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INVERSE SUM LEAP INDEX AND HARMONIC LEAP INDEX OF CERTAIN WINDMILL GRAPHS

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ABSTRACT

We propose two new topological indices the inverse sum leap index and harmonic leap index of a graph. Also we compute the inverse sum leap index and harmonic leap index of Kulli cycle windmill graphs, Kulli path windmill graphs, Dutch windmill graphs and French windmill graphs.

Keywords: *Inverse sum leap index, harmonic leap index, windmill graph.*

Mathematics Subject Classification: *05C05, 05C07, 05C12, 05C35.*

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d(v)$ of a vertex v is the number of edges incident to v. We refer [1] for other undefined notations and terminologies.

The distance $d(u,v)$ between any two vertices u and v is equal to the length of a shortest path connecting them. For a positive integer k, the open k-neighborhood $N_k(v)$ of a vertex v in G is defined as $N_k(v/G)= \{u\in V(G) : d(u, v) = k\}$. The k-distance degree $d_k(v)$ of v in G is defined as the number of k neighbors of v in a graph G.

In [2], Vukičević et al. observed that many topological indices are defined simply as the sum of individual bond contributions. They have proposed a class of discrete Adriatic indices to study whether there other possibly significant topological indices of this form. One of these discrete Adriatic indices is the inverse sum indeg index, which is defined as

$$ISI(G)=\sum_{uv\in E(G)}\frac{d(u)d(v)}{d(u)+d(v)}$$

This index was also studied in [3, 4, 5].

Motivated by the definition of inverse sum indeg index, we propose the inverse sum leap index and harmonic leap index of a graph as follows:

The inverse sum leap index of a graph G is defined as

$$ISL(G)=\sum_{uv\in E(G)}\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)} \qquad (1)$$

The harmonic leap index of a graph G is defined as

$$HL(G)=\sum_{uv\in E(G)}\frac{2}{d_2(u)+d_2(v)}.$$

The general harmonic leap index of a graph G is defined as

$$HL^a(G)=\sum_{uv\in E(G)}\left(\frac{2}{d_2(u)+d_2(v)}\right)^a. \qquad (2)$$

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Recently, some leap indices were introduced and studied such as minus leap and square leap indices [6], leap hyper Zagreb indices [7], sum connectivity leap and geometric-arithmetic leap index and *ABC* leap index [8], *F*-leap indices [9], product connectivity leap index and *ABC* leap index [10], augmented leap index [11]. The harmonic index was studied in [12, 13, 14].

In this paper, the inverse sum leap index, harmonic leap index and general harmonic leap index of four types of windmill graphs are computed. For windmill graphs, see [15, 16, 17, 18].

2. RESULTS FOR KULLI PATH WINDMILL GRAPHS

The Kulli path windmill graph P_{n+1}^m is the graph obtained by taking m copies of the graph $K_1 + P_n$ with a vertex K_1 in common, see Figure 1.

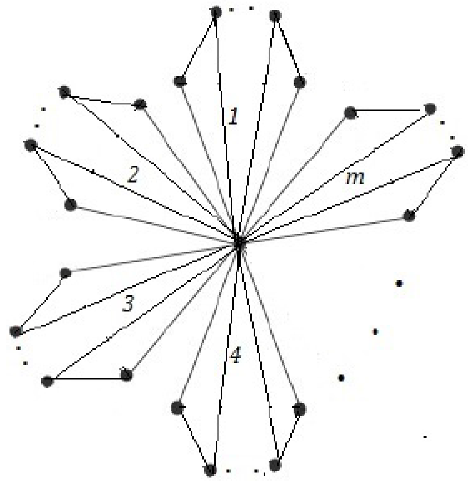

Figure-1: Kulli path windmill graph P_{n+1}^m

Let $G = P_{n+1}^m$ be a Kulli path windmill graph with $mn + 1$ vertices and $2mn - m$ edges, $m \geq 2$, $n \geq 5$. Then G has four types of the 2-distance degree of edges as given in Table 1.

