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MINUS LEAP AND SQUARE LEAP INDICES AND THEIR POLYNOMIALS OF SOME SPECIAL GRAPHS

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ABSTRACT

We propose the minus leap index and square leap index of a graph. In this paper, we compute the minus leap and square leap indices and their polynomials of wheel, gear, helm, flower and sunflower graphs.

Keywords: *minus leap index, square leap index, wheel.*

Mathematics Subject Classification: *05C07, 05C12, 05C76.*

1. INTRODUCTION

We consider only finite, simple connected graphs. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. The distance between two vertices u and v of a graph G is the number of edges in a shortest path connecting u and v; and it is denoted by $d(u, v)$. For a positive integer k and a vertex v in G, the open neighborhood of v is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k-distance degree $d_k(v)$ of a vertex v in G is the number of k neighbors of v in G, see [1].

In [1], the first and second leap Zagreb indices were introduced and defined as

$$LM_1(G) = \sum_{u \in V(G)} d_2^2(u) \qquad\qquad LM_2(G) = \sum_{uv \in E(G)} d_2(u) d_2(v).$$

Recently, some novel variants of leap indices were introduced and studied such as leap hyper-Zagreb indices, [2], sum connectivity leap index and geometric-arithmetic leap index [3], F-leap indices [4], augmented leap index [5].

In [6], Albertson proposed the irrregularity index (called as minus index in [7]), and defined as

$$M_i(G) = \sum_{uv \in E(G)} |d(u) - d(v)|.$$

Recently, the square *ve*-degree index [8] was introduced and defined as

$$Q_{ve}(G) = \sum_{uv \in E(G)} \left[d_{ve}(u) - d_{ve}(v)\right]^2.$$

Very recently, some square indices were proposed and studied such as square reverse index [9] and square Revan index [10].

We now propose the minus leap index and square leap index of a graph G as follows:

The minus leap index of a graph G is defined as

$$ML(G) = \sum_{uv \in E(G)} |d_2(u) - d_2(v)|. \qquad (1)$$

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The square leap index of G is defined as

$$QL(G) = \sum_{uv \in E(G)} \left[d_2(u) - d_2(v)\right]^2. \tag{2}$$

Considering the minus leap and square leap indices, we define the minus leap polynomial and square leap polynomial of G as

$$ML(G,x) = \sum_{uv \in E(G)} x^{\left|d_G(u) - d_G(v)\right|} \tag{3}$$

$$QL(G,x) = \sum_{uv \in E(G)} x^{\left[d_G(u) - d_G(v)\right]^2}. \tag{4}$$

In this paper, exact expressions for the minus leap and square leap indices and their polynomials of some special graphs. For special graphs see [11].

2. WHEELS

The wheel W_n is the join of C_n and K_1. Clearly, $|V(W_n)| = n+1$ and $E(W_n) = 2n$. The vertex K_1 is called apex and the vertices of C_n are called rim vertices. A graph W_n is shown in Figure 1. Throughout this paper, we consider a wheel W_n with n+1 vertices.

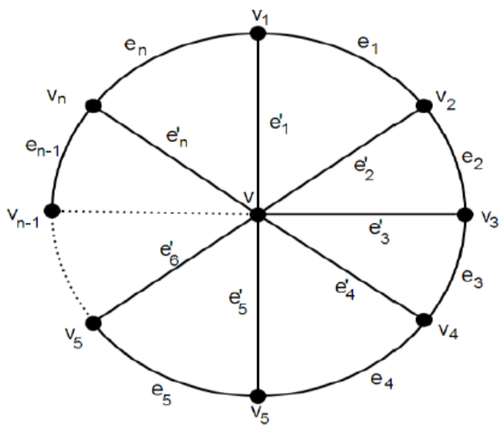

Figure-1: Wheel W_n

Lemma 1: Let W_n be a wheel with $2n$ edges, $n \geq 3$. Then W_n has two types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(W_n) \mid d_2(u) = 0,\ d_2(v) = n-3\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(W_n) \mid d_2(u) = d_2(v) = n-3\}$, $\quad |E_2| = n$.

