International Research Journal of Pure Algebra-9(4), 2019, 44-50
Available online through www.rjpa.info ISSN 2248–9037

ON AUGMENTED LEAP INDEX AND IT'S POLYNOMIAL OF SOME WHEEL TYPE GRAPHS

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(Received On: 26-03-19; Revised & Accepted On: 12-04-19)

ABSTRACT

We introduce the augmented leap index and augmented leap polynomial of a graph. In this paper, we determine the augmented leap index and augmented leap polynomial of wheel, gear, helm, flower, sunflower graphs.

Keywords: *augmented leap index, augmented leap polynomial, wheel, gear, helm, flower, sunflower graphs.*

Mathematics Subject Classification: *05C05, 05C07, 05C12, 05C76.*

1. INTRODUCTION

By a graph, we mean a finite, undirected, connected without loops and multiple edges, Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex v, denoted by $d(v)$, is the number of vertices adjacent to v. The distance $d(u, v)$ between any two vertices u and v of G is the number of edges in a shortest path connecting them. For a positive integer k and a vertex v in G, the open neighborhood of v in G is defined as $N_k(v/G) = \{u \in V(G): d(u, v) = k\}$. The k-distance degree $d_k(v)$ of v in G is the number of k neighbors of v in G, see [1].

The augmented Zagreb index [2] of G is defined as

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d(u)d(v)}{d(u)+d(v)-2} \right)^3.$$

This index was studied in [3, 4] and also other augmented indices were introduced and studied in [5, 6].

We now propose the augmented leap index, defined as

$$ALI(G) = \sum_{uv \in E(G)} \left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2} \right)^3.$$

Also we define the augmented leap polynomial as

$$ALI(G,x) = \sum_{uv \in E(G)} x^{\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2} \right)^3}.$$

Very recently, some different polynomials were studied, for example, in [7, 8, 9, 10, 11, 12, 13].

We consider wheels and some wheel type graphs see [14]. In this article, the augmented leap index and augmented leap polynomial of wheel graphs and some wheel type graphs are computed.

2. RESULTS FOR WHEEL GRAPHS

The wheel W_n is defined to be the join of cycle C_n and complete graph K_1. The wheel W_n has n+1 vertices and $2\underline{n}$ edges, see Figure 1. The vertex K_1 is called apex and the vertices of C_n are called rim vertices.

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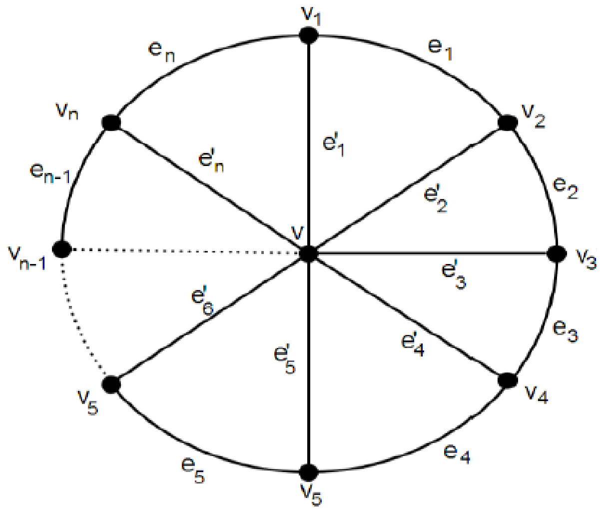

Figure-1: Wheel W_n

In W_n, there are two types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d_2(u) = 0, d_2(v) = n - 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d_2(u) = d_2(v) = n - 3\}$, $|E_2| = n$.

Theorem 1: Let W_n be a wheel with $2n$ edges, $n\geq 3$. Then

(a) $ALI(W_n) = \dfrac{n(n-3)^6}{(2n-8)^3}$.

(b) $ALI(W_n, x) = nx^0 + nx^{\frac{(n-3)^6}{(2n-8)^3}}$.

