

COMPUTATION OF A SUMMATION FORMULA OF HALF ARGUMENT PAIRED WITH CONTIGUOUS RELATION AND HYPERGEOMETRIC FUNCTION

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ABSTRACT

The main objective of the present paper is to establish a summation formula of half argument paired with contiguous relation and hypergeometric function.

Key words: Gaussian Hypergeometric function, Recurrence relation, Bailey summation theorem, Contiguous relation.

2010 MSC NO: 33C05, 33C20, 33C45, 33D50, 33D60.

I. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k}{(b_1)_k (b_2)_k \dots (b_B)_k} \frac{z^k}{k!} \tag{1}$$

or

$${}_A F_B((a_A); (b_B); z) \equiv {}_A F_B((a_j)_{j=1}^A; (b_j)_{j=1}^B; z) = \sum_{k=0}^{\infty} \frac{((a_A))_k}{((b_B))_k} \frac{z^k}{k!} \tag{2}$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relation is defined by:

[Andrews p.363(9.16)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z) \tag{3}$$

Recurrence relation is defined by:

$$\Gamma(\xi+1) = \xi \Gamma(\xi) \tag{4}$$

Bailey summation theorem[Prud, p.491(7.3.7.3):

$${}_2F_1(a, 1-a; c; \frac{1}{2}) = \frac{\Gamma(\frac{c}{2})\Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\Gamma(c)}{2^{c-1}\Gamma(\frac{c+a}{2})\Gamma(\frac{c+1-a}{2})} \tag{5}$$

II MAIN SUMMATION FORMULA:

$${}_2F_1(a, -a-39; c; \frac{1}{2}) = \frac{\sqrt{\pi}\Gamma(c)}{2^{c+39}} \left[\frac{8599156327855309779763200000-15021001352792310915207168000a}{\Gamma(\frac{c+a+39}{2})\Gamma(\frac{c-a}{2})} + \frac{7999533292150223027056742400a^2-1702695111924874537106657280a^3}{\Gamma(\frac{c+a+39}{2})\Gamma(\frac{c-a}{2})} + \frac{120808340483250368309333760a^4+4933642575880165513991040a^5}{\Gamma(\frac{c+a+39}{2})\Gamma(\frac{c-a}{2})} + \frac{-724355874911721577722880a^6-12513864974364030513600a^7+1694387191628313939120a^8}{\Gamma(\frac{c+a+39}{2})\Gamma(\frac{c-a}{2})} \right]$$