Theorem 2: Let W_n be a wheel with n+1 vertices and $2n$ edges, $n \geq 3$. Then

(a) $ML(W_n) = n(n-3)$ (b) $QL(W_n) = n(n-3)^2$.

Proof:
(a) From equation (1) and by Lemma 1, we obtain

$$ML(W_n) = \sum_{uv \in E(W_n)} \left|d_2(u) - d_2(v)\right|$$

$$= n\left|0-(n-3)\right| + n\left|(n-3)-(n-3)\right| = n(n-3).$$

(b) From equation (2) and by Lemma 1, we obtain

$$QL(W_n) = \sum_{uv \in E(W_n)} \left[d_2(u) - d_2(v)\right]^2$$

$$= n\left[0-(n-3)\right]^2 + n\left[(n-3)-(n-3)\right]^2 = n(n-3)^2.$$

Theorem 3: Let W_n be a wheel with $2n$+1 vertices and $2n$ edges. Then

a) $ML(W_n, x) = nx^{n-3} + nx^0$.

b) $QL(W_n, x) = nx^{(n-3)^2} + nx^0$.

Proof:
(a) From equation (3) and by Lemma 1, we have

$$ML(W_n, x) = \sum_{uv\in E(W_n)} x^{|d_2(u)-d_2(v)|}$$

$$= nx^{|0-(n-3)|} + nx^{|(n-3)-(n-3)|} = nx^{(n-3)} + nx^{0}.$$

(b) From equation (4) and by Lemma 1, we have

$$QL(W_n, x) = \sum_{uv\in E(G)} x^{[d_2(u)-d_2(v)]^2}$$

$$= nx^{[0-(n-3)]^2} + nx^{[(n-3)-(n-3)]^2} = nx^{(n-3)^2} + nx^{0}.$$

3. GEAR GRAPHS

The gear graph G_n is a graph obtained from wheel W_n by adding a vertex between each pair of adjacent rim vertices. Clearly, $|V(G_n)| = 2n+1$ and $|E(G_n)| = 3n$. A gear graph G_n is presented in Figure 2.

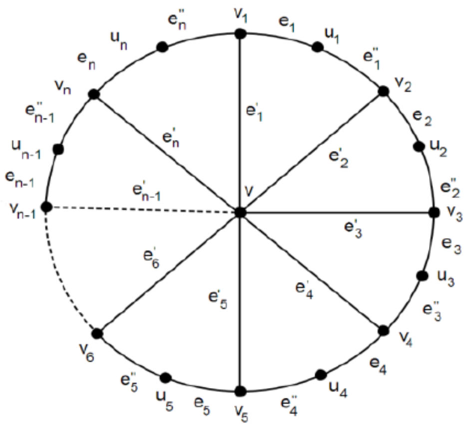

Figure-2: Gear graph G_n

Lemma 4: Let G_n be a gear graph with $3n$ edges, $n\geq 3$. Then G_n has two types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(G_n) \mid d_2(u) = n, d_2(v) = n - 1\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(G_n) \mid d_2(u) = 3, d_2(v) = n - 1\}$, $\quad |E_2| = 2n$.

Theorem 5: Let G_n be a gear graph G_n with $2n$ edges, $n\geq 4$. Then
(a) $ML(G_n) = 2n^2 - 7n.$ (b) $QL(G_n) = 2n^3 - 16n^2 + 33n.$

Proof:
(a) By using Lemma 4 and equation (1), we have

$$ML(G_n) = \sum_{uv\in E(G_n)} |d_2(u) - d_2(v)|$$

$$= n|n-(n-1)| + 2n|3-(n-1)| = 2n^2 - 7n.$$

(b) By using Lemma 4 and equation (2), we deduce

$$QL(G_n) = \sum_{uv\in E(G_n)} [d_2(u) - d_2(v)]^2$$

$$= n[n-(n-1)]^2 + 2n[3-(n-1)]^2 = 2n^2 - 16n^2 + 33n.$$

Theorem 6: Let G_n be a gear graph with $3n$ edges. $n\geq 4$ Then
a) $ML(G_n, x) = nx + 2nx^{n-4}.$
b) $QL(G_n, x) = nx + 2nx^{(n-4)^2}.$

Proof:

(a) From equation (3) and by Lemma 4, we obtain

$$ML(G_n, x) = \sum_{uv \in E(G_n)} x^{|d_2(u) - d_2(v)|}$$

$$= nx^{|n-n+1|} + 2nx^{|3-n+1|} = nx^1 + 2nx^{n-4}, \text{ since } n \geq 4.$$

(b) From equation (4) and by Lemma 4, we have

$$QL(G_n, x) = \sum_{uv \in E(G_n)} x^{[d_2(u) - d_2(v)]^2}$$

$$= nx^{[n-n+1]^2} + 2nx^{(3-n+1)^2} = nx + 2nx^{(n-4)^2}, \text{ since } n \geq 4.$$

4. HELM GRAPHS

The helm graph, denoted by H_n, is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A helm graph H_n is shown in Figure 3.

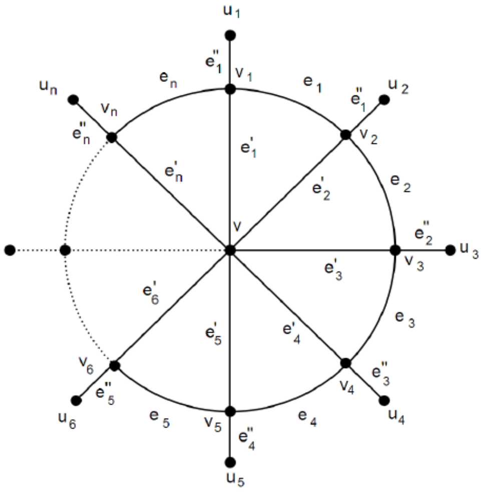

Figure-3: Helm graph H_n

Lemma 7: Let H_n be a helm graph with $3n$ edges, $n \geq 3$. Then H_n has 3 types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(H_n) \mid d_2(u) = n, d_2(v) = n - 1\}$, $|E_1| = n$.

$E_2 = \{uv \in E(H_n) \mid d_2(u) = 3, d_2(v) = n - 1\}$, $|E_2| = n$.

$E_3 = \{uv \in E(H_n) \mid d_2(u) = d_2(v) = n - 1\}$, $|E_3| = n$.

Theorem 8: Let H_n be a helm graph with $3n$ edges, $n \geq 4$. Then

(a) $ML(H_n) = n^2 - 3n.$ (b) $QL(H_n) = n^3 - 8n^2 + 17n.$

Proof:

(a) By using Lemma 7 and equation (1), we deduce

$$ML(H_n) = \sum_{uv \in E(H_n)} |d_2(u) - d_2(v)|$$

$$= n|n-(n-1)| + n|3-(n-1)| + n|(n-1)-(n-1)| = n^2 - 3n.$$

(b) By using equation (2) and Lemma 7, we derive

$$QL(H_n) = \sum_{uv \in E(H_n)} [d_2(u) - d_2(v)]^2$$

$$= n[n-(n-1)]^2 + n[3-(n-1)]^2 + n[(n-1)-(n-1)]^2 = n^3 - 8n^2 + 17n.$$

Theorem 9: Let H_n be a helm graph with $3n$ edges. $n \geq 4$. Then

a) $ML(H_n, x) = nx^1 + nx^{n-4} + nx^0.$

b) $QL(H_n, x) = nx^1 + nx^{(n-4)^2} + nx^0.$

Proof:
(a) From equation (3) and by Lemma 7, we have

$$ML(H_n,x)=\sum_{uv\in E(H_n)} x^{|d_2(u)-d_2(v)|}$$
$$=nx^{|n-(n-1)|}+nx^{|3-(n-1)|}+nx^{|(n-1)-(n-1)|}=nx^1+nx^{n-4}+nx^0.$$

(b) From equation (4) and by Lemma 7, we obtain

$$QL(H_n,x)=\sum_{uv\in E(H_n)} x^{[d_2(u)-d_2(v)]^2}$$
$$=nx^{[n-(n-1)]^2}+nx^{[3-(n-1)]^2}+nx^{[(n-1)-(n-1)]^2}=nx^1+nx^{(n-4)^2}+nx^0.$$

5. FLOWER GRAPHS

The flower graph Fl_n is a graph obtained from a helm graph H_n by joining each endvertex to the apex of H_n. Clearly, $|V(Fl_n)| = 2n+1$ and $|E(Fl_n)| = 4n$. A graph Fl_n is presented in Figure 4.