Proof: (a) From equation (1) and by cardinalities of the 2-distance degree of edge partition of W_n, we obtain

$$ALI(W_n) = \sum_{uv\in E(W_n)} \left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3$$

$$= n\left(\frac{0\times(n-3)}{0+n-3-2}\right)^3 + n\left(\frac{(n-3)(n-3)}{n-3+n-3-2}\right)^3$$

$$= \frac{n(n-3)^6}{(2n-8)^3}.$$

(b) From equation (2) and by cardinalities of the 2-distance degree of edge partition W_n, we have

$$ALI(W_n, x) = \sum_{uv\in E(W_n)} x^{\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3}$$

$$= nx^{\left(\frac{0\times(n-3)}{0+n-3-2}\right)^3} + nx^{\left(\frac{(n-3)\times(n-3)}{n-3+n-3-2}\right)^3}$$

$$= nx^0 + nx^{\frac{(n-3)^6}{(2n-8)^3}}.$$

3. RESULTS FOR GEAR GRAPHS

A gear graph is a graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n. Clearly $|V(G_n)|=2n+1$ and $|E(G_n)|=3n$. A gear graph G_n is shown in Figure 2.

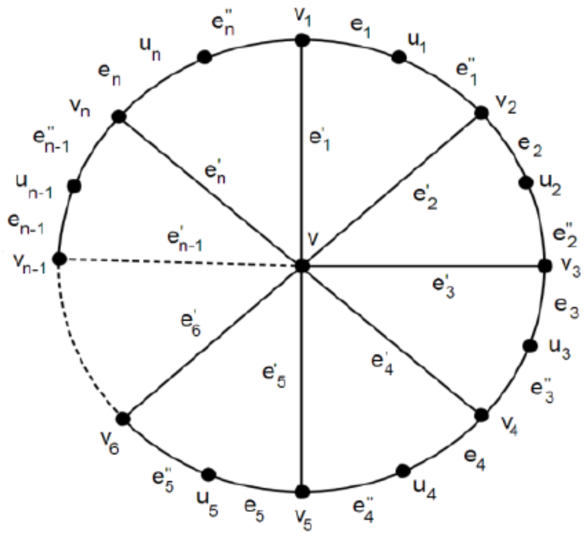

Figure-2: Gear graph G_n

In G_n, there are two types of the 2-distance degree of edges as given below.

$E_1 = \{uv \in E(G_n) \mid d_2(u) = n,\ d_2(v) = n - 1\}$, $|E_1| = n$.

$E_2 = \{uv \in E(G_n) \mid d_2(u) = 3,\ d_2(v) = n - 1\}$, $|E_2| = 2n$.

Theorem 2: Let G_n be a gear graph with $3n$ edges, $n \geq 3$. Then

(a) $ALI(G_n) = (n-1)^3\left[\dfrac{n^4}{(2n-3)^3} + \dfrac{54}{n^2}\right]$.

(b) $ALI(G_n, x) = nx^{\left(\frac{n(n-1)}{2n-3}\right)^3} + 2nx^{\left(\frac{3(n-1)}{n}\right)^3}$.

Proof: (a) By using equation (1) and by cardinalities of the 2-distance degree of edge partition of G_n, we deduce

$$ALI(G_n) = \sum_{uv \in E(G_n)} \left(\frac{d_2(u)d_2(v)}{d_2(u) + d_2(v) - 2}\right)^3$$

$$= n\left(\frac{n(n-1)}{n+n-1-2}\right)^3 + 2n\left(\frac{3(n-1)}{3+n-1-2}\right)^3 = (n-1)^3\left[\frac{n^4}{(2n-3)^3} + \frac{54}{n^2}\right].$$

(b) By using equation (2) and by cardinalities of the 2-distance degree of edge partition of G_n, we derive

$$ALI(G_n, x) = \sum_{uv \in E(G_n)} x^{\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3}$$

$$= nx^{\left(\frac{n(n-1)}{n+n-1-2}\right)^3} + 2nx^{\left(\frac{3(n-1)}{3+n-1-2}\right)^3}$$

$$= nx^{\left(\frac{n(n-1)}{2n-3}\right)^3} + 2nx^{\left(\frac{3(n-1)}{n}\right)^3}.$$

4. RESULTS FOR HELM GRAPHS

Let W_n be a wheel with n+1 vertices. The helm graph, denoted by H_n, is a graph obtained from W_n by attaching an edge to each rim vertex of W_n. Clearly the graph H_n has $2n$+1 vertices and $3n$ edges. A graph H_n is presented in Figure 3.

