International Research Journal of Pure Algebra-9(3), 2019, 26-36
Available online through www.rjpa.info ISSN 2248–9037

UNION OF ANTI-FUZZY SUB – QUADRADIC GROUPS, ANTI-FUZZY SUB – PENTAGROUPS AND ANTI - FUZZY SUB – N GROUPS.

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(Received On: 06-02-19; Revised & Accepted On: 21-02-19)

ABSTRACT

In this paper, union of Anti-Fuzzy subsets, definition of Anti-Fuzzy sub-Quadratic group, definition of Anti-Fuzzy sub_ pendant group and definition of Anti-Fuzzy sub- "n" group are derived. Moreover, some properties and theorems based on these have been derived.

Keywords: *Anti-Fuzzy group, Anti-Fuzzy sub-bigroup, Anti-Fuzzy subgroup, Anti-Fuzzy sub-trigroup union of Anti-Fuzzy subsets, Anti- Fuzzy sub – Quadratic group, Anti- fuzzy sub- Pentagroup and Anti-Fuzzy sub–N groups.*

1 .INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld gave the idea of fuzzy subgroups and Ranjith Biswas gave the idea of anti fuzzy subgroups .Since the paper fuzzy set theory has been considerably developed by zadeh himself and some researchers. The original concept of fuzzy sets was introduced as an extension of crisps (usual) sets, by enlarging the truth value set of “grade of members” from the two value set {0, 1} to unit interval {0, 1} of real numbers. The study of fuzzy group was started by Rosenfeld. It was extended by Roventa who have introduced the fuzzy groups operating on fuzzy sets.

Rosenfield introduced the notion of fuzzy group and showed that many group theory results can be extended in an elementary manner to develop the theory of fuzzy group. The underlying logic of the theory of fuzzy group is to provide a strict fuzzy algebraic structure where level subset of a fuzzy group of a group G is a subgroup of the group.

The notion of bigroup was first introduced by P.L.Maggu in 1994.W.B.Vasanthakandasamy introduced fuzzy sub-bigroup with respect to +ʻʻ and .ʻʻand illustrate it with example. W.B.Vasanthakandasamy was the first one to introduce the notion of bigroups in the year 1994. Several mathematicians have followed them in investigating the fuzzy group theory.

2. PRELIMINARIES

In this section contain some definitions, examples and some results.

2.1 Concept of a Fuzzy set:
The concept of a fuzzy set is an extension of the concept of a crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0,1}, a fuzzy set on a domain U is defined by its membership function from U to [0,1] .

Let U be a non-empty set, to be called the **Universal set** (or) **Universe of discourse or simply a domain**. Then, by a fuzzy set on U is meant a function **A: U →[0, 1]. A** is called **the membership function; A (x)** is called **the membership grade** of x in A. We also write

$A= \{(x, A (x)): x \in U\}$.

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Examples: Consider U={a, b, c, d} and A:U→**1** defined by A(a) = 0, A(b) = 0.7, A(c) = 0.4, and A(d) = 1. Then A is a fuzzy set can also be written as follows:

A= {(a, 0), (b, 0.7), (c, 0.4),(d,1)}.

2.2. Relation between Fuzzy sets: Let U be a domain and A, B be fuzzy sets on U.

Inclusion (or) Containment: A is said to be included (or) contained in B if and only if $A(x) \leq B(x)$ for all x in U. In symbols, we write, $A \subseteq B$. We also say that A is a subset of B.

2.3 Definition: Let S be a set. A fuzzy subset A of S is a function $A: S \to [0, 1]$.

2.4 Definition of Union of Anti - Fuzzy sets: The union of two fuzzy subsets A_1, A_2 is defined by

$(A_1 \cup A_2)(x) = \min\{A_1(x), A_2(x)\}$ for every x in U.

2.5 Definition of Anti Fuzzy Subgroup:
Let G be a group. A fuzzy subset A of a group G is called an anti-fuzzy subgroup of the group G if

i) $A(xy) \leq \max\{A(x), A(y)\}$ for every $x, y \in G$ and

ii) $A(x^{-1}) = A(x)$ for every $x \in G$.

2.6 Definition
A set (G, +, ·) with two binary operations ' + ' and '. ' is called a **bigroup** if there exist two proper subsets G_1 and G_2 of G such that

i. $G = G_1 \cup G_2$

ii. $(G_1, +)$ is a group.

iii. $(G_2, .)$ is a group.

A non-empty subset H of a bigroup (G, +, .) is called a sub-bigroup, if H itself is a bigroup under '+' and '.' operations defined on G.

2.7 Definition of Anti- Fuzzy union of the fuzzy sets A_1 and A_2:

Let A_1 be a fuzzy subset of a set x_1 and A_2 be a fuzzy subset of a set x_2, then an anti-fuzzy union of the fuzzy sets A_1 and A_2 is defined as a function. $A_1 \cup A_2: x_1 \cup x_2 \to [0,1]$ given by

$$(A_1 \cup A_2)(x) = \begin{cases} \min(A_1(x), A_2(x)) & \text{if } x \in x_1 \cap x_2. \\ A_1(x) & \text{if } x \in x_1 \,\&\, x \notin x_2. \\ A_2(x) & \text{if } x \in x_2 \,\&\, x \notin x_1. \end{cases}$$

2.8 Definition of Anti Fuzzy sub- bigroup:
Let G = (G, +, ·) be a bigroup. Then $A : G \to [0, 1]$ is said to be an **anti-Fuzzy sub- bigroup of the bigroup G** if there exists two fuzzy subsets A_1 (of G_1) and A_2 (of G_2) such that

i) $(A_1, +)$ is an anti-fuzzy subgroup of $(G_1, +)$

ii) $(A_2, \cdot)$ is an anti-fuzzy subgroup of $(G_2, \cdot)$ and

iii) $A = A_1 \cup A_2$.