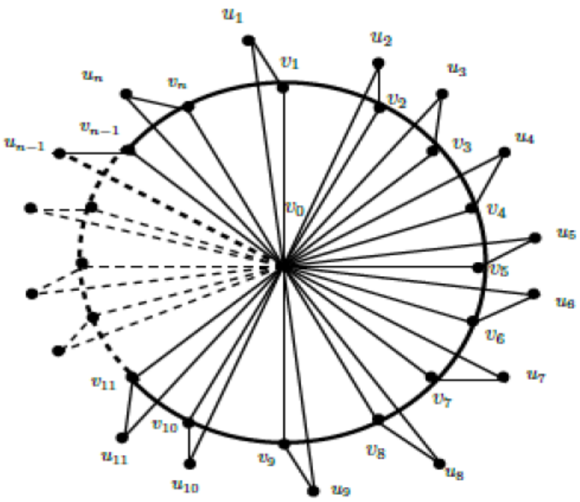

Figure-4: Flower graph Fl_n

Lemma 10: Let Fl_n be a flower graph with $4n$ edges, $n\geq 3$. Then Fl_n has 4 types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(Fl_n) \mid d_2(u) = 0, d_2(v) = n - 5\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(Fl_n) \mid d_2(u) = 0, d_2(v) = n - 2\}$, $\quad |E_2| = n$.
$E_3 = \{uv \in E(Fl_n) \mid d_2(u) = n - 5, d_2(v) = n - 2\}$, $\quad |E_3| = n$.
$E_4 = \{uv \in E(Fl_n) \mid d_2(u) = d_2(v) = n - 5\}$, $\quad |E_4| = n$.

Theorem 11: Let Fl_n be a flower graph with $4n$ edges, $n \geq 3$. Then
(a) $ML(Fl_n)=2n^2-4n.$ (b) $QL(Fl_n)=2n^3-14n^2+38n.$

Proof:
(a) By equation (1) and by Lemma 10, we have

$$ML(Fl_n)=\sum_{uv\in E(Fl_n)}|d_2(u)-d_2(v)|$$
$$=n|0-(n-5)|+n|0-(n-2)|+n|(n-5)-(n-2)|+n|(n-5)-(n-5)|=2n^2-4n.$$

(b) By using equation (2) and Lemma 10, we obtain

$$QL(Fl_n)=\sum_{uv\in E(Fl_n)}[d_2(u)-d_2(v)]^2$$
$$=n[0-(n-5)]^2+n[0-(n-2)]^2+n[(n-5)-(n-2)]^2+n[(n-5)-(n-5)]^2$$
$$=2n^3-14n^2+38n.$$

Theorem 12: Let Fl_n be a flower graph with $4n$ edges. $n\geq 3$. Then

(a) $ML(Fl_n, x) = nx^{n-5} + nx^{n-2} + nx^3 + nx^0.$

(b) $QL(Fl_n, x) = nx^{(n-5)^2} + nx^{(n-2)^2} + nx^9 + nx^0.$

Proof:

(a) From equation (3) and by Lemma 10, we deduce

$$ML(Fl_n, x) = \sum_{uv\in E(Fl_n)} x^{|d_2(u)-d_2(v)|}$$
$$= nx^{|0-(n-5)|} + nx^{|0-(n-2)|} + nx^{|(n-5)-(n-2)|} + nx^{|(n-5)-(n-5)|}$$
$$= nx^{n-5} + nx^{n-2} + nx^3 + nx^0.$$

(b) From equation (4) and by Lemma 10, we obtain

$$QL(Fl_n, x) = \sum_{uv\in E(Fl_n)} x^{[d_2(u)-d_2(v)]^2}$$
$$= nx^{[0-(n-5)]^2} + nx^{[0-(n-2)]^2} + nx^{[(n-5)-(n-2)]^2} + nx^{[(n-5)-(n-5)]^2} = nx^{(n-5)^2} + nx^{(n-2)^2} + nx^9 + nx^0.$$

6. SUNFLOWER GRAPHS

The sunflower graph Sf_n is a graph obtained from the flower graph Fl_n by attaching n end edges to the apex vertex of Fl_n. Then we have $|V(Sf_n)| = 3n+1$ and $|E(Sf_n)| = 5n$. A graph Sf_n is presented in Figure 5.