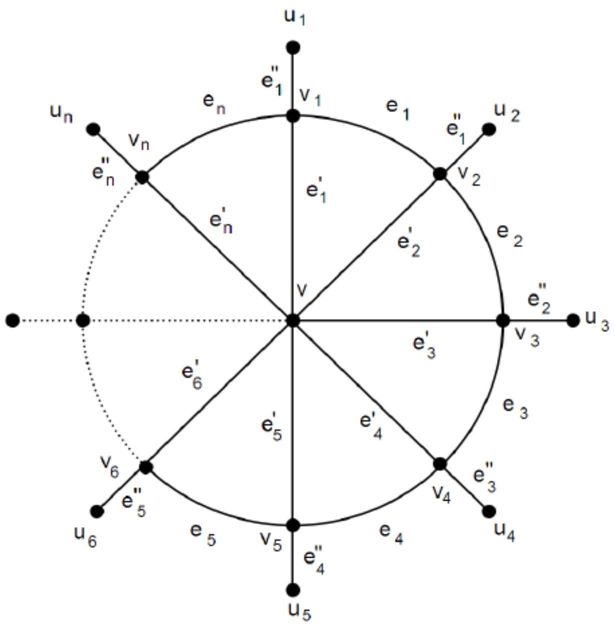

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Figure-3: Helm graph H_n

In H_n, these are three types of the 2-distance degree of edges as follows.

$E_1 = \{uv\in E(H_n) | d_2(u) = n, d_2(v) = n - 1\}$, $|E_1| = n$.
$E_2 = \{uv\in E(H_n) | d_2(u) =3\ d_2(v) = n - 1\}$, $|E_2| = n$.
$E_3 = \{uv\in E(H_n) | d_2(u) = d_2(v) = n - 1\}$, $|E_3| = n$.

Theorem 3: Let H_n be a helm graph with $3n$ edges, $n \geq 3$. Then

(a) $ALI(H_n) = (n-1)^3\left[\frac{n^4}{(2n-3)^3}+\frac{n(n-1)^3}{(2n-4)^3}+\frac{27}{n^2}\right]$.

(b) $ALI(H_n, x) = nx^{\left(\frac{n(n-1)}{2n-3}\right)^3}+nx^{\left(\frac{3(n-1)}{n}\right)^3}+nx^{\left(\frac{n^2-2n+1}{2n-4}\right)^3}$.

Proof: From equation (1) and by cardinalities of the 2-distance degree of edge partition of H_n, we derive

$$ALI(H_n)=\sum_{uv\in E(H_n)}\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3$$
$$=n\left(\frac{n(n-1)}{n+n-1-2}\right)^3+n\left(\frac{3(n-1)}{3+n-1-2}\right)^3+n\left(\frac{(n-1)(n-1)}{n-1+n-1-2}\right)^3$$
$$=(n-1)^3\left[\frac{n^4}{(2n-3)^3}+\frac{n(n-1)^3}{(2n-4)^3}+\frac{27}{n^2}\right].$$

(b) From equation (2) and by cardinalities of the 2-distance degree of edge partition of H_n, we deduce

$$ALI(H_n,x)=\sum_{uv\in E(H_n)}x^{\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3}$$
$$=nx^{\left(\frac{n(n-1)}{n+n-1-2}\right)^3}+nx^{\left(\frac{3(n-1)}{3+n-1-2}\right)^3}+nx^{\left(\frac{(n-1)(n-1)}{n-1+n-1-2}\right)^3}$$
$$=nx^{\left(\frac{n(n-1)}{2n-3}\right)^3}+nx^{\left(\frac{3(n-1)}{n}\right)^3}+nx^{\left(\frac{n^2-2n+1}{2n-4}\right)^3}.$$

5. RESULTS FOR FLOWER GRAPHS

A graph is a flower graph which is obtained from a helm graph H_n by joining an end vertex to the apex of the helm graph and the resulting graph is denoted by Fl_n. A flower graph Fl_n has $2n$+1 vertices and $4n$ edges. A graph Fl_n is presented in Figure 4.