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$$\begin{aligned}
 &+ \frac{4331514292355434360a^9 - 1381663130521759800a^{10} - 62211712407118560a^{11} - 268903069851520a^{12}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{21020666481360a^{13} + 385792685040a^{14} + 1327753440a^{15} - 25221360a^{16} - 266760a^{17} - 760a^{18}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{21102606390025756241952768000c - 26605931711014116361093939200ac}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{10744334898628110441060372480a^2c - 1684664088428954701110362112a^3c}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{63596959693232506924527360a^4c + 6985034875129382858876544a^5c - 370792150532378646499072a^6c}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-18932101882421252406336a^7c + 555823442863466921520a^8c + 33257928756951416232a^9c}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{4952459588031384a^{10}c - 21649859385197472a^{11}c - 346407655472320a^{12}c + 1269535713264a^{13}c}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{7888578448a^{14}c + 664578720a^{15}c + 578640a^{16}c - 14040a^{17}c - 40a^{18}c + 20645406035256664809249177600c^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-19927666387405469519038218240ac^2 + 6205691479310384982485741568a^2c^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{702698908837028850238482432a^3c^2 + 5943333682195807817317632a^4c^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{3118644664717423322557440a^5c^2 - 48744550598541826793856a^6c^2 - 6969596364192336690816a^7c^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{2278233238210362576a^8c^2 + 7310823908263349760a^9c^2 + 109443095390785728a^{10}c^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-1856118816744192a^{11}c^2 - 60044006070048a^{12}c^2 - 366771525120a^{13}c^2 + 3234116160a^{14}c^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{46949760a^{15}c^2 + 150480a^{16}c^2 + 11316966227152040415193989120c^3 - 8587504636600186361666174976ac^3}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{2069592700573958435360944128a^2c^3 - 161979853483551250023032832a^3c^3}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-3286808149309039989152512a^4c^3 + 682671130793301392870400a^5c^3 + 5680143965894334378624a^6c^3}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-1135234930101247527552a^7c^3 - 19391073044864300208a^8c^3 + 631419193422251520a^9c^3}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{19036903539130048a^{10}c^3 + 39044686576896a^{11}c^3 - 3561491789856a^{12}c^3 - 39737617920a^{13}c^3}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-65250240a^{14}c^3 + 823680a^{15}c^3 + 2640a^{16}c^3 + 4010063768250013747997835264c^4}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-2421188423955762187850612736ac^4 + 449070365728037839615688704a^2c^4}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-22235213656483275184988160a^3c^4 - 1219887087817886235238400a^4c^4}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{83190547890777449041920a^5c^4 + 2459735813070498586624a^6c^4}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-90400785909517923840a^7c^4 - 3215319916181767680a^8c^4 + 11059886666949120a^9c^4}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{1336306183500288a^{10}c^4 + 12697656491520a^{11}c^4 - 56819642880a^{12}c^4 - 1314593280a^{13}c^4 - 4815360a^{14}c^4}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{992996463376124096098074624c^5 - 478943457122578941639524352ac^5}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{67427081093747059785728000a^2c^5 - 1707782282241111152713728a^3c^5}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-203320851526609834967040a^4c^5 + 5474392433642565328896a^5c^5 + 331521226391379920896a^6c^5}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-245973174742423424a^7c^5 - 243135053007475200a^8c^5 - 1847220023480832a^9c^5 + 41142632154624a^{10}c^5}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{653186806272a^{11}c^5 + 1622100480a^{12}c^5 - 13837824a^{13}c^5 - 50688a^{14}c^5 + 180282500712485167619375104c^6}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 &+ \frac{-69358460559591884974325760ac^6 + 7240554906654816150290432a^2c^6}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{-31598749169248488235008a^3c^6 - 20600331258861802319872a^4c^6 + 104099626599155066880a^5c^6}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{24513972913152859136a^6c^6 + 157337505818228736a^7c^6 - 9634212270498816a^8c^6 - 143804235294720a^9c^6}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{240073946112a^{10}c^6 + 13671770112a^{11}c^6 + 58426368a^{12}c^6 + 24762397191146209860911104c^7}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-7554657168203919614410752ac^7 + 563651294108595103137792a^2c^7 + 8329477068541816332288a^3c^7}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1369410024740239978496a^4c^7 - 13336890763832064000a^5c^7 + 1078388410232646656a^6c^7}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{17163647414065152a^7c^7 - 172613199762432a^8c^7 - 4544749946880a^9c^7 - 15956548608a^{10}c^7 + 102795264a^{11}c^7}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{439296a^{12}c^7 + 2627600160959125518286848c^8 - 629481821956418825551872ac^8}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{31619462174581386952704a^2c^8 + 1080850818520490434560a^3c^8 - 60243054776930488320a^4c^8}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1334029052616099840a^5c^8 + 26591317212235776a^6c^8 + 710248457318400a^7c^8 + 553881968640a^8c^8}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-68358850560a^9c^8 - 350558208a^{10}c^8 + 218367061287840742637568c^9 - 40504667291307307106304ac^9}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1230006926493424009216a^2c^9 + 72195929860788264960a^3c^9 - 1653182074768773120a^4c^9}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-61522767623116800a^5c^9 + 221647063013376a^6c^9 + 15698550067200a^7c^9 + 77245808640a^8c^9}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-399759360a^9c^9 - 2050048a^{10}c^9 + 14327507754584352227328c^{10} - 2018964798513932206080ac^{10}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{28942661352968929280a^2c^{10} + 3141338617082265600a^3c^{10} - 20977370837463040a^4c^{10}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1690559946915840a^5c^{10} - 6155023114240a^6c^{10} + 182290268160a^7c^{10} + 1168527360a^8c^{10}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{744577124096601489408c^{11} - 77717267194461093888ac^{11} + 124089871107604480a^2c^{11}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{9356554082430976a^3c^{11} + 251572455493632a^4c^{11} - 28240164126720a^5c^{11} - 201683722240a^6c^{11}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{872202240a^7c^{11} + 5591040a^8c^{11} + 30604492219101478912c^{12} - 2287151352269242368ac^{12}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-18949907972882432a^2c^{12} + 1901210161643520a^3c^{12} + 15757131120640a^4c^{12} - 265149480960a^5c^{12}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-2266234880a^6c^{12} + 988508362104635392c^{13} - 50477418289299456ac^{13} - 791011151314944a^2c^{13}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{25265059921920a^3c^{13} + 289022935040a^4c^{13} - 1073479680a^5c^{13} - 9175040a^6c^{13} + 24772834630828032c^{14}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-808187272888320ac^{14} - 16859143864320a^2c^{14} + 198133678080a^3c^{14} + 2540175360a^4c^{14}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{471525639585792c^{15} - 8859008630784ac^{15} - 213597552640a^2c^{15} + 695205888a^3c^{15} + 8912896a^4c^{15}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{6583118462976c^{16} - 59440103424ac^{16} - 1524105216a^2c^{16} + 63534268416c^{17} - 184025088ac^{17} - 4718592a^2c^{17}}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{378535936c^{18} + 1048576c^{19} - 33437550590250813751320576000a + 34797369598383800208858624000a^2}{\Gamma\left(\frac{c+a+39}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-11943047499272678944749934080a^3 + 1511154442079582941867032576a^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-12174593640573656531584800a^5 - 9176746768296957048511440a^6 + 164066382326104606030800a^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{29237214468361141838296a^8 - 1308392886998937250a^9 - 47060119476348419805a^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-890778487466837970a^{11} + 20479102880378181a^{12} + 890698199979900a^{13} + 7697494278430a^{14}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-107257459140a^{15} - 2631459054a^{16} - 17321850a^{17} - 1185a^{18} + 390a^{19} + a^{20}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{33437550590372458851729408000c - 94111231731558487988561510400ac}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{61301126494584828290604072960a^2c - 14787830196254701109822238720a^3c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1182878502102537827326525440a^4c + 39327517749489439142050560a^5c - 706995776345437370523200a^6c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-102330794805287436984960a^7c + 16921415244528199566720a^8c + 408835746049165372080a^9c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-14175209101293205680a^{10}c - 615871007439779520a^{11}c - 2489150032432960a^{12}c + 211423101795360a^{13}c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{3847123991520a^{14}c + 13121035200a^{15}c - 252715200a^{16}c - 2667600a^{17}c - 7600a^{18}c}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{59313862133606252926520524800c^2 - 95304589804285655783883079680ac^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{44131706337418429390654758912a^2c^2 - 7633748547751654333052835840a^3c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{332116884548647005620666880a^4c^2 + 31576352386104964399178880a^5c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1878549356278911294518144a^6c^2 - 89013912443003481635520a^7c^2 + 2873244951678734536800a^8c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{163111231577144153160a^9c^2 - 69941398739746248a^{10}c^2 - 108436905157220640a^{11}c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1714211916603680a^{12}c^2 + 6546205214640a^{13}c^2 + 394753552080a^{14}c^2 + 3318775200a^{15}c^2 + 2880000a^{16}c^2}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-70200a^{17}c^2 - 200a^{18}c^2 + 45946510809642116168370094080c^3 - 51818459319187364433974722560ac^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1785646272275732102276587520a^2c^3 - 219497720260633228360007680a^3c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{27471291708753221133432320a^4c^3 + 9843489211470482382796800a^5c^3 - 178311752851840686624000a^6c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-22625742391054056334080a^7c^3 + 28804703787004310880a^8c^3 + 24292334435938483200a^9c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{355907260103824000a^{10}c^3 - 6271394757788160a^{11}c^3 - 199766525999040a^{12}c^3 - 1213807795200a^{13}c^3}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{10812489600a^{14}c^3 + 156499200a^{15}c^3 + 501600a^{16}c^3 + 20963428736554169287015661568c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1777376014663332839327416320ac^4 + 4634724344937205047369062400a^2c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-390400049734941434318607360a^3c^4 - 6578084170823754089694080a^4c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1660532273818658294745600a^5c^4 + 11481647807096817197120a^6c^4 - 2816893575259128805440a^7c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-46458629469429394360a^8c^4 + 1593756030387206400a^9c^4 + 