2.9 Definition of the Anti-fuzzy union of the fuzzy sets A_1 and A_2, A_3:
Let A_1 be a fuzzy subset of a set X_1 and A_2 be a fuzzy subset of a set X_2 . A_3 be a fuzzy subset of a set X_3. then an anti- fuzzy union of the fuzzy subsets A_1 and A_2 , A_3 is defined as a function.
$A_1 \cup A_2 \cup A_3: X_1 \cup X_2 \cup X_3 \to [0, 1]$ given by

$$(A_1 \cup A_2 \cup A_3)(x) = \begin{cases} \min(A_1(x), A_2(x), A_3(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 . \\ \min(A_1(x), A_2(x)) \text{ if } x \in X_1 \cap X_2 \,\&\, x \notin X_3 . \\ \min(A_2(x), A_3(x)) \text{ if } x \in X_2 \cap X_3 \,\&\, x \notin X_1 \\ \min(A_3(x), A_1(x)) \text{ if } x \in X_3 \cap X_1 \,\&\, x \notin X_2 \\ A_1(x) \text{ if } x \in X_1 \,\&\, x \notin X_2, X_3 \\ A_2(x) \text{ if } x \in X_2 \,\&\, x \notin X_1, X_3. \\ A_3(x) \text{ if } x \in X_3 \,\&\, x \notin X_1, X_2 . \end{cases}$$

2.10. Definition of anti-fuzzy sub-trigroup of the trigroup G:
Let (G,+ , .,*) be a trigroup with three binary operations +(addition) , . (Multiplications), * (ab/2). Then A : G → [0,1] is said to be a Anti - fuzzy sub-trigroup of the trigroup G under ,+ , .,* operations defined on G. if there exists three proper fuzzy subsets A_1 of G_1 and A_2 of G_2, A_3 of G_3 such that

(i) (A_1,+) is an anti- fuzzy subgroup of (G_1,+)
(ii) (A_2,.) is an anti- fuzzy subgroup of (G_2, .)
(iii) (A_3,*) is an anti- fuzzy subgroup of (G_3 ,*)
(iv) A = ($A_1 \cup A_2 \cup A_3$).

Note: A fuzzy subset A of a group G is said to be a union of three anti- fuzzy subgroups of the group G if there exists three anti-fuzzy subgroups A_1 and A_2, A_3 of A (A_1= A, A_2= A and A_3= A) such that ($A_1 \cup A_2 \cup A_3$). Here by the term Anti-fuzzy subgroup B of A we mean that B is an anti- fuzzy subgroup of the group G and B ⊆ A (where A is also an anti- fuzzy subgroup of G).

Example:
Let X_1 = {1, 2, 3, 4, 5} and X_2 = {2, 4, 6, 8, 10}, X_3= {1, 3, 5, 7, 9} be three sets
Define A_1: X_1→ [0,1] by

$$A_1 (x) = \begin{cases} 1 \text{ if } x = 1, 2 \\ 0.6 \text{ if } x =3 \\ 0.2 \text{ if } x = 4, 5. \end{cases}$$

Define A_2 : → [0, 1] by

$$A_2 (x) = \begin{cases} 1 \text{ if } x = 2, 4 \\ 0.6 \text{ if } x =6 \\ 0.2 \text{ if } x = 8, 10. \end{cases}$$

And Define A_3: X_3 → [0, 1] by

$$A_3 (x) = \begin{cases} 1 \text{ if } x = 1 \\ 0.6 \text{ if } x =3, 5 \\ 0.2 \text{ if } x = 7, 9 \end{cases}$$

Hence

$$(A_1 \cup A_2 \cup A_3) (x) = \begin{cases} 1 \text{ if } x = 1, 2, 4. \\ 0.6 \text{ if } x =3, 5, 6 \\ 0.2 \text{ if } x = 4, 5, 7, 8, 9, 10. \end{cases}$$

Similarly, we define the definitions of Anti-fuzzy union of the Quadratic anti fuzzy sets, and soon Anti-fuzzy union of the n anti fuzzy sets .

2.11. Definition of the Anti-fuzzy union of the fuzzy sets $A_1$and A_2, A_3, A_4:
Let A_1 be a fuzzy subset of a set X_1 and A_2 be a fuzzy subset of a set X_2, A_3 be a fuzzy subset of a set X_3 , A_4 be a fuzzy subset of a set X_4. then an anti- fuzzy union of the fuzzy subsets A_1 and A_2, A_3 ,A_4 , is defined as a function.
$A_1 \cup A_2 \cup A_3 \cup A_4$: $X_1 \cup X_2 \cup X_3 \cup X_4$ → [0, 1] given by