$d_2(u), d_2(v) \setminus uv \in E(G)$	$(0, mn-2)$	$(0, mn-3)$	$(mn-2, mn-3)$	$(mn-3, mn-3)$
Number of edges	$2m$	$mn-2m$	$2m$	$mn-3m$

Table-1: 2-distance degree edge partition of P_{n+1}^m

Theorem 1: The inverse sum leap index of a Kulli path windmill graph P_{n+1}^m is

$$ISL\left(P_{n+1}^m\right) = m(mn-3)\left[\frac{2mn-4}{2mn-5} + \frac{n-3}{2}\right]$$

Proof: Let $G = P_{n+1}^m$. By using equation (1), we have

$$ISL\left(P_{n+1}^m\right) = \sum_{uv \in E(G)} \frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)}$$

Then by using Table 1, we obtain

$$ISL\left(P_{n+1}^m\right) = \left(\frac{0\times(mn-2)}{0+mn-2}\right)2m + \left(\frac{0\times(mn-3)}{0+mn-3}\right)(mn-2m)$$
$$+\left(\frac{(mn-2)\times(mn-3)}{mn-2+mn-3}\right)2m + \left(\frac{(mn-3)\times(mn-3)}{mn-3+mn-3}\right)(mn-3m)$$
$$= m(mn-3)\left[\frac{2mn-4}{2mn-5} + \frac{n-3}{2}\right].$$

Theorem 2: The general harmonic leap index of a Kulli path windmill graph P_{n+1}^m is

$$HL^a\left(P_{n+1}^m\right) = \left(\frac{2}{mn-2}\right)^a 2m + \left(\frac{2}{mn-3}\right)^a m(n-2) + \left(\frac{2}{2mn-5}\right)^a 2m + \left(\frac{1}{mn-3}\right)^a m(n-3) \tag{3}$$

Proof: Let $G = P_{n+1}^{m}$. From equation (2), we have

$$HL\left(P_{n+1}^{m}\right) = \sum_{uv\in(G)} \left(\frac{2}{d_2(u)+d_2(v)}\right)^{a}.$$

Then by using Table 1, we derive

$$HL^{a}\left(P_{n+1}^{m}\right) = \left(\frac{2}{mn-2}\right)^{a} 2m + \left(\frac{2}{mn-3}\right)^{a} m(n-2) + \left(\frac{2}{2mn-5}\right)^{a} 2m + \left(\frac{1}{mn-3}\right)^{a} m(n-3)$$

Corollary 2.1: The harmonic leap index of P_{n+1}^{m} is

$$HL\left(P_{n+1}^{m}\right) = \left(\frac{2}{mn-2}\right)2m + \left(\frac{2}{mn-3}\right)m(n-2) + \left(\frac{2}{2mn-5}\right)2m + \left(\frac{1}{mn-3}\right)m(n-3)$$

Proof: Put $a = 1$ in equation (3), we get the desired result.

3. RESULTS FOR KULLI CYCLE WINDMILL GRAPHS

The Kulli cycle windmill graph C_{n+1}^{m} is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geq 3$ with a vertex K_1 is common, see Figure 2.

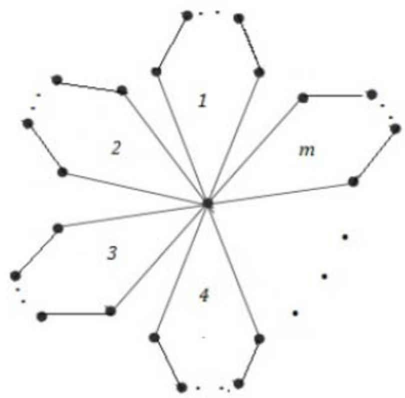

Figure-2: Kulli cycle windmill graph C_{n+1}^{m}

Let $G = C_{n+1}^{m}$ be a Kulli cycle windmill graph with $mn + 1$ vertices and $2mn$ edges, $m \geq 2$, $n \geq 5$. Then G has two types of the 2-distance degree of edges as given in Table 2.

$d_2(u), d_2(v) \setminus uv \in E(G)$	$(0, mn-2)$	$(mn-2, mn-2)$
Number of edges	mn	mn

Table-2: 2-distance degree edge partition of C_{n+1}^{m}

Theorem 3: The inverse sum leap index of a Kulli cycle windmill graph C_{n+1}^{m} is

$$ISL\left(C_{n+1}^{m}\right) = \frac{1}{2}mn(mn-2).$$

Proof: Let $G = C_{n+1}^{m}$. From equation (1), we have

$$ISL\left(C_{n+1}^{m}\right) = \sum_{uv\in E(G)} \frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)}$$

