈ K implies $I_A(xy, p) = I_A(e, p)$ and $I_A(yx, p) = I_A(e, p)$ implies $I_A(xy, p) = I_A(yx, p)$. Since 'B' is an intuitionistic P-fuzzy soft normal subgroup of G implies $I_B(xy, p) = I_B(yx, p)$.

Consider $t_{A\cap B}(xy, p) = T\{t_A(xy, p), t_B(xy, p)\} = T\{t_A(yx, p), t_B(yx, p)\} = t_{A\cap B}(yx, p)$.

Also, $f_{A\cap B}(xy, p) = S\{f_A(xy, p), f_B(xy, p)\} = S\{f_A(yx, p), f_B(yx, p)\} = f_{A\cap B}(yx, p)$.

Therefore $I_{A\cap B}(xy, p) = I_{A\cap B}(yx, p)$. Thus A∩B is an intuitionistic P-fuzzy soft normal subgroup of K.

Proposition 3.5: Let 'A' be a intuitionistic P- fuzzy soft subgroup of G. Then 'A' is an intuitionistic P- fuzzy soft normal subgroup of G iff $I_A([x, y]_p) \geq I_A(x, p)$ for all x, y in G, where $[x, y]_p = (x^{-1}y^{-1}xy, p)$.

Proof: Suppose 'A' is an intuitionistic P-fuzzy soft normal subgroup of G. For all x, y ∈ G,
$I_A([x, y]_p) = I_A(x^{-1}y^{-1}xy, p) = I_A(x^{-1}(y^{-1}xy), p) \geq T\{I_A(x^{-1}, p), I_A(y^{-1}xy, p)\}$
$= T\{I_A(x, p), I_A(x, p)\} = I_A(x, p)$.

Therefore $I_A([x, y]_p) \geq I_A(x, p)$.

Conversely, suppose $I_A([x, y]_p) \geq I_A(x, p)$. For x, z in G,

We have $I_A(x^{-1}zx, p) = I_A(ex^{-1}zx, p) = I_A(zz^{-1}x^{-1}zx, p) = I_A(z[z, x]_p) \geq T\{I_A(z, p), I_A([z, x]_p)\} = I_A(z, p)$.

Therefore $I_A(x^{-1}zx, p) \geq I_A(z, p)$ for all x, z in G and p ∈ P. Again we get that
$I_A(z, p) = I_A(xx^{-1}zxx^{-1}, p) \geq T\{I_A(x, p), I_A(x^{-1}zx, p)\}$.

Now if $T\{I_A(x, p), I_A(x^{-1}zx, p)\} = I_A(x, p)$, then we obtain $I_A(z, p) \geq I_A(x, p)$ for all x, z in G. Implying the constant set and in this case the result is holds trivially.

If $T\{I_A(x, p), I_A(x^{-1}zx, p)\} = I_A(x^{-1}zx, p)$, then we get $I_A(z, p) \geq I_A(x^{-1}zx, p)$ for all x, z in G.

This implies $I_A(z, p) = I_A(x^{-1}zx, p)$. Thus 'A' is an intuitionistic P- fuzzy soft normal subgroup of G.

Proposition 3.6: Let ' A' be an intuitionistic P-fuzzy soft normal subgroup of G. Then
(i) P-normalizer N(A) is a crisp subgroup of G.
(ii) 'A' is an intuitionistic P- fuzzy soft normal subgroup of N(A).

