



NEW KULLI-BASAVA INDICES OF GRAPHS

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

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ABSTRACT

A topological index or graph index is a numerical parameter mathematically derived from the graph structure. In this paper we introduce the total Kulli-Basava index, modified inverse first Kulli-Basava inverse degree, Kulli-Basava Zeroth order index, F-Kulli-Basava index and general Kulli-Basava index of a graph. Also we introduce the total Kulli-Basava polynomial and F-Kulli-Basava polynomial of a graph. Furthermore, we exact formulas for regular graphs, complete graphs, cycles, wheel graphs, gear graphs and helm graphs.

Keywords: F-Kulli-Basava index, general Kulli-Basava index, regular, wheel, gear, helm graphs.

Mathematics Subject Classification: 05C05, 05C12, 05C35.

1. INTRODUCTION

Let $G = (V(G), E(G))$ be a finite, simple, connected graph. The degree $dG(v)$ of a vertex v is the number of edges incident to v . The degree of an edge $e = uv$ in a graph G is defined by $dG(e) = dG(u) + dG(v) - 1$. The set of all edges incident to v is called the edge neighborhood of v and denoted by $Ne(v)$. Let $S_e(v)$ denote the sum of degrees of all edges incident to a vertex v . For undefined term and notation, we refer the reader to [1].

A topological index or graph index is a numerical parameter mathematically derived from the graph structure. The Graph indices have their applications in various disciplines of Science and Technology.

The modified first Kulli-Basava index was defined by Basavanagoud *et.al* [2], as

$$KB_1^*(G) = \sum_{u \in V(G)} S_e(u)^2.$$

In [3], Kulli introduced the modified first Kulli-Basava polynomial of a graph, defined as

$$KB_1^*(G, x) = \sum_{u \in V(G)} x^{S_e(u)^2}.$$

Recently, the hyper Kulli-Basava indices [3], connectivity Kulli-Basava indices [4], square Kulli-Basava index [5], multiplicative Kulli-Basava and multiplicative hyper Kulli-Basava indices [6] were introduced and studied.

We propose the following the Kulli-Basava indices:

The total Kulli-Basava index of a graph G is defined as

$$TKB(G) = \sum_{u \in V(G)} S_e(u).$$

The modified inverse first Kulli-Basava index of a graph G is defined as

$${}^m KB_1^*(G) = \sum_{u \in V(G)} \frac{1}{S_e(u)^2}.$$

Corresponding Author: V. R. Kulli*

Department of Mathematics, Gulbarga University, Gulbarga - 585106, India.

The Kulli-Basava inverse degree of a graph G is defined as

$$KBID(G) = \sum_{u \in V(G)} \frac{1}{S_e(u)}.$$

The Kulli-Basava zeroth order index of a graph G is defined as

$$KBZ(G) = \sum_{u \in V(G)} \frac{1}{\sqrt{S_e(u)}}$$

The F -Kulli-Basava index of a graph G is defined as

$$FKB(G) = \sum_{u \in V(G)} S_e(u)^3.$$

The general Kulli-Basava index of a graph G is defined as

$$KB^a(G) = \sum_{u \in V(G)} S_e(u)^a, \tag{1}$$

where a is a real number.

We also introduce the total Kulli-Basava polynomial and F -Kulli-Basava Polynomial of a graph, defined as

$$TKB(G, x) = \sum_{u \in V(G)} x^{S_e(u)}, \tag{2}$$

$$FKB(G, x) = \sum_{u \in V(G)} x^{S_e(u)^3}. \tag{3}$$

In recent years, some topological indices were studied, for example, in [7, 8, 9, 10, 11, 12, 13, 14].

In this study, we derive explicit formulas for computing the total Kulli-Basava index, modified inverse first Kulli-Basava index, Kulli-Basava inverse degree, Kulli-Basava zeroth order index, F -Kulli-Basava index and general Kulli-Basava index of regular, wheel, gear, helm graphs, Also we compute the total Kulli-Basava Polynomial and F -Kulli-Basava polynomial of regular, wheel, gear and helm graphs. For wheel, gear, helm graphs, see [15]

2. RESULTS FOR REGULAR GRAPHS

A graph G is r -regular if the degree of each vertex of G is r .

