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ASSOCIATOR IN THE CENTER OF NONASSOCIATIVE RINGS

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ABSTRACT

We present some results on associators in the center of nonassociative rings. In this paper we show that if R is simple, characteristic $\neq$ 2, 3 and satisfies $(R,(R,R,R))=0$, then R must be either associative or commutative.

Keywords: *Nonassociative ring, center, associator, commutator, characteristic and simple ring.*

1. INTRODUCTION

The great mathematician Thedy whose contribution towards the rings is great ppreciable. He has introduced that rings which satisfy the identity $(R,(R,R,R))=0$ (1)

Now by using his results we show that R is a simple ring of char. $\neq$ 2, 3 and satisfies $(R,(R,R,R))=0$ then R must be either associative or commutative.

2. PRELIMINARIES

In this paper we consider a nonassociative ring R, which satisfies $(R,(R,R,R))=0$ ..(2) Let R be a nonassociative ring. We shall denote commutator and the associator by $(x,y)=xy-yx$ and $(x,y,z)=(xy)z-x(yz)$ for all x,y,z in R respectively. By the center C of R, c in N such that $(c,R)=0$. It is easily verified that N is a subring of R and C is a subring of N. Obviously, we note that $N=R$ if and only if R is an associative ring and $C=R$ if and only if R is associative and commutative. A ring R is said to be char. $\neq n$ if $nx=0 \Rightarrow x=0$, for all $x\in R$ and $n\in N$. A ring R is said to be simple if whenever A is an ideal of R, then either $A=R$ or $A=0$.

3. MAIN RESULTS

In every ring the so called semi-Jacobi identity

$$(xy,z)=x(y,z)+(x,z)y+(x,y,z)+(z,x,y)-(x,z,y) \tag{3}$$

***Lemma* 3.1:** V is an ideal of R.

***Proof*:** Since $V\subset U$, it is sufficient to show that V is a right ideal. Let $v\in V$. Then for all $r,s\in R$, $vr\in U$ follows from the definition of V. Since (2) implies $(v,r,s)\in U$ and $vr.s=(v,r,s)+vr.s\in U$, it follows that $vr\in V$.

***Theorem* 3.1:** *If R is a simple ring of char. $\neq$ 2, 3 and satisfies $(R,(R,R,R))=0$ then R is either associative or commutative.*

***Proof*:** Now very first to prove the theorem, assume that R is not commutative. Hence V is not equals to R.

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Since R is simple and by the lemma.3.1 we reduce the case, where V is equals to zero. Then in every ring the Teichmuller identity is

$$(wx,y,z)-(w,xy,z)+(w,x,yz)=w(x,y,z)+(w,x,y)z \tag{4}$$

It follows that on expanding each side and using the associator definition.

Now by using (2) and every term of (4) is commute by r we have

$$[r,(wx,y,z)]-[r,(w,xy,z)]+[r,(w,x,yz)]=[r,w(x,y,z)]+[r,(w,x,y)z]$$

$$\Rightarrow [r,w(x,y,z)]+[r,(w,x,y)z]=0$$

So that $$[r,w(x,y,z)]=-[r,(w,x,y)z]=-[r,z(w,x,y)]$$

Using (2). By permuting cyclically $(wzyx)$ we get

$$[r,w(x,y,z)]=-[r,z(w,x,y)]=-[r,y(z,w,x)]=-[r,x(y,z,w,)] \tag{5}$$

Let the associator of R is a, which is an arbitrary

Let $y=x$ and $z=a$ in (3) and use (2).

Thus $(x^2,a)=x(x,a)+(x,a)x+(x,x,a)+(a,x,x)-(x,a,x)$

So that $$(x,x,a)+(a,x,x)-(x,a,x)=0 \tag{6}$$

Now by using (6), multiplying with x on left and simultaneously commutating by z.
Then we get

$$(z,x((x,x,a)+(a,x,x)-(x,a,x))=0 \tag{7}$$

Using (5) and (7), we see that

$$-(z,a(x,x,x))-(z,a(x,x,x))-(z,a(x,x,x))=0$$

$$\Rightarrow -3(z,a(x,x,x))=0$$

Thus $$(z,a(x,x,x))=0$$

Now we change a with (b,c,d). Because of an arbitrary associator a, we obtain

$$(z,(b,c,d)(x,x,x))=0 \tag{8}$$

$$(z,(x,x,x)(b,c,d))=0 \tag{9}$$

Applying (5) to (9), we obtain

$$-(z,((x,x,x),b,c)d)=0$$

Then $$-(z,d((x,x,x),b,c))=0$$

$$\Rightarrow (z,d((x,x,x),b,c))=0$$

$$\Rightarrow -(z,c(d,(x,x,x),b))=0$$

$$\Rightarrow (z,b(c,d,(x,x,x)))=0$$

Thus $$\Rightarrow (z,b(c,d,(x,x,x)))=0=(z,c(d,(x,x,x),b))=0=(z,d((x,x,x),b,c)) \tag{10}$$

By using (2) in the above we get

$$(b(c,d,(x,x,x)))=((c,d,(x,x,x))b)$$

But (10) and (2) prove that

$$(c,d,(x,x,x))\in V,\ (d,(x,x,x),b)\in V \text{ and } ((x,x,x),b,c)\in V$$

Since $V=0,(x,x,x)$ must be in the nucleus of R.
This is combined with (2) prove that (x,x,x) is in the center of R.
Next we apply (5) to $(z,x(x,x,x))$.
Thus $(z,x(x,x,x))=-(z,x(x,x,x))$

This leads to $2(z, x(x,x,x)) = 0$

So that $(z, x(x,x,x)) = 0$ (11)

Expanding $(x,(x,x,x),z) = 0$, by using the semi-Jacobi identity we have

$$0 = x((x,x,x),z) + (x,z)(x,x,x) + (x,(x,x,x),z) + (z,x,(x,x,x)) - (x,z,(x,x,x))$$

Which implies that (x,x,x) is in the center. Hence we have left one term and which gives

$$(x,z)(x,x,x) = 0 \tag{12}$$

Now let $z = -x^2$ in (12) we get

$$(x,-x^2)(x,x,x) = 0$$

Since
$$\begin{aligned}(x,-x^2) &= -(x,x^2)\\ &= -(xx^2 - x^2x)\\ &= -x(xx) + (xx)x\\ &= (x,x,x)\end{aligned}$$

We obtain

$$\begin{aligned}&(z, x(x,x,x)) = 0\\ &\Rightarrow (x,-x^2)(x,x,x) = 0\\ &\Rightarrow (x,x,x)(x,x,x) = 0\\ &\Rightarrow (x,x,x)^2 = 0\end{aligned} \tag{13}$$

Let $q = (x,x,x)$.

That is the center element is q.

So from (13) we get $q^2 = 0$.

Now it is clear that the ideal $q^2 = 0$ belongs to R. Which concludes that R is commutative as well as associative.

By our assumption we said that R is not commutative.

That is the ideal generated by q is zero.

$$\Rightarrow q = 0$$

So $q = (x,x,x) = 0$

Hence the proof.

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