$$(A_1 \cup A_2 \cup A_3 \cup A_4) (x) = \begin{cases} \min(A_1 (x), A_2 (x), A_3 (x) , A_4 (x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 \cap X_4. \\ \min(A_1 (x), A_2 (x), A_3 (x)) & \text{if } x \in X_1 \cap X_2 \cap X_3, \ \& \ x \notin X_4 . \\ \min(A_2 (x), A_3 (x), A_4 (x)) & \text{if } x \in X_2 \cap X_3 \cap X_4, \ \& \ x \notin X_1 . \\ \min(A_3 (x), A_4 (x), A_1 (x)) & \text{if } x \in X_3 \cap X_4 \cap X_1, \ \& \ x \notin X_2 . \\ \min(A_4 (x), A_1 (x), A_2 (x)) & \text{if } x \in X_4 \cap X_1 \cap X_2, \ \& \ x \notin X_3 . \\ \min(A_1 (x), A_2 (x)) & \text{if } x \in X_1 \cap X_2 \ \& \ x \notin X_3, X_4 . \\ \min(A_2 (x), A_3 (x)) & \text{if } x \in X_2 \cap X_3 \ \& \ x \notin X_1, X_4 . \\ \min(A_3 (x), A_4 (x)) & \text{if } x \in X_3 \cap X_4 \ \& \ x \notin X_1, \ X_2. \\ \min(A_4 (x), A_1 (x)) & \text{if } x \in X_4 \cap X_1 \ \& \ x \notin A_2, \ A_3. \\ A_1 (x) \text{ if } x \in A_1 \ \& \ x \notin A_2, A_3, A_4 . \\ A_2 (x) \text{ if } x \in A_2 \ \& x \notin A_1, A_3, A_4 . \\ A_3(x) \text{ if } x \in A_3 \ \& \ x \notin A_1, A_2, A_4 . \\ A_4(x) \text{ if } x \in A_4 \ \& \ x \notin A_1, A_2, A_3 . \end{cases}$$

2.12. Definition of the Anti-fuzzy union of the fuzzy sets $A_1$and A_2, A_3 , A_4, A_5:
Let A_1 be a fuzzy subset of a set X_1 and A_2 be a fuzzy subset of a set X_2, A_3 be a fuzzy subset of a set X_3 , A_4 be a fuzzy subset of a set X_4, A_5 be a fuzzy subset of a set X_5. then an anti- fuzzy union of the fuzzy subsets A_1 and A_2, A_3,A_4,A_5 is defined as a function.
$A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$: $X_1 \cup X_2 \cup X_3 \cup X_4 \cup X_5 \to [0, 1]$ given by

$$
(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = \begin{cases}
\min(A_1(x), A_2(x), A_3(x), A_4(x), A_5(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 \cap X_4. \\
\min(A_1(x), A_2(x), A_3(x), A_4(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3 \cap X_4 \text{ \& } x \notin X_5. \\
\min(A_2(x), A_3(x), A_4(x), A_5(x)) \text{ if } x \in X_2 \cap X_3 \cap X_4 \cap X_5 \text{ \& } x \notin X_1. \\
\min(A_3(x), A_4(x), A_5(x), A_1(x)) \text{ if } x \in X_3 \cap X_4 \cap X_5 \cap X_1 \text{ \& } x \notin X_2. \\
\min(A_4(x), A_5(x), A_1(x), A_2(x)) \text{ if } x \in X_4 \cap X_5 \cap X_1 \cap X_2 \text{ \& } x \notin X_3. \\
\min(A_5(x), A_1(x), A_2(x), A_3(x)) \text{ if } x \in X_5 \cap X_1 \cap X_2 \cap X_3 \text{ \& } x \notin X_4. \\
\min(A_1(x), A_2(x), A_3(x)) \text{ if } x \in X_1 \cap X_2 \cap X_3, \text{ \& } x \notin X_4, X_5. \\
\min(A_2(x), A_3(x), A_4(x)) \text{ if } x \in X_2 \cap X_3 \cap X_4, \text{ \& } x \notin X_1, X_5. \\
\min(A_3(x), A_4(x), A_5(x)) \text{ if } x \in X_3 \cap X_4 \cap X_5, \text{ \& } x \notin X_2, X_1. \\
\min(A_4(x), A_5(x), A_1(x)) \text{ if } x \in X_4 \cap X_5 \cap X_1, \text{ \& } x \notin X_2, X_3. \\
\min(A_5(x), A_1(x), A_2(x)) \text{ if } x \in X_5 \cap X_1 \cap X_2, \text{ \& } x \notin X_3, X_4. \\
\min(A_1(x), A_2(x)) \text{ if } x \in X_1 \cap X_2 \text{ \& } x \notin X_3, X_4, X_5. \\
\min(A_2(x), A_3(x)) \text{ if } x \in X_2 \cap X_3 \text{ \& } x \notin X_1, X_4, X_5. \\
\min(A_3(x), A_4(x)) \text{ if } x \in X_3 \cap X_4 \text{ \& } x \notin X_1, X_2, X_5. \\
\min(A_4(x), A_5(x)) \text{ if } x \in X_4 \cap X_5 \text{ \& } x \notin X_2, X_3, X_1. \\
\min(A_5(x), A_1(x)) \text{ if } x \in X_5 \cap X_1 \text{ \& } x \notin X_2, X_3, X_4. \\
A_1(x) \text{ if } x \in X_1 \text{ \& } x \notin X_2, X_3, X_4, X_5. \\
A_2(x) \text{ if } x \in X_2 \text{ \&} x \notin X_1, X_3, X_4, X_5. \\
A_3(x) \text{ if } x \in X_3 \text{ \& } x \notin X_1, X_2, X_4, X_5. \\
A_4(x) \text{ if } x \in X_4 \text{ \& } x \notin X_1, X_2, X_3, X_5. \\
A_5(x) \text{ if } x \in X_5 \text{ \& } x \notin X_1, X_2, X, X_4.
\end{cases}
$$