Theorem 1: Let G be an r -regular graph with n vertices and m edges. Then the general Kulli-Basava index of G is

$$KB^a(G) = n[2r(r-1)]^a. \tag{4}$$

Proof: Let G be an r -regular graph with n vertices and m edges. Then $m = \frac{nr}{2}$, $S_e(u) = 2r(r-1)$ for each vertex u of G . Thus

$$KB^a(G) = \sum_{u \in V(G)} S_e(u)^a = n[2r(r-1)]^a.$$

Corollary 1.1: If G is an r -regular graph with n vertices, then

$$(i) \quad TKB(G) = 2nr(r-1). \quad (ii) \quad {}^m KB_1^*(G) = \frac{n}{4r^2(r-1)^2}.$$

$$(iii) \quad KBID(G) = \frac{n}{2r(r-1)}. \quad (iv) \quad KBZ(G) = \frac{n}{\sqrt{2r(r-1)}}.$$

$$(v) \quad FKB(G) = 8nr^3(r-1)^3.$$

Proof: Put $a = 1, -2, -1, -\frac{1}{2}, 3$ in equation (4), we get the desired results.

Corollary 1.2: If K_n is a complete graph with n vertices then

$$(i) \quad TKB(K_n) = 2n(n-1)(n-2). \quad (ii) \quad {}^m KB_1^*(K_n) = \frac{n}{4(n-1)^2(n-2)^2}.$$

$$(iii) \quad KBID(K_n) = \frac{n}{2(n-1)(n-2)}. \quad (iv) \quad KBZ(K_n) = \frac{n}{\sqrt{2(n-1)(n-2)}}.$$

$$(v) \quad FKB(K_n) = 8n(n-1)^3(n-2)^3.$$

Proof: Put $r = n - 1$ and $a = 1, -1, -\frac{1}{2}, 3$ in equation (4), we obtain the desired results.

Corollary 1.3: If C_n is a cycle with n vertices, then

- (i) $TKB(C_n) = 4n.$ (ii) ${}^m KB_1^*(C_n) = \frac{n}{16}.$
- (iii) $KBID(C_n) = \frac{n}{4}.$ (iv) $KBZ(C_n) = \frac{n}{2}.$
- (v) $FKB(C_n) = 64n.$

Proof: Put $r = 2$, Also put $a = 1, -2, -1, -\frac{1}{2}, 3$ in equation (4), we get the desired results.

Theorem 2: Let G be an r -regular graph with n vertices. Then

- (i) $TKB(G, x) = nx^{2r(r-1)}.$ (ii) $FKB(G, x) = nx^{8r^3(r-1)^3}.$

Proof: Let G be an r -regular graph with n vertices. Then $S_e(u) = 2r(r - 1)$ for each vertex u of G . Thus

- (i) $TKB(G, x) = \sum_{u \in V(G)} x^{S_e(u)} = nx^{2r(r-1)}.$ (ii) $FKB(G, x) = \sum_{u \in V(G)} x^{S_e(u)^3} = nx^{8r^3(r-1)^3}.$

Corollary 2.1: Let K_n be a complete graph with n vertices. Then

- (i) $TKB(K_n, x) = nx^{2(n-1)(n-2)}.$ (ii) $FKB(K_n, x) = nx^{8(n-1)^3(n-2)^3}.$

Corollary 2.2: Let C_n be a cycle with n vertices. Then

- (i) $TKB(C_n, x) = nx^4.$ (ii) $FKB(C_n, x) = nx^{64}.$

3. RESULTS FOR WHEEL GRAPHS

A wheel W_n is the join of C_n and K_1 . Clearly W_n has $n+1$ vertices and $2n$ edges. The vertices of C_n are called rim vertices and the vertex of K_1 is called apex. A graph W_n is shown in Figure 1.

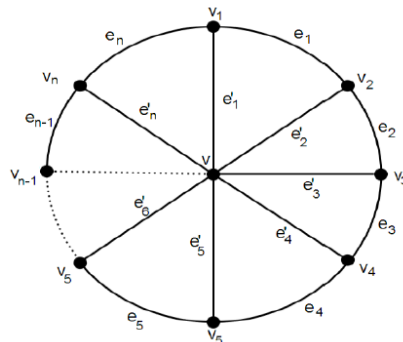


Figure-1: Wheel W_n

Lemma 3: Let W_n be a wheel with $n+1$ vertices and $2n$ edges. Then W_n has two types of vertices as given below:

$$V_1 = \{u \in V(W_n) \mid S_e(u) = n(n+1)\}, \quad |V_1| = 1.$$

$$V_2 = \{u \in V(W_n) \mid S_e(u) = n+9\}, \quad |V_2| = n.$$

Theorem 4: The general Kulli-Basava index of a wheel W_n is

$$KB^a(W_n) = [n(n+1)]^a + n(n+9)^a. \tag{5}$$

Proof: Let W_n be a wheel with $n + 1$ vertices and $2n$ edges. Then by using equation (1) and Lemma 3, we obtain

