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NEW KULLI-BASAVA INDICES OF GRAPHS

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ABSTRACT

A topological index or graph index is a numerical parameter mathematically derived from the graph structure. In this paper we introduce the total Kulli-Basava index, modified inverse first Kulli-Basava inverse degree, Kulli-Basava Zeroth order index, F-Kulli-Basava index and general Kulli-Basava index of a graph. Also we introduce the total Kulli-Basava polynomial and F-Kulli-Basava polynomial of a graph. Furthermore, we exact formulas for regular graphs, complete graphs, cycles, wheel graphs, gear graphs and helm graphs.

Keywords: *F-Kulli-Basava index, general Kulli-Basava index, regular, wheel, gear, helm graphs.*

Mathematics Subject Classification: *05C05, 05C12, 05C35.*

1. INTRODUCTION

Let $G = (V(G), E(G)$ be a finite, simple, connected graph. The degree $dG(v)$ of a vertex v is the number of edges incident to v. The degree of an edge $e = uv$ in a graph G is defined by $dG(e) = dG(u) + dG(v) – 1$. The set of all edges incident to v is called the edge neighborhood of v and denoted by $N_{e(v)}$. Let $S_{e(v)}$ denote the sum of degrees of all edges incident to a vertex v. For undefined term and notation, we refer the reader to [1].

A topological index or graph index is a numerical parameter mathematically derived from the graph structure. The Graph indices have their applications in various disciplines of Science and Technology.

The modified first Kulli-Basava index was defined by Basavanagoud *et.al* [2], as

$$KB_1^*(G) = \sum_{u\in V(G)} S_e(u)^2.$$

In [3], Kulli introduced the modified first Kulli-Basava polynomial of a graph, defined as

$$KB_1^*(G,x) = \sum_{u\in V(G)} x^{S_e(u)^2}.$$

Recently, the hyper Kulli-Basava indices [3], connectivity Kulli-Basava indices [4], square Kulli-Basava index [5], multiplicative Kulli-Basava and multiplicative hyper Kulli-Basava indices [6] were introduced and studied.

We propose the following the Kulli-Basava indices:

The total Kulli-Basava index of a graph G is defined as

$$TKB(G) = \sum_{u\in V(G)} S_e(u).$$

The modified inverse first Kulli-Basava index of a graph G is defined as

$${}^m KB_1^*(G) = \sum_{u\in V(G)} \frac{1}{S_e(u)^2}.$$

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The Kulli-Basava inverse degree of *a* graph *G* is defined as

$$KBID(G)=\sum_{u\in V(G)}\frac{1}{S_e(u)}.$$

The Kulli-Basava zeroth order index of *a* graph *G* is defined as

$$KBZ(G)=\sum_{u\in V(G)}\frac{1}{\sqrt{S_e(u)}}$$

The *F*-Kulli-Basava index of *a* graph *G* is defined as

$$FKB(G)=\sum_{u\in V(G)}S_e(u)^3.$$

The general Kulli-Basava index of *a* graph *G* is defined as

$$KB^a(G)=\sum_{u\in V(G)}S_e(u)^a, \qquad (1)$$

where *a* is *a* real number.

We also introduce the total Kulli-Basava polynomial and *F*-Kulli-Basava Polynomial of *a* graph, defined as

$$TKB(G,x)=\sum_{u\in V(G)}x^{S_e(u)}, \qquad (2)$$

$$FKB(G,x)=\sum_{u\in V(G)}x^{S_e(u)^3}. \qquad (3)$$

In recent years, some topological indices were studied, for example, in [7, 8, 9, 10, 11, 12, 13, 14].

In this study, we derive explicit formulas for computing the total Kulli-Basava index, modified inverse first Kulli-Basava index, Kulli-Basava inverse degree, Kulli-Basava zeroth order index, *F*-Kulli-Basava index and general Kulli-Basava index of regular, wheel, gear, helm graphs, Also we compute the total Kulli-Basava Polynomial and *F*-Kulli-Basava polynomial of regular, wheel, gear and helm graphs. For wheel, gear, helm graphs, see [15]

2. RESULTS FOR REGULAR GRAPHS

A graph *G* is *r*-regular if the degree of each vertex of *G* is *r*.