Proof:
(i) 'A' is an intuitionistic P-fuzzy soft normal subgroup of G and $N(A) = \{a \in G / I_A(ax^{-1}a, p)\} = I_A(x, p)$, for all $x \in G$. Now let $x, y \in N(A)$ implies $I_A(xax^{-1}, p) = I_A(a, p)$ and $I_A(yby^{-1}, p) = I_A(b, p)$.
Consider $I_A(xy^{-1}a(xy^{-1})^{-1}, p) = I_A(xy^{-1}ax^{-1}y, p) = I_A(y^{-1}ay, p) = I_A(a, p) = xy^{-1} \in N(A)$.
Therefore, N(A) is a crisp subgroup of G.
(ii) Suppose 'A' is an intuitionistic P-fuzzy soft normal subgroup of G. Let $a \in G$, $x \in A$, $I_A(xax^{-1}, p) = I_A(a, p)$ implies $a \in N(A)$ implies $G \subseteq N(A) \subseteq G$. This implies N(A) = G. Conversely, N(A) = G implies $I_A(axa^{-1}, p) = I_A(x, p)$ for all $x \in G$ implies 'A' is an intuitionistic P-fuzzy soft normal subgroup of G.
Let $x \in A$ and $\alpha \in N(A) \subseteq G$ implies $I_A(x\alpha x^{-1}, p) = I_A(\alpha, p)$ implies 'A' is an intuitionistic P-fuzzy soft normal subgroup of N(A).

Proposition 3.7: Let 'A' be an intuitionistic P-fuzzy soft normal subgroup of G and $K = \{x \in G / I_A(x, p) = I_A(e, p)\}$. Then $K \subseteq$ **C(**A).

Proof: Let $x \in K$ implies $I_A(x, p) = I_A(e, p)$. For all $y \in G$,
we consider $I_A([x, y]_p) = I_A(x^{-1}y^{-1}xy, p) = I_A(x^{-1}(y^{-1}xy), p) \geq T\{I_A(x, p), I_A(y, p)\} = I_A(x, p) = I_A(e, p)$ implies $I_A([x, y]_p) \geq I_A(e, p)$. But $I_A([x, y]_p) \leq I_A(e, p)$ implies $I_A([x, y]_p) = I_A(e, p)$ implies $x \in$ **C(**A). Thus $K \subseteq$ **C(**A).

Proposition 3.8: Let 'A' be an intuitionistic P-fuzzy soft subgroup of a group G. Then $K = \{x \in G / I_A(x, p) = I_A(e, p)\}$ is a normal subgroup of N(A) .

Proof: Let $x \in K$, $y \in G$ implies $I_A(x, p) = I_A(e, p)$.

Consider, $I_A(xyx^{-1}, p) \geq T\{I_A(x, p), I_A(y, p)\} = I_A(y, p) = I_A(eye, p) = I_A(x^{-1}(xyx^{-1}x), p) \geq T\{I_A(x^{-1}, p), I_A(xyx^{-1}x, p)\} = T\{I_A(e, p), I_A(xyx^{-1}, p)\} = I_A(xyx^{-1}, p)$, for all $y \in G$ implies $I_A(xyx^{-1}, p) = I_A(y, p)$, for all $y \in G$ implies $x \in N(A)$ implies $K \subseteq N(A)$. Now, for all $a \in N(A)$ implies $I_A(axa^{-1}, p) = I_A(x, p)$ for all $x \in G$. we have $I_A(aya^{-1}, p) = I_A(y, p) = I_A(e, p)$ for all $y \in K$. This implies $aya^{-1} \in K$. Therefore K is a normal subgroup of N(A).

Proposition 3.9: Let 'A' be an intuitionistic P-fuzzy soft subgroup of G. Then C(A) is a normal soft subgroup of G.

Proof: $C(A) = \{a \in G / I_A([a, x]_p) = I_A(e, p)$, for all $x \in G\}$

Let $a \in C(A)$ implies $I_A([a, x]_p) = I_A(e, p)$ implies $I_A(a^{-1}x^{-1}ax, p) = I_A(e, p)$ implies $I_A((xa)^{-1}ax, p) = I_A(e, p)$ implies $I_A(xa, p) = I_A(ax, p)$.

Let $a, b \in C(A)$ implies $I_A([a, x]_p) = I_A(e, p)$ and $I_A([b, x]_p) = I_A(e, p)$.