$$KB^a(W_n) = \sum_{u \in V(W_n)} S_e(u)^a$$

$$= 1 \times [n(n+1)]^a + n(n+9)^a = [n(n+1)]^a + n(n+9)^a.$$

Theorem 5: Let W_n be a wheel with $n+1$ vertices and $2n$ edges. Then

(i) $TKB(W_n, x) = x^{n(n+1)} + nx^{n+9}$. (ii) $FKB(W_n, x) = x^{n^3(n+1)^3} + nx^{(n+9)^3}$.

Proof: (i) from equation (2) and by using Lemma (30), we deduce

$$TKB(W_n, x) = \sum_{u \in V(W_n)} x^{S_e(u)} = 1 \times x^{2n(n+1)} + n \times x^{n+9} = x^{n(n+1)} + nx^{n+9}$$

(ii) By using equation (3) and Lemma 3, we have

$$FKB(W_n, x) = \sum_{u \in V(W_n)} x^{S_e(u)^3} = x^{n^3(n+1)^3} + nx^{(n+9)^3}$$

4. RESULTS FOR GEAR GRAPHS

A gear graph G_n is a graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices. Clearly G_n has $2n+1$ vertices and $3n$ edges. A gear graph G_n is presented in Figure 2.

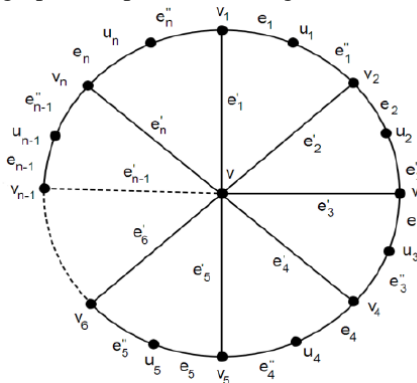


Figure-2: Gear graph G_n

Lemma 6: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then G_n has three types of vertices as follows:

$$\begin{aligned} V_1 &= \{u \in V(G_n) \mid S_e(u) = n(n+1)\}, & |V_1| &= 1. \\ V_2 &= \{u \in V(G_n) \mid S_e(u) = n+7\}, & |V_2| &= n. \\ V_3 &= \{u \in V(G_n) \mid S_e(u) = 6\}, & |V_3| &= n. \end{aligned}$$

Theorem 7: The general Kulli-Basava index of a gear graph G_n , is given by

$$KB^a(G_n) = [n(n+1)]^a + n(n+7)^a + 6^a n. \tag{6}$$

Proof: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then from equation (1) and by using Lemma 6, we deduce

$$\begin{aligned} KB^a(G_n) &= \sum_{u \in V(G_n)} S_e(u)^a = |V_1| \times [n(n+1)]^a + |V_2| \times (n+7)^a + |V_3| 6^a \\ &= [n(n+1)]^a + n(n+7)^a + n6^a. \end{aligned}$$

Corollary 7.1: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then

(i) $TKB(G_n) = 2n^2 + 14n$. (ii) ${}^m KB_1^*(G_n) = \frac{1}{n^2(n+1)^2} + \frac{n}{(n+7)^2} + \frac{n}{36}$

(iii) $KBID(G_n) = \frac{1}{n(n+1)} + \frac{n}{n+7} + \frac{n}{6}$. (iv) $KBZ(G_n) = \frac{1}{\sqrt{n(n+2)}} + \frac{n}{\sqrt{n+7}} + \frac{n}{\sqrt{6}}$

(v) $FKB(G_n) = n^3(n+1)^3 + n(n+7)^3 + 216n$.

Proof: Put $a = 1, -2, -1, \frac{1}{2}, 3$ in equation (6) we get the desired results.

Theorem 8: Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges. Then

$$(i) \quad TKB(G_n, x) = x^{n(n+1)} + nx^{n+7} + nx^6. \quad (ii) \quad FKB(G_n, x) = x^{n^3(n+1)^3} + nx^{(n+7)^3} + nx^{216}.$$

Proof:

(i) By using equation (2) and Lemma 6, we derive

$$\begin{aligned} TKB(G_n, x) &= \sum_{u \in V(G_n)} x^{S_e(u)} = |V_1| \times x^{n(n+1)} + |V_2| \times x^{n+7} + |V_3| \times x^6 \\ &= x^{n(n+1)} + nx^{n+7} + nx^6. \end{aligned}$$

(ii) By using equation (3) and Lemma 6, we deduce

$$\begin{aligned} FKB(G_n, x) &= \sum_{u \in V(G_n)} x^{S_e(u)^3} = |V_1| \times x^{n^3(n+1)^3} + |V_2| \times x^{(n+7)^3} + |V_3| \times x^{6^3} \\ &= x^{n^3(n+1)^3} + nx^{(n+7)^3} + nx^{216}. \end{aligned}$$

5. RESULTS FOR HELM GRAPHS

A helm graph, denoted by H_n , is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly H_n has $m+1$ vertices and $3n$ edges. A graph H_n is depicted in Figure 3.

