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SOME THEOREMS ON CHARACTERISTICS OF UP ALGEBRA

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ABSTRACT

In this paper, anti fuzzy UP ideals and anti fuzzy UP sub algebras on UP-Algebra are studied. The notions of cartesian product and dot product of fuzzy sets are used to derive some properties of UP-Algebra.

Key words: *UP-Algebra, anti fuzzy UP ideals, anti fuzzy UP sub algebras.*

1. INTRODUCTION

UP Algebra is introduced by IAMPAN.A [4] where UP denotes University of Phayao.Also he derived the concept of UP-ideals, UP-subalgebras, congruences and UP homomorphisms in UP-algebras, and investigated some related properties of them. He also described connections between UPideals, UP-subalgebras, congruences and UP-homomorphisms. In this paper, anti fuzzy UP ideals and anti fuzzy UP sub algebras are studied and proved some theorems.

2. PRELIMINARIES

Definition 2.1: An algebra $A = (A,\cdot,0)$ of type (2,0) is called a UP-algebra if it satisfies the following axioms : for any $x, y, z \in A$,
(UP-1) $(y \cdot z)\cdot((x \cdot y)\cdot(x \cdot z)) = 0$,
(UP-2) $0\cdot x = x$,
(UP-3) $x \cdot 0 = 0$, and
(UP-4) $x \cdot y = y \cdot x = 0$ implies $x = y$.

Definition 2.2: A non empty subset B of A is called a UP-ideal of A if it satisfies the following properties:
(1) The constant 0 of A is in B, and
(2) For any $x, y, z \in A$, $x \cdot(y \cdot z) \in B$ and $y \in B$ imply $x \cdot z \in B$.
Clearly, A and 0 are up-ideal of A.

Definition 2.3: A subset S of A is called a UP-sub algebra of A if the constant 0 of A is in S, and $(S, \cdot, 0)$ itself forms a UP-algebra. Clearly, A and $\{0\}$ are UP-subalgebra of A.

Definition 2.4: A fuzzy set f in A is called an anti-fuzzy UP-ideal of A if it satisfies the following properties: for any $x, y, z \in A$,
(1) $f(0) \le f(x)$, and
(2) $f(x \cdot z) \le \max\{f(x \cdot(y \cdot z)), f(y)\}$.

Definition 2.5: A fuzzy set f in A is called an anti-fuzzy UP-sub algebra of A if for any $x, y \in A$, $f(x \cdot y) \le \max\{f(x), f(y)\}$.

Definition 2.6: If f is a fuzzy set in a non empty set X, the strongest fuzzy relation on X is $\mu_f : X\times X \to [0,1]$ defined by $\mu_{f(x,y)} = \max\{f(x),g(y)\}$, for all $x, y \in X$. For $x, y \in X$, we have $f(x), f(y) \in [0,1]$. Thus $\mu_{f(x,y)} = \max\{f(x), f(y)\} \in [0,1]$. Hence, μ_f is a fuzzy relation on X . We note that if f is a fuzzy set in a non empty set X, then $f \times f = \mu_f$.

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3. CHARACTERISTICS OF UP ALGEBRA

Theorem 3.1: If f is an anti-fuzzy UP-ideal of A if and only if μ_f is an anti-fuzzy UP-ideal of **A×A.**

Proof: Assume that f is an anti-fuzzy UP-ideal of A.

Let $x, y \in$ A×A.
Then,

$$\begin{aligned}(f\times f)(0,0) &= \max\{f(0), f(0)\}\\ &= \mu_f(0, 0)\\ &\le \mu_f(x, y)\\ &= \max\{f(x),f(y)\}\\ &= (f\times f)(x, y).\end{aligned}$$