Theorem 1: Let *G* be an *r*-regular graph with *n* vertices and *m* edges. Then the general Kulli-Basava index of *G* is

$$KB^a(G)=n\left[2r(r-1)\right]^a. \qquad (4)$$

Proof: Let *G* be an *r*- regular graph with *n* vertices and *m* edges. Then $m=\frac{nr}{2}$, *Se*(*u*) = 2*r*(*r* – 1) for each vertex *u* of *G*. Thus

$$KB^a(G)=\sum_{u\in V(G)}S_e(u)^a=n\left[2r(r-1)\right]^a.$$

Corollary 1.1: If *G* is an *r*-regular graph with *n* vertices, then

(i) $TKB(G)=2nr(r-1).$ (ii) ${}^{m}KB_1^*(G)=\frac{n}{4r^2(r-1)^2}.$

(iii) $KBID(G)=\frac{n}{2r(r-1)}.$ (iv) $KBZ(G)=\frac{n}{\sqrt{2r(r-1)}}.$

(v) $FKB(G)=8nr^3(r-1)^3.$

Proof: Put *a* = 1, –2, –1, $-\frac{1}{2}$, 3 in equation (4), we get the desired results.

Corollary 1.2: If K_n is *a* complete graph with *n* vertices then

(i) $TKB(K_n)=2n(n-1)(n-2).$ (ii) ${}^{m}KB_1^*(K_n)=\frac{n}{4(n-1)^2(n-2)^2}.$

(iii) $KBID(K_n)=\frac{n}{2(n-1)(n-2)}.$ (iv) $KBZ(K_n)=\frac{n}{\sqrt{2(n-1)(n-2)}}.$

(v) $FKB(K_n)=8n(n-1)^3(n-2)^3.$

Proof: Put $r = n - 1$ and $a = 1, -1, -\frac{1}{2}, 3$ in equation (4), we obtain the desired results.

Corollary 1.3: If C_n is *a* cycle with *n* vertices, then

(i) $TKB(C_n) = 4n.$ (ii) ${}^{m}KB_1^*(C_n) = \frac{n}{16}.$

(iii) $KBID(C_n) = \frac{n}{4}.$ (iv) $KBZ(C_n) = \frac{n}{2}.$

(v) $FKB(C_n) = 64n.$

Proof: Put $r = 2$, Also put $a = 1, -2, -1, -\frac{1}{2}, 3$ in equation (4), we get the desired results.

Theorem 2: Let *G* be an *r*-regular graph with *n* vertices. Then

(i) $TKB(G,x) = nx^{2r(r-1)}.$ (ii) $FKB(G,x) = nx^{8r^3(r-1)^3}.$

Proof: Let *G* be an *r*-regular graph with *n* vertices. Then $S_e(u) = 2r(r-1)$ for each vertex *u* of *G*. Thus

(i) $TKB(G,x) = \sum_{u\in V(G)} x^{S_e(u)} = nx^{2r(r-1)}.$ (ii) $FKB(G,x) = \sum_{u\in V(G)} x^{S_e(u)^3} = nx^{8r^3(r-1)^3}.$

Corollary 2.1: Let K_n be *a* complete graph with *n* vertices. Then

(i) $TKB(K_n,x) = nx^{2(n-1)(n-2)}.$ (ii) $FKB(K_n,x) = nx^{8(n-1)^3(n-2)^3}.$

Corollary 2.2: Let C_n be *a* cycle with *n* vertices. Then

(i) $TKB(C_n,x) = nx^4.$ (ii) $FKB(C_n,x) = nx^{64}.$

3. RESULTS FOR WHEEL GRAPHS

A wheel W_n is the join of C_n and K_1. Clearly W_n has $n+1$ vertices and $2n$ edges. The vertices of C_n are called rim vertices and the vertex of K_1 is called apex. A graph W_n is shown in Figure 1.