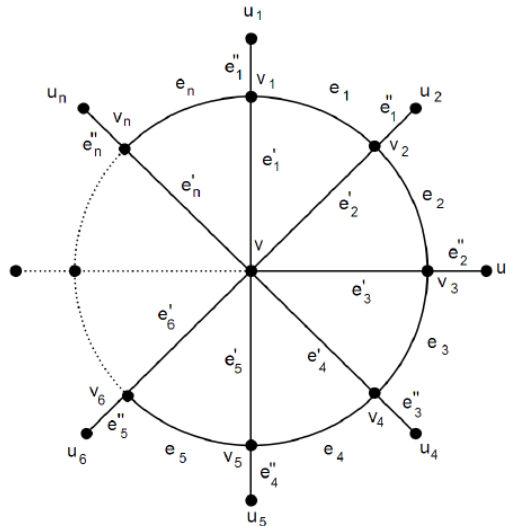


Figure-3: Helm graph H_n

Lemma 9: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then H_n has three types of vertices as follows:

$$\begin{aligned} V_1 &= \{u \in V(H_n) \mid S_e(u) = n(n+2)\}, & |V_1| &= 1. \\ V_2 &= \{u \in V(H_n) \mid S_e(u) = n+17\}, & |V_2| &= n. \\ V_3 &= \{u \in V(H_n) \mid S_e(u) = 3\}, & |V_3| &= n. \end{aligned}$$

Theorem 10: The general Kulli-Basava index of a helm graph H_n is given by

$$KB^a(H_n) = [n(n+2)]^a + n(n+17)^a + 3^a n. \quad (7)$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. From equation (1) and by using Lemma 9, we derive

$$\begin{aligned} KB^a(H_n) &= \sum_{u \in V(H_n)} S_e(u)^a = |V_1| \times [n(n+2)]^a + |V_2| \times (n+17)^a + |V_3| \times 3^a \\ &= [n(n+2)]^a + n(n+17)^a + n3^a. \end{aligned}$$

Corollary 10.1: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

$$(i) \quad TKB(H_n) = 2n^2 + 22n. \quad (ii) \quad {}^m KB_1^*(H_n) = \frac{1}{n^2(n+2)^2} + \frac{n}{(n+17)^2} + \frac{n}{9}$$

$$(iii) \quad KBID(H_n) = \frac{1}{n(n+2)} + \frac{n}{n+17} + \frac{n}{3}. \quad (iv) \quad KBZ(H_n) = \frac{1}{\sqrt{n(n+2)}} + \frac{n}{\sqrt{n+17}} + \frac{n}{\sqrt{3}}$$

$$(v) \quad FKBZ(H_n) = n^3(n+2)^3 + n(n+17)^3 + 27n.$$

Proof: Put $a = 1, -2, -1, -\frac{1}{2}, 3$ in equation (7) we obtain desired results.

Theorem 11: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

- (i) $TKB(H_n, x) = x^{n(n+2)} + nx^{n+17} + nx^3.$
- (ii) $FKB(H_n, x) = x^{n^3(n+2)^3} + nx^{(n+17)^3} + nx^{27}.$

Proof: (i) From equation (2) and using Lemma 9, we obtain

$$TKB(H_n, x) = \sum_{u \in V(H_n)} x^{S_e(u)} = |V_1| \times x^{n(n+2)} + |V_2| \times x^{n+17} + |V_3| \times x^3$$

$$= x^{n(n+2)} + nx^{n+17} + nx^3.$$

(ii) By using equation (3) and Lemma 9, we have

$$FKB(H_n, x) = \sum_{u \in V(H_n)} x^{S_e(u)^3} = |V_1| \times x^{n^3(n+2)^3} + |V_2| \times x^{(n+17)^3} + |V_3| \times x^{3^3}$$

$$= x^{n^3(n+2)^3} + nx^{(n+17)^3} + nx^{27}.$$

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