Now, let $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in$ A×A.
Then,

$$\begin{aligned}(f\times f)((x_1, x_2)\blacklozenge(z_1, z_2)) &= (f\times f)(x_1 \cdot z_1, x_2 * z_2)\\ &= \max\{f(x_1 \cdot z_1), f(x_2 * z_2)\}\\ &\le \max\{\max\{f(x_1\cdot (y_1 \cdot z_1)), f(y_1)\}, \max\{f(x_2 * (y_2 * z_2)), f(y_2\}\}\\ &= \max\{\max\{f(x_1 \cdot(y_1 \cdot z_1)), f(x_2 *(y_2 * z_2))\},\max\{f(y_1), f(y_2\}\}\\ &= \max\{(f\times f)(x_1 \cdot(y_1 \cdot z_1), x_2 *(y_2 * z_2)), (f\times f)(y_1, y_2)\}\\ &= \max\{(f\times f)((x_1, x_2)\blacklozenge((y_1, y_2)\blacklozenge(z_1, z_2))),(f\times f)(y_1, y_2)\}.\end{aligned}$$

We have μ_f = f × f is an anti-fuzzy UP-ideal of A × A.

Hence, f × f is an anti-fuzzy UP-ideal of A × A.

Conversely,

Assume that μ_f is an anti-fuzzy UP-ideal of A × A.

Since f × f = μ_f,

Suppose that f is not an anti-fuzzy UP-ideal of A.

Assume that $f(0_A) \le f(x)$, for all $x \in$ A.
Then from (2) either $f(0_A) \le f(y)$, for all $y \in$ A or $f(0_A) \le f(x)$, for all $x \in$ A. If $f(0_A) \le f(x)$ for all $x \in$ A, then for all $x \in$ A, $(f \times f)(x, 0_A) = \max\{f(x), f(0_A)\} = f(x)$.

Since f × f is an anti-fuzzy UP-ideal of A × A, we have for any $x, y, z \in$ A,

$$\begin{aligned}f(x \cdot z) &= (f \times f)(x \cdot z, 0_A)\\ &= (f \times f)(x \cdot z, 0_A * 0_A)\\ &= (f \times f)((x, 0_A) \blacklozenge (z, 0_A))\\ &\le \max\{(f \times f)((x, 0_B) \blacklozenge [(x, 0_A) \blacklozenge (z, 0_A)]), (f \times f)(y, 0_A)\}\\ &= \max\{(f \times f)(x \cdot(y \cdot z), 0_A * (0_A * 0_A), (f \times f)(y, 0_A)\}\\ &= \max\{(f \times f)(x \cdot(y \cdot z), (f \times f)(y, 0_A\}\\ &= \max\{\max\{f(x \cdot(y \cdot z)), f(0_A)\}, \max\{f(y), f(0_A)\}\}\\ &= \max\{f(x \cdot(y \cdot z)),f(y)\}.\end{aligned}$$

Hence, f is an anti-fuzzy UP-ideal of A.

Which is a contradiction.

Assume that $f(0_A) \le f(y)$, for all $y \in$ A.

Then, either $f(0_A) \le f(x)$, for all $x \in$ A or $f(0_A) \le f(y)$, for all $y \in$ A.
Then for all $y \in$ A,

$$\begin{aligned}(f \times f)(0_A, y) &= \max\{f(0_A), f(y)\}\\ &= f(y).\end{aligned}$$

Since f × f is an anti-fuzzy UP-ideal of A × A. We have for any $x, y, z \in A$,

$$\begin{aligned} f(x * z) &= (f \times f)(0_A, x * z) \\ &= (f \times f)(0_A \cdot 0_A, x * z) \\ &= (f \times f)((0_A, x) \blacklozenge (0_A, z)) \\ &\leq \max\{f \times f)((0_A, x) \blacklozenge [(0_A, y) \blacklozenge (0_A, z)]), (f \times f)(0_A, y)\} \\ &= \max\{(f \times f)(0_A \cdot (0_A \cdot 0_A)), (x * (y * z)), (f \times f)(0_A, y)\} \\ &= \max\{(f \times f)(0_A, x * (y * z)), (f \times f)(0_A, y)\} \\ &= \max\{\max\{f(0_A), f(x * (y * z))\}, \max\{f(x * (y * z)), f(y)\} \end{aligned}$$

Hence, f is an anti-fuzzy UP-ideal of A.
Which is a contradiction.