Similarly for n,

2.13. Definition of the Anti-fuzzy union of the fuzzy sets $A_1$and A_2, A_3 ……… , A_n:
Let A_1 be a fuzzy subset of a set X_1 and A_2 be a fuzzy subset of a set X_2, A_3 be a fuzzy subset of a set X_3, and soon …… A_n be a fuzzy subset of a set X_n. then an anti- fuzzy union of the fuzzy subsets A_1 and A_2 , ………, A_n is defined as a function.
$A_1 \cup A_2 \cup \ldots\ldots \cup A_n$: $X_1 \cup X_2 \cup \ldots\ldots \cup X_n \to [0, 1]$ given by

$$
(A_1 \cup A_2 \cup \ldots\ldots\ldots \cup A_n)(x) = \begin{cases}
\min(A_1(x), A_2(x), \ldots, A_n(x)) \text{ if } x \in X_1 \cap X_2 \cap \ldots \cap X_n. \\
\min(A_1(x), A_2(x), \ldots, A_{n-1}(x)) \text{ if } x \in X_1 \cap X_2 \cap \ldots \cap X_{n-1} \text{ \& } x \notin X_n. \\
\min(A_2(x), A_3(x), \ldots, A_n(x)) \text{ if } x \in X_2 \cap X_3 \cap \ldots \cap X_n \text{ \& } x \notin X_1. \\
\min(A_3(x), \ldots, A_n(x), A_1(x)) \text{ if } x \in X_3 \cap \ldots \cap X_n \cap X_1 \text{ \& } x \notin X_2. \\
\min(A_4(x), \ldots A_n(x), A_1(x), A_2(x)) \text{ if } x \in X_4 \cap \ldots X_n \cap X_1 \cap X_2 \text{ \& } x \notin X_3 \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\min(A_n(x), A_1(x), \ldots, A_{n-2}(x)) \text{ if } x \in X_n \cap X_1 \cap \ldots \cap X_{n-2} \text{ \& } x \notin X_{n-1}. \\
\min(A_1(x), \ldots, A_{n-2}(x)) \text{ if } x \in X_1 \cap \ldots \cap X_{n-2}, \text{ \& } x \notin X_{n-1}, X_n. \\
\min(A_2(x), \ldots, A_{n-1}(x)) \text{ if } x \in X_2 \cap \ldots \cap X_{n-1}, \text{ \& } x \notin X_1, X_n. \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\min(A_1(x), A_2(x)) \text{ if } x \in X_1 \cap X_2 \text{ \& } x \notin X_3, X_4, \ldots, X_n. \\
\min(A_2(x), A_3(x)) \text{ if } x \in X_2 \cap X_3 \text{ \& } x \notin X_1, X_4, \ldots\ldots X_n. \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\min(A_n(x), A_1(x)) \text{ if } x \in X_n \cap X_1 \text{ \& } x \notin X_2, X_3, \ldots X_{n-1}. \\
A_1(x) \text{ if } x \in X_1 \text{ \& } x \notin X_2, X_3, \ldots\ldots\ldots X_n. \\
A_2(x) \text{ if } x \in X_2 \text{ \&} x \notin X_1, X_3, \ldots\ldots\ldots X_n. \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
A_n(x) \text{ if } x \in X_n \text{ \& } x \notin X_1, X_2, X_3 \ldots\ldots\ldots, X_{n-1}.
\end{cases}
$$

2.14. Definition of anti-fuzzy sub-quadratic group of the Quadratic group G:
Let (G,+ , ˙,* ,**) be a Quadratic group with Four binary operations +(addition) , . (multiplications), *, ** . Then A : G → [0,1] is said to be a Anti - fuzzy sub- quadratic group of the Quadratic group G under ,+ , .,*,**operations defined on G. if there exists four proper fuzzy subsets A_1 of G_1 and A_2 of G_2, A_3 of G_3, A_4 of G_4 such that

(i) (A_1,+) is an anti- fuzzy subgroup of (G_1,+)
(ii) (A_2,.) is an anti- fuzzy subgroup of (G_2, .)
(iii) (A_3,*) is an anti- fuzzy subgroup of (G_3 ,*)
(iv) (A_4,**) is an anti- fuzzy subgroup of (G_4 ,**)
(v) A = ($A_1 \cup A_2 \cup A_3 \cup A_4$).

2.15. Definition of anti-fuzzy sub- penta group of the pentagroup G :
Let (G,+ , ˙,* ,**,***) be a Penta group with Five binary operations +(addition),. (multiplications), * ,**,*** . Then A: G → [0,1] is said to be a Anti - fuzzy sub – penta group of the Pentagroup G under ,+ , .,*,** ,***,operations defined on G. if there exists five proper fuzzy subsets A_1 of G_1 and A_2 of G_2, A_3 of G_3 , A_4 of G_4 , A_5 of G_5 such that

(i) (A_1,+) is an anti- fuzzy subgroup of (G_1,+)
(ii) (A_2,.) is an anti- fuzzy subgroup of (G_2, .)
(iii) (A_3,*) is an anti- fuzzy subgroup of (G_3 ,*)
(iv) (A_4,**) is an anti- fuzzy subgroup of (G_4 ,**)
(v) (A_5,***) is an anti- fuzzy subgroup of (G_5 ,***)
(vi) A = ($A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$).

2.16. Definition of anti-fuzzy sub- n group of the n group G:
Let (G,+ , ˙,* ,**,***,….,$^{(n-2)}$*) be a n group with n binary operations +(addition),. (multiplications) , * ,**,***,…., $^{(n-2)}$* . Then A : G→ [0,1] is said to be a Anti - fuzzy sub - n group of the n group G under ,+ , .,*,** ,***,,…., $^{(n-2)}$*operations defined on G. if there exists ”n” proper fuzzy subsets A_1 of G_1 and A_2 of G_2, A_3 of G_3, A_4 of G_4 , A_5 of G_5 … … . , A_n of G_n such that

(i) (A_1,+) is an anti- fuzzy subgroup of (G_1,+)
(ii) (A_2,.) is an anti- fuzzy subgroup of (G_2, .)
(iii) (A_3,*) is an anti- fuzzy subgroup of (G_3 ,*)
(iv) (A_4,**) is an anti- fuzzy subgroup of (G_4 ,**)
(v) (A_5,***) is an anti- fuzzy subgroup of (G_5 ,***)
(vi) ………………………………………….
(vii) ………………………………………….
(viii) (A_n,$^{(n-2)}$*) is an anti- fuzzy subgroup of (G_n ,$^{(n-2)}$*)
(ix) A = ($A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \dots . \cup A_n$.).