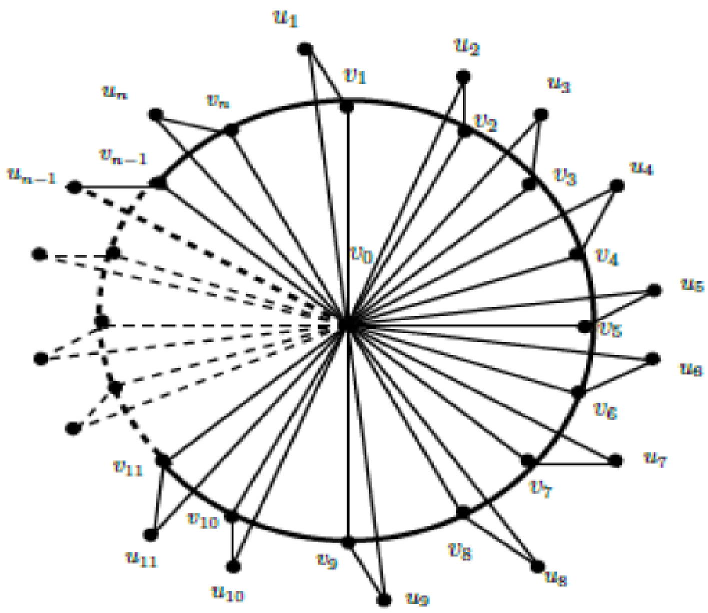

Figure-4: A flower graph Fl_n

In Fl_n, there are four types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(Fl_n) \mid d_2(u) = 0,\ d_2(v) = n-5\}$, $|E_1| = n$.
$E_2 = \{uv \in E(Fl_n) \mid d_2(u) = 0,\ d_2(v) = n-2\}$, $|E_2| = n$.
$E_3 = \{uv \in E(Fl_n) \mid d_2(u) = n-5,\ d_2(v) = n-2\}$, $|E_3| = n$.
$E_4 = \{uv \in E(Fl_n) \mid d_2(u) = d_2(v) = n-5\}$, $|E_4| = n$.

Theorem 4: Let Fl_n be a flower graph with $4n$ edges, $n \geq 3$. Then

(a) $ALI(Fl_n) = n(n-5)^3\left[\left(\frac{n-2}{2n-9}\right)^3 + \left(\frac{n-5}{2n-12}\right)^3\right]$.

(b) $ALI(Fl_n, x) = 2nx^0 + nx^{\left(\frac{n^2-7n+10}{2n-9}\right)^3} + nx^{\left(\frac{n^2-10n+25}{2n-12}\right)^3}$.

Proof: From equation (1) and by cardinalities of the 2-distance degree of edge partition of Fl_n, we derive

$$ALI(Fl_n) = \sum_{uv \in E(Fl_n)} \left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3$$

$$= n\left(\frac{0(n-5)}{0+n-5-2}\right)^3 + n\left(\frac{0(n-2)}{0+n-2-2}\right)^3 + n\left(\frac{(n-5)(n-2)}{n-5+n-2-2}\right)^3 + n\left(\frac{(n-5)(n-5)}{n-5+n-5-2}\right)^3$$

$$= n(n-5)^3\left[\left(\frac{n-2}{2n-9}\right)^3 + \left(\frac{n-5}{2n-12}\right)^3\right].$$