Since f × f is not an anti-fuzzy UP-ideal of A × A $f(0_A) \leq f(x)$, for all $x \in A$ and $f(0_A \leq f(y)$, for all $y \in A$, there exist $x, y, z \in A$, $x', y', z' \in A$ such that

$$f(x \cdot z) > \max\{f(x \cdot (y \cdot z)), f(y)\} \text{ and } f(x' * z') > \max\{f(x' * (y' * z')), f(y)\}.$$
$$\max\{f(x \cdot z), f(x' * z')\} > \max\{\max\{f(x \cdot (y \cdot z)), f(y)\}, \max\{f(x' * z'), f(y')\}\}.$$

Since f × f is an anti-fuzzy UP-ideal of A × A, we have

$$\begin{aligned} \{f(x \cdot z), f(x' * z')\} &= (f \times f)(x \cdot z, x' * z') \\ &= f(x \cdot z)((x, x') \blacklozenge (z, z')) \\ &\leq \max\{(f \times f)((x, x') \blacklozenge [(y, y') \blacklozenge (z, z')]), (f \times f)(y, y')\} \\ &= \max\{(f \times f)(x \cdot (y \cdot z)), f(x' * (y' * z')), (f \times f)(y, y') \\ &= \max\{\max f(x \cdot (y \cdot z)), f(x' * (y' * z'))\}, \max\{f(y), f(y')\}\}. \end{aligned}$$
$$\max\{f(x \cdot z), f(x' * z')\} \not< \max\{\max\{f(x \cdot (y \cdot z)), f(y)\}, \max\{f(x' * (y' * z))\}\}$$

Which is a contradiction.

Similarly, by (1),

If $f(0_A) \leq f(y)$, for all $y \in A$, we have a contradiction.

Hence, either f is an anti-fuzzy UP-ideal of A.

Theorem 3.2: If f is an anti-fuzzy UP-sub algebra of A if and only if μ_f is an anti-fuzzy UP-sub algebra of A × A.

Proof: Assume that f is an anti-fuzzy UP-subalgebra of A.

Let $(x_1, x_2), (y_1, y_2) \in A \times A$.
Then,

$$\begin{aligned} (f \times f)((x_1, x_2) \blacklozenge (y_1, y_2)) &= (f \times f)(x_1 \cdot y_1, x_2 * y_2) \\ &= \max\{f(x_1 \cdot y_1), f(x_2 * y_2)\} \\ &\leq \max\{\max\{(f(x_1), f(y_1)\}, \max\{(f(x_2), f(y_2)\}\} \max\{\max\{(f(x_1), f(y_2)\}, \max\{(f(y_1), f(y_2)\}\} \\ &= \max\{(f \times f)(x_1, x_2), (f \times f)(y_1, y_2)\}. \end{aligned}$$

Hence, $\mu_f = f \times f$ is an anti-fuzzy UP-sub algebra of A × A.

Conversely,

Assume that μ_f is an anti-fuzzy UP-sub algebra of A × A.

Since $f \times f = \mu_f$, Suppose that f is not an anti-fuzzy UP-sub algebra of A.

Then there exist $x, y \in A$ and $a, b \in A$ such that $f(x \cdot y) > \max\{f(x), f(y)\}$ and $f(a * b) > \max\{f(a), f(b)\}$

Thus $\max\{f(x \cdot y), f(a * b)\} > \max\{\max\{f(x), f(y)\}, \max\{f(a), g(b)\}\}$.

Since, f×f is an anti- fuzzy UP-sub algebra of A×A, we have

$$\begin{aligned} \max\{f(x \cdot y), f(a * b)\} &= (f \times f)((x, a) \blacklozenge (y, b)) \\ &\leq \max\{(f \times f)((x, a), (f \times f)(y, b)\} \\ &= \max\{\max\{f(x), f(a)\}, \max\{f(y), f(b)\}\} \\ &= \max\{\max\{f(x), f(y)\}, \max\{f(a), f(b)\}\}. \end{aligned}$$

Thus, $\max\{f(x \cdot y), f(a * b)\} \not> \max\{\max\{f(x), f(y)\}\}, \max\{f(a), f(b)\}$
Which is a contradiction.

Hence, either f is an anti-fuzzy UP-sub algebra of A.

4. CONCLUSION

In this paper, anti fuzzy UP ideals and anti fuzzy UP sub algebras are studied. The characteristics of UP-Algebra and therelation between Anti fuzzy UP ideal and anti fuzzy UP sub algebra are explained and related theorems proved. In future more theorems can be derived in this topic.

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