3. THEOREMS

3.1. Theorem: A is an Anti - fuzzy subgroup of a group S if and only if $A(\mathrm{xy}^{-1}) \leq \max\{\mathrm{A(x)}, \mathrm{A(y)}\}$ for all x, y in S.

Proof:

Necessary part: Let A is an anti- fuzzy subgroup,

To prove:

$\mathrm{A(xy^{-1})} \leq \max\{\mathrm{A(x)}, \mathrm{A(y)}\}$
$\mathrm{A(xy^{-1})} \leq \max\{\mathrm{A(x)}, \mathrm{A(y^{-1})}\}$
$\mathrm{A(xy^{-1})} \leq \max\{\mathrm{A(x)}, \mathrm{A(y)}\}$, since $\mathrm{A(y)} = \mathrm{A(y^{-1})}$

Sufficient part:
$\mathrm{A(xy^{-1})} \leq \max\{\mathrm{A(x)}, \mathrm{A(y)}\}$.

To prove:

A is an anti fuzzy subgroup,
$\mathrm{A(y^{-1})} = \mathrm{A(ey^{-1})} \leq \max\{\mathrm{A(e)}, \mathrm{A(y)}\}= \mathrm{A(y)}$.
$\mathrm{A(xy)} = \mathrm{A(x(y^{-1}){-}^{1})} \leq \max\{\mathrm{A(x)}, \mathrm{A(y^{-1})}\}$.
$\mathrm{A(xy))} \leq \max\{\mathrm{A(x)}, \mathrm{A(y)}\}$.

Therefore A is an anti - fuzzy subgroup,

Hence proved.

3.2 Theorem: Every anti - fuzzy sub – bigroup of a group G is an anti - fuzzy subgroup of the group G but not conversely.

Proof: It follows from the definition of an anti- fuzzy sub –bigroup of a group G that every an anti- fuzzy sub – bigroup of a group G is an anti -fuzzy subgroup of the group G. Since the union of any two anti-fuzzy subgroup is an anti-fuzzy subgroup. Converse part is not true.

3.3 Main Theorem: The union of two anti - fuzzy subgroups of a group G is an anti- fuzzy subgroup if and only if one is contained in the other.

Proof:
Necessary part:
Let A_1 and A_2 be two anti - fuzzy subgroups of G such that one is contained in the other.

Hence either $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

To prove: $A_1 \cup A_2$ is an anti - fuzzy subgroup of G.
Let the union of two fuzzy subsets A_1, A_2 is defined by

$A_1 \cup A_2$ (x) = min $\{A_1(x), A_2 (x)\}$

So $A_1 \cup A_2(xy)$ = min $\{A_1(xy), A_2 (xy)\}$ (1)

Since A_1 and A_2 are the two fuzzy subgroup of G.

$A_1 \cup A_2(xy) = A_1(xy)$ (or) $A_1 \cup A_2(xy) = A_2(xy)$ (2)

Since $A_1(xy)$ and $A_2(xy)$ are anti- fuzzy subgroups of G.

From (1) and (2).

$A_1 \cup A_2(xy)$ is also an anti - fuzzy subgroup of G.

Hence $A_1 \cup A_2$ is an anti- fuzzy subgroup of G.

SUFFICIENT PART:

Suppose $A_1 \cup A_2$ is an anti - fuzzy subgroup of G.

To Claim:
$A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

Since A_1, and A_2 are anti - fuzzy subgroups of G.

$A_1(xy) \leq \max \{A_1(x), A_1(y)\}$ (by condition (i) of definition of fuzzy subgroup of G)). (3)

There are two cases,

i) $A_1(xy) \leq A_1(x)$,

ii) $A_1(xy) \leq A_1(y)$.

Case i)

$A_1(xy) \leq A_1(x)$ (4)

By an anti - fuzzy union of fuzzy sets A_1 and A_2 , we have

$A_1 \cup A_2(x)=A_1(x)$ (5)

Sub (5) in (4), we get

$A_1(xy) \leq A_1 \cup A_2(x) = A_2(x)$.

$A_1(xy) \leq A_2(x)$ (6)

From (4) and (6), we get

$A_1(x) \leq A_2(x)$.

$A_1 \subseteq A_2$ (7)

Similarly case ii):

$A_1(xy) \leq A_1(y)$ (8)

By definition of an anti - fuzzy union of fuzzy sets of G, we have

$$A_1 \cup A_2(y) = A_1(y).. \tag{9}$$

Sub (9) in (8), we get,

$$A_1(xy) \le A_1 \cup A_2(y) = A_2(y).$$
$$A_1(xy) \le A_2(y) \tag{10}$$

From (8) and (10) we have,

$$A_2(y) \le A_1(y)$$
$$A_2 \subseteq A_1 \tag{11}$$

From (7) and (11),

i.e $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

Hence A_1 is contained in A_2 and A_2 is contained in A_1.

Therefore The Union of two anti -fuzzy subgroups of a group G is an anti - fuzzy subgroup. if and only if one contained in other.

3.4 Main Theorem: The union of two Anti-fuzzy sub-bigroups of a bigroup G is an anti -fuzzy sub-bigroup if and only if one is contained in the other.