By using Table 2, we deduce

$$ISL\left(C_{n+1}^{m}\right) = \left(\frac{0\times(mn-2)}{0+mn-2}\right)mn + \left(\frac{(mn-2)\times(mn-2)}{mn-2+mn-2}\right)mn$$
$$= \frac{1}{2}mn(mn-2).$$

Theorem 4: The general harmonic leap index of a Kulli cycle windmill graph C_{n+1}^m is

$$HL^a\left(C_{n+1}^m\right)=\left(\frac{2}{mn-2}\right)^a mn+\left(\frac{1}{mn-2}\right)^a mn. \tag{4}$$

Proof: Let $G = C_{n+1}^m$. By using equation (2), we get

$$HL^a\left(C_{n+1}^m\right)=\sum_{uv\in E(G)}\left(\frac{2}{d_2(u)+d_2(v)}\right)^a$$

From Table 2, we deduce

$$HL\left(C_{n+1}^m\right)=\left(\frac{2}{0+mn-2}\right)^a mn+\left(\frac{2}{mn-2+mn-2}\right)^a mn.$$
$$=\left(\frac{2}{mn-2}\right)^a mn+\left(\frac{1}{mn-2}\right)^a mn.$$

Corollary 4.1: The harmonic leap index of C_{n+1} is $\frac{3mn}{mn-2}$.

Proof: Put a = 1 is equation (4); we get the desired result.

4. RESULTS FOR FRENCH WINDMILL GRAPHS

The French windmill graph F_{n+1}^m is the graph obtained by taking $m \geq 2$ copies of the graph K_n, $n \geq 2$ with a vertex in common, see Figure 3.

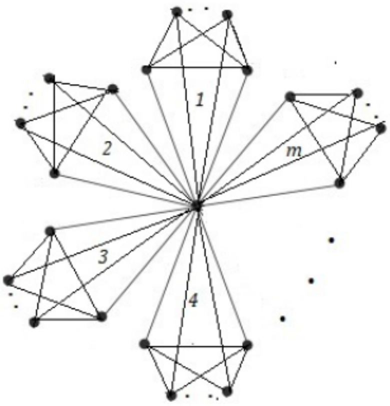

Figure-3: French windmill graph F_{n+1}^m

Let $G = F_{n+1}^m$ be a French windmill graph which has $1+ m(n-1)$ vertices and $\frac{1}{2}mn(n-1)$ edges, $m \geq 2$, $n \geq 2$. Then there are two types of the 2-distance degree of edges as given in Table 3.

$d_2(u), d_2(v) \setminus uv \in E(G)$	$(0, (n-1)(m-1))$	$((n-1)(m-1), (n-1)(m-1))$
Number of edges	$m(n-1)$	$\frac{1}{2}m(n-1)(n-2)$

Table 3. 2-distance degree edge partition of F_{n+1}^m

Theorem 5: The inverse sum leap index of a French windmill graph F_{n+1}^m is

$$ISL\left(F_{n+1}^m\right)=\frac{1}{4}m(m-1)(n-1)^2(n-2).$$

Proof: Let $G = F_{n+1}^m$. From equation (1), we obtain

$$ISL\left(F_{n+1}^m\right)=\sum_{uv\in E(G)}\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)}.$$

By using Table 3, we deduce

$$IS\ L\left(F_{n+1}^{m}\right)=\left(\frac{0\times(n-1)(m-1)}{0+(n-1)(m-1)}\right)m(m-1)+\left(\frac{(n-1)(m-1)\times(n-1)(m-1)}{(n-1)(m-1)+(n-1)(m-1)}\right)\frac{1}{2}m(n-1)(m-2)$$
$$=\frac{1}{4}m(m-1)(n-1)^{2}(n-2).$$

Theorem 6: The general harmonic leap index of a French windmill graph F_{n+1}^{m} is

$$HL^{a}\left(F_{n+1}^{m}\right)=\left(\frac{2}{(n-1)(m-1)}\right)^{a}m(n-1)+\left(\frac{1}{(n-1)(m-1)}\right)^{a}\frac{1}{2}m(n-1)(n-2). \qquad (5)$$

Proof: Let $G = F_{n+1}^{m}$. From equation (2), we have

$$HL^{a}\left(F_{n+1}^{m}\right)=\sum_{uv\in E(G)}\left(\frac{2}{d_2(u)+d_2(v)}\right)^{a}$$