Consider $I_A([ab^{-1}, x]_p) = I_A((ab^{-1})^{-1}x^{-1}ab^{-1}x, p) = I_A(b(a^{-1}x^{-1}ab^{-1}x), p) = I_A((a^{-1}x^{-1}ax, x^{-1}b^{-1}xb), p) = I_A([a, x]_p, [x, b]_p) \geq T\{I_A([a, x]_p), I_A([x, b]_p)\} = T\{I_A(e, p), I_A(e, p)\} = I_A(e, p)$.

Therefore, $I_A([ab^{-1}, x]_p) \geq I_A(e, p)$.

Implies $I_A([ab^{-1}, x]_p) = I_A(e, p)$ implies $ab^{-1} \in C(A)$. Therefore C(A) is a subgroup of G.

Now, for all $a \in C(A)$ and $g \in G$, we have
$I_A([g^{-1}ag, x]_p) = I_A((g^{-1}ag)^{-1}x^{-1}g^{-1}agx, p) = I_A((g^{-1}a^{-1}gx, a^{-1}(gx)^{-1}a(gx)), p) = I_A([g, a]_p, I_A[a, gx]_p) \geq T\{I_A[g, a]_p, I_A[a, gx]_p\} = T\{I_A(e, p), I_A(e, p)\} = I_A(e, p)$ implies $I_A([g^{-1}ag, x]_p) \geq I_A(e, p)$ implies $I_A([g^{-1}ag, x]_p) = I_A(e, p)$ implies $g^{-1}ag \in C(A)$. Therefore, C(A) is a crisp normal soft subgroup of G.

Proposition 3.10: If 'A' is an intuitionistic P-fuzzy soft subgroup of G and θ is a homomorphism of G, then an intuitionistic P-fuzzy soft set A^{θ} is also an intuitionistic P-fuzzy soft subgroup of G.

Proof: Let $x, y \in G$ and $p \in P$. Then
$t_A^{\theta}(xy, p) = t_A(\theta(xy, p)) = t_A(\theta(x, p), \theta(y, p)) \geq T\{t_A(\theta(x, p)), t_A(\theta(y, p))\} = T\{t_A^{\theta}(x, p), t_A^{\theta}(y, p)\}$.

Also $f_A^{\theta}(xy, p) = f_A(\theta(xy, p)) = f_A(\theta(x, p), \theta(y, p)) \leq S\{f_A(\theta(x, p)), f_A(\theta(y, p))\} = S\{f_A^{\theta}(x, p), f_A^{\theta}(y, p)\}$.
Again $t_A^{\theta}(x^{-1}, p) = t_A(\theta(x^{-1}, p)) \geq t_A(\theta(x, p)) = t_A^{\theta}(x, p)$ for all $x \in G$.

Similarly, we see that $f_A^{\theta}(x^{-1}, p) = f_A(\theta(x, p)) = f_A^{\theta}(x, p)$ for all $x \in G$. Thus A^{θ} is a intuitionistic P-fuzzy subgroup of G.

Proposition 3.11: If 'A' is an intuitionistic P-fuzzy soft characteristic group of G, then A is a intuitionistic P-fuzzy soft normal subgroup of G.

Proof: Let $x, y \in G$ and $p \in P$. Consider the map $\theta: G \to G'$ given by $\theta(g, p) = (x^{-1}gx, p)$ for all $g \in G$. Clearly, θ is an automorphism of G.

Now $t_A(xy, p) = t_A^{\theta}(xy, p) = t_A(\theta(xy, p)) = t_A(x^{-1}xyx, p) = t_A(yx, p)$.

Similarly, we find that $f_A(xy, p) = f_A(yx, p)$ for all $x, y \in G$.

Therefore 'A' is an intuitionistic P-fuzzy soft normal subgroup of G.