47248752656316640a^{10}c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{92172690529920a^{11}c^4 - 8917229040720a^{12}c^4 - 99228729600a^{13}c^4 - 162703200a^{14}c^4 + 2059200a^{15}c^4}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{6600a^{16}c^4 + 6380342063518675686653952000c^5 - 4188685675696797474572206080ac^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{827782886570907363625533440a^2c^5 - 44085755959174456769249280a^3c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2237693604973457273610240a^4c^5 + 165253988875476567306240a^5c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{4676072993519418050560a^6c^5 - 182328656997972925440a^7c^5 - 6334262247190225920a^8c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{23553349575275520a^9c^5 + 2670188724403200a^{10}c^5 + 25258595281920a^{11}c^5 - 114223549440a^{12}c^5}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2629186560a^{13}c^5 - 9630720a^{14}c^5 + 1386670877997514904379064320c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-713983460298843507161825280ac^6 + 105922922072121237600075776a^2c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-2936511729352267724636160a^3c^6 - 323053928140906974269440a^4c^6 + 9273587297246343751680a^5c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{539976129692297346048a^6c^6 - 4298257999114759680a^7c^6 - 403203703947799040a^8c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3025717132838400a^9c^6 + 68757012420096a^{10}c^6 + 1087445399040a^{11}c^6 + 2698375680a^{12}c^6 - 23063040a^{13}c^6}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-84480a^{14}c^6 + 224458870801408564784005120c^7 - 90968651822056491641733120ac^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{9926145324486111885066240a^2c^7 - 59116324529521142906880a^3c^7 - 28628771368706280611840a^4c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{166264644589523251200a^5c^7 + 34664553121025484800a^6c^7 + 215323406661242880a^7c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-13770381885511680a^8c^7 - 204523170508800a^9c^7 + 347636889600a^{10}c^7 + 19531100160a^{11}c^7}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{83466240a^{12}c^7 + 27817723618032457097412608c^8 - 8850552794666080568279040ac^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{686140391180927504506880a^2c^8 + 9346838804264846776320a^3c^8 - 1686917286669563520000a^4c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-15753704772940992000a^5c^8 + 1344622011781185280a^6c^8 + 21219248915504640a^7c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-216924110557440a^8c^8 - 5674941043200a^9c^8 - 19914935040a^{10}c^8 + 128494080a^{11}c^8 + 549120a^{12}c^8}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{2688497867134097109811200c^9 - 666357895248726259138560ac^9 + 34645577106726909706240a^2c^9}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1149076711650001305600a^3c^9 - 66583072864085606400a^4c^9 - 1454146558824960000a^5c^9}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{29647910065213440a^6c^9 + 786126781440000a^7c^9 + 595948953600a^8c^9 - 75954278400a^9c^9 - 389509120a^{10}c^9}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{205042842250378047651840c^{10} - 39096627942546185256960ac^{10} + 1227469588627677970432a^2c^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{70488986341191475200a^3c^{10} - 1657697864984944640a^4c^{10} - 61005389465180160a^5c^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{225341536516096a^6c^{10} + 15682559692800a^7c^{10} + 77143306240a^8c^{10} - 399759360a^9c^{10} - 2050048a^{10}c^{10}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{12423366783259335720960c^{11} - 1790022088470431662080ac^{11} + 26683365342961500160a^2c^{11}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{2817822316701450240a^3c^{11} - 19384401019576320a^4c^{11} - 1531569689395200a^5c^{11} - 5550150860800a^6c^{11}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{165718425600a^7c^{11} + 1062297600a^8c^{11} + 599312590380485050368c^{12} - 63675411787851202560ac^{12}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{120433488005242880a^2c^{12} + 77424254475509760a^3c^{12} + 203386280581120a^4c^{12} - 23510211379200a^5c^{12}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-167870976000a^6c^{12} + 726835200a^7c^{12} + 4659200a^8c^{12} + 22968177274755481600c^{13}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1740528325220106240ac^{13} - 14184468012072960a^2c^{13} + 1457846235955200a^3c^{13} + 12061599334400a^4c^{13}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-203961139200a^5c^{13} - 1743257600a^6c^{13} + 694339634368348160c^{14} - 3583396955537920ac^{14}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-559671769825280a^2c^{14} + 18029091225600a^3c^{14} + 206222131200a^4c^{14} - 766771200a^5c^{14} - 6553600a^6c^{14}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{16340033627750400c^{15} - 537206445834240ac^{15} - 11198786437120a^2c^{15} + 132089118720a^3c^{15}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1693450240a^4c^{15} + 292904572551168c^{16} - 5531666350080ac^{16} - 133364776960a^2c^{16} + 434503680a^3c^{16}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{5570560a^4c^{16} + 3861066547200c^{17} - 34964766720ac^{17} - 896532480a^2c^{17} + 35263610880c^{18} - 102236160ac^{18}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2621440a^2c^{18} + 199229440c^{19} + 524288c^{20}}{\Gamma\left(\frac{c+a+40}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \quad] \tag{6}
 \end{aligned}$$

III DERIVATIONS OF FORMULA (6):

Putting $b = -a-39$, $z = \frac{1}{2}$ in known formula (3), we get

$$(2a+39) {}_2F_1\left(a, -a-39; c; \frac{1}{2}\right) = a {}_2F_1\left(a+1, -a-39; c; \frac{1}{2}\right) + (a+39) {}_2F_1\left(a, -a-38; c; \frac{1}{2}\right)$$

Now using the same parallel method of Ref[5], the main Formula is developed.

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