By using Table 3, we derive

$$HL^{a}\left(F_{n+1}^{m}\right)=\left(\frac{2}{0+(n-1)(m-1)}\right)^{a}m(n-1)+\left(\frac{2}{(n-1)(m-1)+(n-1)(m-1)}\right)^{a}\frac{1}{2}m(n-1)(n-2)$$
$$=\left(\frac{2}{(n-1)(m-1)}\right)^{a}m(n-1)+\left(\frac{1}{(n-1)(m-1)}\right)^{a}\frac{1}{2}m(n-1)(n-2).$$

Corollary 6.1: The harmonic leap index of F_{n+1}^{m} is $\dfrac{m(n+2)}{2(m-1)}$.

Proof: Put a = 1 in equation (5), we get the desired result.

5. RESULTS FOR DUTCH WINDMILL GRAPHS

The Dutch windmill graph D_n^m $m \geq 2$, $n \geq 5$ is the graph obtained by taking m copies of the cycle C_n with a vertex in common, see Figure 4.

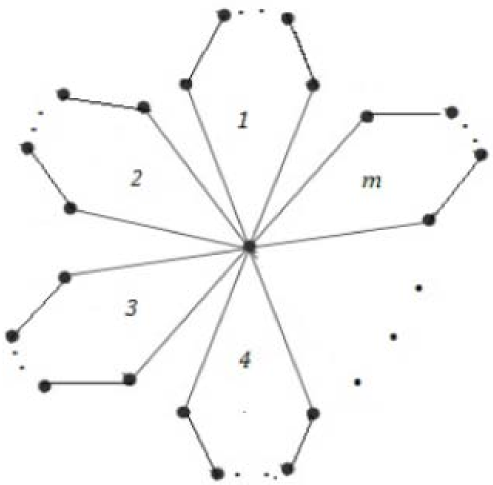

Figure-4: Dutch windmill graph D_n^m

Let $G = D_n^m$ be a Dutch windmill graph with $1 + mn - m$ vertices and mn edges, $m \geq 2$, $n \geq 5$. Then G has three types of 2-distance degree of edges as given in Table 4.

$d_2(u), d_2(v) \setminus uv \in E(G)$	$(2m, 2m)$	$(2m, 2)$	$(2, 2)$
Number of edges	$2m$	$2m$	$m(n-4)$

Table-4: 2-distance degree edge partition of D_n^m

Theorem 7: The inverse sum leap index of a Dutch windmill graph D_n^m is

$$ISL\left(D_n^m\right) = m^2\left(\frac{m+3}{m+1}\right) + m(n-4).$$

Proof: Let $G = D_n^m$. From equation (1), we have

$$ISL\left(D_n^m\right) = \sum_{uv\in E(G)} \frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)}.$$

By using Table 4, we deduce

$$ISL\left(D_n^m\right) = \left(\frac{2m\times 2m}{2m+2m}\right)2m + \left(\frac{2m\times 2}{2m+2}\right)2m + \left(\frac{2\times 2}{2+2}\right)m(n-4)$$

$$= m^2\left(\frac{m+3}{m+1}\right) + m(n-4).$$

Theorem 8: The general harmonic leap index of a Dutch windmill graph D_n^m is

$$HL^a\left(D_n^m\right) = \left(\frac{1}{2m}\right)^a 2m + \left(\frac{1}{m+1}\right)^a 2m + \left(\frac{1}{2}\right)^a 2m(n-4) \qquad (6)$$

Proof: Let $G = D_n^m$. By using equation (2), we get

$$HL^a\left(D_n^m\right) = \sum_{uv\in E(G)} \left(\frac{2}{d_2(u)+d_2(v)}\right)^a$$

By using Table 4, we derive

$$HL^a\left(D_n^m\right) = \left(\frac{2}{2m+2m}\right)^a 2m + \left(\frac{2}{2m+2}\right)^a 2m + \left(\frac{2}{2+2}\right)^a m(n-4)$$

$$= \left(\frac{1}{2m}\right)^a 2m + \left(\frac{1}{m+1}\right)^a 2m + \left(\frac{1}{2}\right) m(n-4).$$

Corollary 8.1: The harmonic leap index of D_n^m is

$$\frac{1}{2}mn - 2m + \frac{2m}{m+1} + 1.$$

Proof: Put a = 1 in equation (6), we obtain the desired result.

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