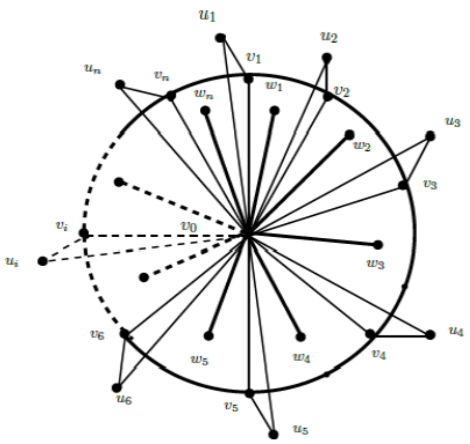

Figure-5: Sunflower graph Sf_n

Lemma 13: Let Sf_n be a sunflower graph with $5n$ edges, $n\geq 3$. Then Sf_n has five types of the 2-distance degree of edges as given below:

$E_1 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 4\}$, $|E_1| = n$.
$E_2 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 2\}$, $|E_2| = n$.
$E_3 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 1\}$, $|E_3| = n$.
$E_4 = \{uv \in E(Sf_n) \mid d_2(u) = d_2(v) = 3n - 4\}$, $|E_4| = n$.
$E_5 = \{uv \in E(Sf_n) \mid d_2(u) = 3n - 4, d_2(v) = 3n - 2\}$, $|E_5| = n$.

Theorem 14: Let Sf_n be a sunflower graph with $5n$ edges. Then

(a) $ML(Sf_n) = 9n^2 - 5n.$ (b) $QL(Sf_n) = 27n^3 - 42n^2 + 25n.$

Proof:

(a) By equation (1) and by Lemma 13, we deduce

$$ML(Sf_n) = \sum_{uv\in E(Sf_n)} |d_2(u) - d_2(v)|$$
$$= n|0-(3n-4)| + n|0-(3n-2)| + n|0-(3n-1)| + n|(3n-4)-(3n-4)|$$
$$+n|(3n-4)-(3n-2)|$$
$$= 9n^2 - 5n.$$

(b) By using equation (2) and Lemma 13, we derive

$$QL(Sf_n) = \sum_{uv \in E(Sf_n)} \left[d_2(u) - d_2(v)\right]^2$$

$$= n[0-(3n-4)]^2 + n[0-(3n-2)]^2 + n[0-(3n-1)]^2 + n[(3n-4)-(3n-4)]^2$$
$$+ n[(3n-4)-(3n-2)]^2$$
$$= 27n^3 - 42n^2 + 25n.$$

Theorem 15: Let Sf_n be a sunflower graph with $5n$ edges. Then

a) $ML(Sf_n, x) = nx^{3n-4} + nx^{3n-2} + nx^{3n-1} + nx^0 + nx^2.$

b) $QL(Sf_n, x) = nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2} + nx^0 + nx^4.$

Proof:

(a) From equation (3) and by Lemma 13, we have

$$ML(Sf_n, x) = \sum_{uv \in E(Sf_n)} x^{|d_2(u) - d_2(v)|}$$

$$= nx^{|0-(3n-4)|} + nx^{|0-(3n-2)|} + nx^{|0-(3n-1)|} + nx^{|(3n-4)-(3n-4)|} + nx^{|(3n-4)-(3n-2)|}$$
$$= nx^{3n-4} + nx^{3n-2} + nx^{3n-1} + nx^0 + nx^2.$$

(b) From equation (4) and by Lemma 13, we obtain

$$QL(Sf_n, x) = \sum_{uv \in E(Sf_n)} x^{\left[d_2(u) - d_2(v)\right]^2}$$

$$= nx^{[0-(3n-4)]^2} + nx^{[0-(3n-2)]^2} + nx^{[(3n-4)-(3n-4)]^2} + nx^{[(3n-4)-(3n-2)]^2}$$
$$= nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2} + nx^0 + nx^4.$$

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