Proof: Let A_1 and A_2 are anti –fuzzy sub-bigroups and $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

To prove:

$A_1 \cup A_2$ is an anti - fuzzy sub-bigroup .

By definition of Anti - Fuzzy Union of the fuzzy sets A_1 and A_2 .

$A_1 \cup A_2(xy) = A_1(xy)$ (or) $A_1 \cup A_2(xy) = A_2(xy)$.

Since $A_1(xy)$ and $A_2(xy)$ are anti - fuzzy sub - bigroups of G.

Hence, $A_1 \cup A_2$ is an anti - fuzzy sub-bigroup .

Conversely, Let $A_1 \cup A_2$ is an anti - fuzzy sub-bigroup .

To prove:

$A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

Since $A_1 \cup A_2$ is an anti - fuzzy sub-bigroup.

By Theorem: 3.2
Every anti - fuzzy sub –bigroup of a group G is an anti - fuzzy subgroup of the group G but not conversely.

Hence $A_1 \cup A_2$ is a fuzzy subgroup.and By Main Theorem: 3.3

The union of two anti -fuzzy subgroups of a group G is an anti - fuzzy subgroup if and only if one is contained in the other.
Hence $A_1 \subseteq A_2$ and $A_2 \subseteq A_1$.

Therefore, the union of two anti -fuzzy sub-bigroups of a bigroup G is an anti - fuzzy sub-bigroup if and only if one is contained in the other.

3.5 Main Theorem: The union of the three Anti - fuzzy sub-trigroups of a group G is an anti - fuzzy sub-trigroups if and only if one is contained in the other.

Proof:
Necessary part: Let A_1 and A_2, A_3 be three anti - fuzzy sub - trigroups of G such that one is contained in the other.
Hence either $A_1 \subseteq A_2$, $A_2 \subseteq A_1$, $A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2$ and $A_2 \subseteq A_3$

To prove: $A = A_1 \cup A_2 \cup A_3$ is an anti - fuzzy sub-trigroup of G.

By Definition of an anti - fuzzy union of the fuzzy sets A_1 and A_2, A_3:

$A(x) = (A_1 \cup A_2 \cup A_3)(x) = \min(A_1(x), A_2(x), A_3(x))$.

which implies either

$A(x) = (A_1 \cup A_2 \cup A_3)(x) = A_1(x)$ (1)

(or) $A(x) = (A_1 \cup A_2 \cup A_3)(x) = A_2(x)$ (2)

(or) $A(x) = (A_1 \cup A_2 \cup A_3)(x) = A_3(x)$ (3)

From (1), (2), (3), since A_1 and A_2, A_3 be three anti - fuzzy sub – trigroups.

A = $(A_1 \cup A_2 \cup A_3)$ is an anti - fuzzy sub-trigroup of G.

Sufficient part: Let A = $(A_1 \cup A_2 \cup A_3)$ is an anti - fuzzy sub-trigroup of G.

To prove:

$A_1 \subseteq A_2$, $A_2 \subseteq A_1$, $A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2$ and $A_2 \subseteq A_3$.

A fuzzy subset A of a group G is said to be a union of three anti- fuzzy sub-groups of the group G if there exists three anti-fuzzy subgroups A_1 and A_2, A_3 of A (A_1 = A, A_2 = A and A_3 = A) such that A=$(A_1 \cup A_2 \cup A_3)$. Here by the term Anti-fuzzy subgroup B of A we mean that B is an anti- fuzzy subgroup of the group G and B $\subseteq$ A (where A is also an anti- fuzzy subgroup of G).

$A_1 \subseteq A = A_1 \cup A_2 \cup A_3$ (4)

$A_2 \subseteq A = A_1 \cup A_2 \cup A_3$ (5)

$A_3 \subseteq A = A_1 \cup A_2 \cup A_3$ (6)

Substitute (5), (6) in (4), we get,

$A_1 \subseteq A_2$ and $A_1 \subseteq A_3$ (7)

Similarly Substitute (4) and (6) in (5), we get,

$A_2 \subseteq A_1$ and $A_2 \subseteq A_3$ (8)

Substitute (4) and (5) in 6, we get,

$A_3 \subseteq A_1$ and $A_3 \subseteq A_2$ (9)

From (7), (8), and (9), We have, $A_1 \subseteq A_2$, $A_2 \subseteq A_1$, $A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2$ and $A_2 \subseteq A_3$.

Hence the union of the three anti - fuzzy sub-trigroups of a group G is an anti – fuzzy sub-trigroups if and only if one is contained in the other.

3.6. Main Theorem: The union of the Four Anti - Fuzzy Sub- Quadratic groups of a group G is an Anti - Fuzzy Sub – Quadratic groups if and only if one is contained in the other.

Proof:

Necessary part: Let A_1 and A_2, A_3, A_4 be Four Anti - Fuzzy Sub - Quadratic groups of G such that one is contained in the other.

Hence either $A_1 \subseteq A_2$, $A_2 \subseteq A_1$, $A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2$, $A_2 \subseteq A_3$
$A_3 \subseteq A_4$, $A_4 \subseteq A_3$, $A_1 \subseteq A_4$, $A_4 \subseteq A_1$, $A_2 \subseteq A_4$ and $A_4 \subseteq A_2$.

To prove:

$A = A_1 \cup A_2 \cup A_3 \cup A_4$ is an anti - fuzzy sub- Quadratic group of G.

By Definition of an anti - fuzzy union of the fuzzy sets $A_1$and A_2, A_3, A_4.