(b) By using equation (2) and by cardinalities of the 2-distance degree of edge partition of Fl_n, we derive

$$ALI(Fl_n, x) = \sum_{uv \in E(Fl_n)} x^{\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3}$$

$$= nx^{\left(\frac{0(n-5)}{0+n-5-2}\right)^3} + nx^{\left(\frac{0(n-2)}{0+n-2-2}\right)^3} + nx^{\left(\frac{(n-5)(n-2)}{n-5+n-2-2}\right)^3} + nx^{\left(\frac{(n-5)(n-5)}{n-5+n-5-2}\right)^3}$$

$$= 2nx^0 + nx^{\left(\frac{n^2-7n+10}{2n-9}\right)^3} + nx^{\left(\frac{n^2-10n+25}{2n-12}\right)^3}.$$

6. RESULTS FOR SUNFLOWER GRAPHS

A graph is a sunflower graph which is obtained from the flower graph Fl_n by attaching n end edges to the apex vertex and it is denoted by Sf_n. A sunflower graph Sf_n has $3n+1$ vertices and $5n$ vertices. A graph Sf_n is depicted in Figure 5.

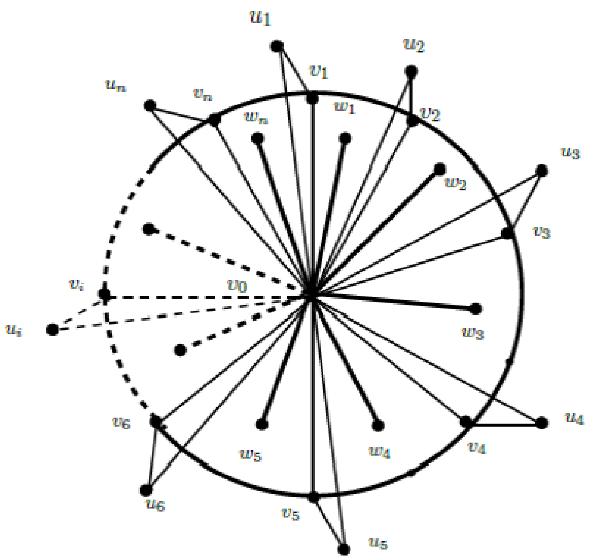

Figure-5: A sunflower graph Sf_n

In Sf_n, there are five types of the 2-distance degree of edges as follows:

$E_1 = \{uv \in E(Sf_n) \mid d_2(u) = 0,\ d_2(v) = 3n - 4\}$, $\quad |E_1| = n$.
$E_2 = \{uv \in E(Sf_n) \mid d_2(u) = 0,\ d_2(v) = 3n - 2\}$, $\quad |E_2| = n$.
$E_3 = \{uv \in E(Sf_n) \mid d_2(u) = 0,\ d_2(v) = 3n - 1\}$, $\quad |E_3| = n$.
$E_4 = \{uv \in E(Sf_n) \mid d_2(u) = d_2(v) = 3n - 4\}$, $\quad |E_4| = n$.
$E_5 = \{uv \in E(Sf_n) \mid d_2(u) = 3n - 4,\ d_2(v) = 3n - 2\}$, $\quad |E_5| = n$.

Theorem 5: Let Sf_n be a sunflower graph with $5n$ edges, $n \geq 3$. Then

(a) $ALI(Sf_n) = n(3n-4)^3\left[\left(\frac{3n-4}{6n-10}\right)^3 + \left(\frac{3n-2}{6n-8}\right)^3\right]$.

(b) $ALI(Sf_n, x) = 3nx^0 + nx^{\left(\frac{9n^2-24n+16}{6n-10}\right)^3} + nx^{\left(\frac{9n^2-18n+8}{6n-8}\right)^3}$.