Definition 3.12: Let A and B be two intuitionistic P-fuzzy soft subgroups of a group G. If there exists $\Phi \in Aut(G)$ such that $I_A(x, p) = I_B(\Phi(x, p))$ for all $x \in G$ (i.e.) $t_A(x, p) = t_B(\Phi(x, p))$ and $f_A(x, p) = f_B(\Phi(x, p))$, then A and B are intuitionistic P- fuzzy soft homologous subgroups of G.

Proposition 3.13: Let 'B' be an intuitionistic P- fuzzy soft subgroup of G and $\Phi \in Aut(G)$. Let 'A' be an intuitionistic P-fuzzy soft set of G such that $I_A(x, p) = I_B(\Phi(x, p))$ for all $x \in G$. Then A and B are intuitionistic P-fuzzy soft homologous subgroups of G.

Proof: $I_A(xy, p) = I_B(\Phi(xy, p)) = I_B(\Phi(x, p), \Phi(y, p)) \geq T\{I_B(\Phi(x, p)), I_B(\Phi(y, p))\} = T\{I_A(x, p), I_A(y, p)\}$. Also $I_A(x^{-1}, p) = I_B(\Phi(x^{-1}, p)) \geq I_B(\Phi(x, p)) = I_A(x, p)$. Therefore 'A' is an intuitionistic P-fuzzy soft subgroup of G. Hence A and B are intuitionistic P-fuzzy soft homologous subgroups of G.

Proposition 3.14: Let A and B be intuitionistic P-fuzzy soft homologous subgroups of G. Then C(A) and N(A) are soft homologous P- subgroups of G.

Proof: Since A and B are two intuitionistic P-fuzzy soft homologous subgroups of G, then C(A), C(B) are subgroups of G. Now we will show that C(A), C(B) are homologous soft subgroups of G. It is enough to prove that, there exists an automorphism Φ of G such that $\Phi(C(A)) = C(B)$. Since A and B are intuitionistic P-fuzzy soft homologous subgroups of G implies $\Phi \in Aut(G)$ such that $I_A(x, p) = I_B(\Phi(x, p))$, $I_B(x, p) = I_A(\Phi^{-1}(x, p))$, for $x \in G$. So for $a \in C(A)$. We have

$I_B([\Phi(a), x]_p) = I_B((\Phi(a))^{-1}x^{-1}\Phi(a)x, p) = I_B(\Phi(a^{-1})x^{-1}\Phi(a)x, p) = I_B(\Phi(a^{-1})\Phi^{-1}(x^{-1})\, a\, \Phi^{-1}(x), p) = I_A([a, \Phi^{-1}(x)]_p) = I_A(e, p) = I_A(\Phi^{-1}(x, p)) = I_B(e, p)$ for all $x \in G$.

Therefore $\Phi(C(A)) \subseteq C(B)$ (1)

On the other hand, for all $a \in C(B)$,

$$I_B([\Phi^{-1}(a), x]_p) = I_A(\Phi^{-1}(a^{-1})x^{-1}\Phi^{-1}(a)x, p) = I_A(\Phi^{-1}(a^{-1})\Phi(x^{-1})\, a\, \Phi(x), p)$$
$$= I_A(\Phi^{-1}([a, \Phi(x)]_p)) = I_B([a, \Phi(x)]_p) = I_B(e, p) = I_B(\Phi(e, p)) = I_A(e, p) .$$

Therefore $\Phi^{-1}(a, p) \in C^{-1}(A)$ implies $C(B) \subseteq \Phi(C(A))$ (2)

Thus from (1) and (2), $\Phi(C(A)) = C(B)$. Hence C(A) and C(B) are intuitionistic P-fuzzy soft homologous subgroups of G.

CONCLUSION

In this paper, we investigate a new kind of intuitionistic P-fuzzy soft normal group and its characterizations. One can obtain the similar idea in the field of bipolar fuzzy sets and vague sets.

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