$A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4)(x) = \min(A_1(x), A_2(x), A_3(x), A_4(x))$.

which implies either

$A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4)(x) = A_1(x)$ (1)

(or) $A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4)(x) = A_2(x)$ (2)

(or) $A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4)(x) = A_3(x)$ (3)

(or) $A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4)(x) = A_4(x)$ (4)

From (1), (2), (3), (4),

Since A_1 and A_2, A_3, A_4 be Four Anti - Fuzzy Sub – Quadratic groups. A = $(A_1 \cup A_2 \cup A_3 \cup A_4)$ is an anti - fuzzy sub-quadratic group of G.

Sufficient part: Let A = ($A_1 \cup A_2 \cup A_3 \cup A_4$) is an anti - fuzzy sub-quadratic group of G.

To prove:
$A_1 \subseteq A_2, A_2 \subseteq A_1, A_1 \subseteq A_3, A_3 \subseteq A_1, A_3 \subseteq A_2, A_2 \subseteq A_3, A_3 \subseteq A_4, A_4 \subseteq A_3, A_1 \subseteq A_4, A_4 \subseteq A_1, A_2 \subseteq A_4$ and $A_4 \subseteq A_2$.

By **2.14. Definition of anti-fuzzy sub-quadratic group of the Quadratic group G and Note**

(i) (A_1,+) is an anti- fuzzy subgroup of (G_1+)
(ii) (A_2,.) is an anti- fuzzy subgroup of (G_2, .)
(iii) (A_3,*) is an anti- fuzzy subgroup of (G_3 *)
(iv) (A_4,**) is an anti- fuzzy subgroup of (G_4 , **)
(v) A = ($A_1 \cup A_2 \cup A_3 \cup A_4$).

Which implies,

$$A_1 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \quad (5)$$
$$A_2 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \quad (6)$$
$$A_3 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \quad (7)$$
$$A_4 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \quad (8)$$

Substitute (6), (7) in (5) We get ,

$$A_1 \subseteq A_2 \text{ and } A_1 \subseteq A_3 \quad (9).$$

Similarly Substitute (5) and (7) in (6), we get,

$$A_2 \subseteq A_1 \text{ and } A_2 \subseteq A_3 \quad (10)$$

Substitute (5) and (6) in (7), we get,

$$A_3 \subseteq A_1 \text{and } A_3 \subseteq A_2 \quad (11)$$

Similarly we get,

$$A_3 \subseteq A_4 \text{ and } A_4 \subseteq A_3 \quad (12)$$
$$A_1 \subseteq A_4 \text{ and } A_4 \subseteq A_1 \quad (13)$$
$$A_2 \subseteq A_4 \text{ and } A_4 \subseteq A_2 \quad (14)$$

From (9), (10), and (11), (12), (13), (14). We get,
$A_1 \subseteq A_2, A_2 \subseteq A_1, A_1 \subseteq A_3, A_3 \subseteq A_1, A_3 \subseteq A_2, A_2 \subseteq A_3, A_3 \subseteq A_4, A_4 \subseteq A_3, A_1 \subseteq A_4, A_4 \subseteq A_1, A_2 \subseteq A_4$ and $A_4 \subseteq A_2$.

Hence the union of the four anti - fuzzy sub- Quadratic groups of a group G is an anti – fuzzy sub-Quadratic groups if and only if one is contained in the other.

3.7. Main Theorem: The union of the Five Anti - Fuzzy Sub- Pentagroups of a group G is an Anti - Fuzzy Sub – Pentagroups if and only if one is contained in the other.

Proof:
Necessary part: Let A_1 and A_2, A_3, A_4, A_5 be Five Anti - Fuzzy Sub - Pentagroups of G such that one is contained in the other.

Hence either $A_1 \subseteq A_2$, $A_2 \subseteq A_1$, $A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2$, $A_2 \subseteq A_3$, $A_3 \subseteq A_4$, $A_4 \subseteq A_3$ $A_1 \subseteq A_4$, $A_4 \subseteq A_1$, $A_2 \subseteq A_4$, $A_4 \subseteq A_2$, $A_1 \subseteq A_5$, $A_5 \subseteq A_1$, $A_5 \subseteq A_2$, $A_2 \subseteq A_5$, $A_5 \subseteq A_3$, $A_3 \subseteq A_5$, $A_5 \subseteq A_4$, $A_4 \subseteq A_5$.

To prove: $A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ is an anti - fuzzy sub- pentagroup of G.

By Definition of an anti - fuzzy union of the fuzzy sets A_1 and A_2, A_3, A_4, A_5:

$$A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = \min(A_1(x), A_2(x), A_3(x), A_4(x), A_5(x)).$$

which implies either

$$A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = A_1(x) \quad (1)$$
$$\text{(or)} \quad A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = A_2(x) \quad (2)$$
$$\text{(or)} \quad A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = A_3(x) \quad (3)$$
$$\text{(or)} \quad A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = A_4(x) \quad (4)$$
$$\text{(or)} \quad A(x) = (A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)(x) = A_5(x) \quad (5)$$

From (1), (2), (3), (4), (5).

Since $A_1$and A_2, A_3, A_4, A_5 be Five Anti - Fuzzy Sub – Pentagroups.
A = ($A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$) is an anti - fuzzy sub- Pentagroup of G.

Sufficient part: Let A = ($A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$) is an anti - fuzzy sub-Pentagroup of G.