Proof: (a) From equation (1) and by using cardinalities of the 2-distance degree of edge partition of Sf_n, we obtain

$$ALI(Sf_n) = \sum_{uv \in E(Sf_n)} \left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3$$

$$= n\left(\frac{0(3n-4)}{0+3n-4-2}\right)^3 + n\left(\frac{0(3n-2)}{0+3n-2-2}\right)^3 + n\left(\frac{0(3n-1)}{0+3n-1-2}\right)^3$$

$$+ n\left(\frac{(3n-4)(3n-4)}{3n-4+3n-4-2}\right)^3 + n\left(\frac{(3n-4)(3n-2)}{3n-4+3n-2-2}\right)^3$$

$$= n(3n-4)^3\left[\left(\frac{3n-4}{6n-10}\right)^3 + \left(\frac{3n-2}{6n-8}\right)^3\right].$$

(b) By using equation (2) and by cardinalities of the 2-distance degree of edge partition of Sf_n, we deduce

$$ALI(Sf_n, x) = \sum_{uv \in E(Sf_n)} x^{\left(\frac{d_2(u)d_2(v)}{d_2(u)+d_2(v)-2}\right)^3}$$

$$= nx^{\left(\frac{0(3n-4)}{0+3n-4-2}\right)^3} + nx^{\left(\frac{0(3n-2)}{0+3n-2-2}\right)^3} + nx^{\left(\frac{0(3n-1)}{0+3n-1-2}\right)^3} + nx^{\left(\frac{(3n-4)(3n-4)}{3n-4+3n-4-2}\right)^3} + nx^{\left(\frac{(3n-4)(3n-2)}{3n-4+3n-2-2}\right)^3}$$

$$= 3nx^0 + nx^{\left(\frac{9n^2-24n+16}{6n-10}\right)^3} + nx^{\left(\frac{9n^2-18n+8}{6n-8}\right)^3}.$$

REFERENCES

1. A.M. Naji, N.D. Soner and I. Gutman, On leap Zagreb indices of graphs, *Commun. Comb. Optim.* 2 (2017) 99-117.
2. B. Furtula, A. Graovac and D. Vukičević, Augmented Zagreb index, *J. Math. Chem.* 48 (2010) 370-380.
3. V.R.Kulli, Some topological indices of certain nanotubes, *Journal of Computer and Mathematical Sciences*, 8(1) (2017) 1-7.
4. V.R. Kulli, Computation of some topological indices of certain networks, *International Journal of Mathematical Archive,* 8(2) (2017) 99-106.
5. V.R.Kulli, On augmented reverse index and its polynomial of certain nanostar dendrimers, *International Journal of Engineering Sciences and Research Technology*, 7(8) (2018) 237-243.
6. V.R. Kulli, On augmented Revan index and its polynomial of certain families of benzenoid systems, *Internationals Journal of Mathematics and its Applications*, 6(4) (2018) 43-50.
7. V.R.Kulli, Computing the F-ve-degree index and its polynomial of dominating oxide and regular triangulate oxide networks, *International Journal of Fuzzy Mathematical Archive*, 16(1) (2018) 1-6.
8. V.R.Kulli, On the square ve-degree index and its polynomial of certain network, *Journal of Global Research in Mathematical Archives,* 5(10) (2018) 1-4.
9. V.R. Kulli, Square reverse index and its polynomial of certain networks, *International Journal of Mathematical Archive*, 9(10) (2018) 22-33.
10. V.R.Kulli, Reduced second hyper-Zagreb index and its polynomial of certain silicate networks, *Journal of Mathematics and Informatics*, 14 (2018) 11-16.
11. V.R. Kulli, On *KV* indices and their polynomials of two families of dendrimers, *International Journal of Current Research in Life Sciences*, 7(9) (2018) 2739-2744.
12. V.R. Kulli, Computing F-reverse index and F-reverse polynomial of certain networks, *International Journal of Mathematical Archive*, 9(8) (2018) 27-33.
13. V.R.Kulli, *F*-Revan index and *F*-Revan polynomial of some families of benzenoid systems, *Journal of Global Research in Mathematical Archives* 5(11) (2018) 1-6.
14. V.R.Kulli, Leap hyper-Zagreb indices and their polynomials of certain graphs, *International Journal of Current Research in Life Sciences*, 7(10) (2018) 2783-2791.

Source of Support: Nil, Conflict of interest: None Declared