To prove:
$A_1 \subseteq A_2$, $A_2 \subseteq A_1$,$A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2$, $A_2 \subseteq A_3$, $A_3 \subseteq A_4$, $A_4 \subseteq A_3 A_1 \subseteq A_4$, $A_4 \subseteq A_1$, $A_2 \subseteq A_4$, $A_4 \subseteq A_2$, $A_1 \subseteq A_5$, $A_5 \subseteq A_1$, $A_5 \subseteq A_2$, $A_2 \subseteq A_5$, $A_5 \subseteq A_3 A_3 \subseteq A_5$, $A_5 \subseteq A_4$ and $A_4 \subseteq A_5$.

By **2.15. Definition of anti-fuzzy sub-Penta group of the Penta group G and Note**
(i) $(A_1,+)$ is an anti- fuzzy subgroup of $(G_1,+)$
(ii) $(A_2,.)$ is an anti- fuzzy subgroup of $(G_2,.)$
(iii) $(A_3,*)$ is an anti- fuzzy subgroup of $(G_3,*)$
(iv) $(A_4,**)$ is an anti- fuzzy subgroup of $(G_4,**)$
(v) $(A_5,***)$ is an anti- fuzzy subgroup of $(G_5,***)$
(vi) A = ($A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$).

Which implies,

$$A_1 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \quad (6)$$
$$A_2 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \quad (7)$$
$$A_3 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \quad (8)$$
$$A_4 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \quad (9)$$
$$A_5 \subseteq A = A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5 \quad (10)$$

Substitute (6) and (7), We get,
$$A_1 \subseteq A_2 \text{ and } A_1 \subseteq A_2 \quad (11)$$

Similarly Substitute (6) and (8), We get ,
$$A_3 \subseteq A_1 \text{ and } A_1 \subseteq A_3 \quad (12)$$

Substitute 6 and 9 we get,
$$A_2 \subseteq A_3 \text{ and } A_3 \subseteq A_2 \quad (13)$$

Similarly we get,
$$A_3 \subseteq A_4 \text{ and } A_4 \subseteq A_3 \quad (14)$$
$$A_1 \subseteq A_4 \text{ and } A_4 \subseteq A_1 \quad (15)$$
$$A_2 \subseteq A_4 \text{ and } A_4 \subseteq A_2 \quad (16)$$
$$A_1 \subseteq A_5 \text{ and } A_5 \subseteq A_1 \quad (17)$$
$$A_5 \subseteq A_2 \text{ and } A_2 \subseteq A_5 \quad (18)$$
$$A_5 \subseteq A_3 \text{ and } A_3 \subseteq A_5 \quad (19)$$
$$A_5 \subseteq A_4 \text{ and } A_4 \subseteq A_5 \quad (20)$$

From (11), (12), (13), (14), (15), (16), (17), (18), (19), (20), we get,

$A_1 \subseteq A_2$, $A_2 \subseteq A_1$, $A_1 \subseteq A_3$, $A_3 \subseteq A_1$, $A_3 \subseteq A_2 A_2 \subseteq A_3$, $A_3 \subseteq A_4$, $A_4 \subseteq A_3$, $A_1 \subseteq A_4$, $A_4 \subseteq A_1$, $A_2 \subseteq A_4$,$A_4 \subseteq A_2$,$A_1 \subseteq A_5$, $A_5 \subseteq A_1$, $A_5 \subseteq A_2$, $A_2 \subseteq A_5$, $A_5 \subseteq A_3$,$A_3 \subseteq A_5$, $A_5 \subseteq A_4$, $A_4 \subseteq A_5$.

Hence the union of the five anti - fuzzy sub- Pentagroups of a group G is ananti - fuzzy sub- Penta groups if and only if one is contained in the other.

Similarly,

3.8. Main Theorem: The union of the "n" Anti - Fuzzy Sub- "n" groups of a group G is an Anti – Fuzzy Sub – "n" groups if and only if one is contained in the other.

Proof: Proof is Similar like above Theorems.

3.9. Main Theorem: Every anti - fuzzy sub –Tri group of a group G is an anti - fuzzy subgroup of the group G but not conversely.

Proof: It follows from the definition of an anti- fuzzy sub – Tri group of a group G that every an anti- fuzzy sub – Tri group of a group G is an anti -fuzzy subgroup of the group G. Converse part is not true.

3.10. Main Theorem: Every anti - fuzzy sub – Quadratic group of a group G is an anti - fuzzy subgroup of the group G but not conversely.

Proof: It follows from the definition of an anti- fuzzy sub – Quadratic group of a group G that every an anti- fuzzy sub – Quadratic group of a group G is an anti -fuzzy subgroup of the group G. Converse part is not true.

3.11. Main Theorem: Every anti - fuzzy sub – Pentagroup of a group G is an anti - fuzzy subgroup of the group G but not conversely.

Proof: It follows from the definition of an anti- fuzzy sub – Pentagroup of a group G that every an anti- fuzzy sub – Pentagroup of a group G is an anti -fuzzy subgroup of the group G . Converse part is not true.

3.12. Main Theorem: Every anti - fuzzy sub – “n” group of a group G is an anti - fuzzy subgroup of the group G but not conversely .

Proof: It follows from the definition of an anti- fuzzy sub – “n” group of a group G that every an anti- fuzzy sub – “n” group of a group G is an anti -fuzzy subgroup of the group G. Converse part is not true.

CONCLUSION

In the paper, We derive the union of the Anti-Fuzzy sub "n" groups of a group G for n = 1, 2, 3,....n. and definition of Anti-Fuzzy sub- Quadratic groups and Anti - Fuzzy sub -pendant groups and soon upto definition of Anti-Fuzzy sub - "n" groups. Shows that every Anti-Fuzzy sub- bigroups, Anti-Fuzzy sub- trigroups and soon Anti-Fuzzy sub- "n" groups are Anti-Fuzzy subgroups.

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Source of Support: Nil, Conflict of interest: None Declared

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