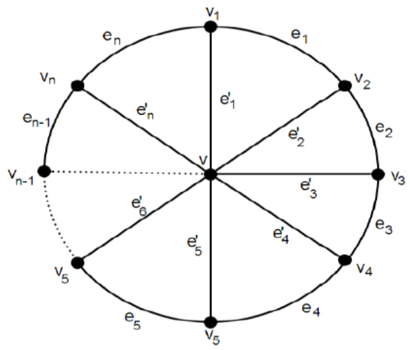

Figure-1: Wheel W_n

Lemma 3: Let W_n be a wheel with $n+1$ vertices and $2n$ edges. Then W_n has two types of vertices as given below:

$V_1 = \{u \in V(W_n) \mid S_e(u) = n(n+1)\}$, $|V_1| = 1.$

$V_2 = \{u \in V(W_n) \mid S_e(u) = n+9\}$, $|V_2| = n.$

Theorem 4: The general Kulli-Basava index of *a* wheel W_n is

$$KB^a(W_n) = [n(n+1)]^a + n(n+9)^a. \tag{5}$$

Proof: Let W_n be a wheel with $n+1$ vertices and $2n$ edges. Then by using equation (1) and Lemma 3, we obtain

$$KB^a(W_n) = \sum_{u\in V(W_n)} S_e(u)^a$$

$$= 1\times[n(n+1)]^a + n(n+9)^a = [n(n+1)]^a + n(n+9)^a.$$

Theorem 5: Let W_n be a wheel with n+1 vertices and 2n edges. Then

(i) $TKB(W_n, x) = x^{n(n+1)} + nx^{n+9}$. (ii) $FKB(W_n, x) = x^{n^3(n+1)^3} + nx^{(n+9)^3}$.

Proof: (i) from equation (2) and by using Lemma (30), we deduce

$$TKB(W_n, x) = \sum_{u \in V(W_n)} x^{S_e(u)} = 1 \times x^{2n(n+1)} + n \times x^{n+9} = x^{n(n+1)} + nx^{n+9}$$

(ii) By using equation (3) and Lemma 3, we have

$$FKB(W_n, x) = \sum_{u \in V(W_n)} x^{S_e(u)^3} = x^{n^3(n+1)^3} + nx^{(n+9)^3}$$

4. RESULTS FOR GEAR GRAPHS

A gear graph G_n is a graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices. Clearly G_n has 2n+1 vertices and 3n edges. A gear graph G_n is presented in Figure 2.

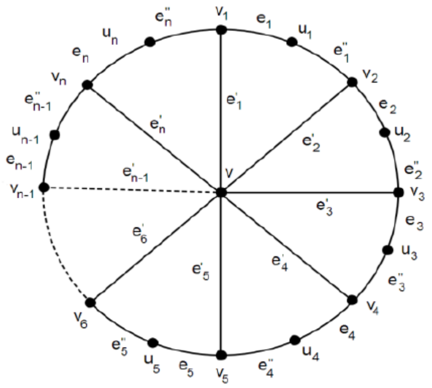

Figure-2: Gear graph G_n

Lemma 6: Let G_n be a gear graph with 2n+1 vertices and 3n edges. Then G_n has three types of vertices as follows:

$V_1 = \{u \in V(G_n) \mid S_e(u) = n(n+1)\}$, $|V_1| = 1$.
$V_2 = \{u \in V(G_n) \mid S_e(u) = n+7\}$, $|V_2| = n$.
$V_3 = \{u \in V(G_n) \mid S_e(u) = 6\}$, $|V_3| = n$.

Theorem 7: The general Kulli-Basava index of a gear graph G_n, is given by

$$KB^a(G_n) = [n(n+1)]^a + n(n+7)^a + 6^a n. \tag{6}$$

Proof: Let G_n be a gear graph with 2n+1 vertices and 3n edges. Then from equation (1) and by using Lemma 6, we deduce

$$KB^a(G_n) = \sum_{u \in V(G_n)} S_e(u)^a = |V_1| \times [n(n+1)]^a + |V_2| \times (n+7)^a + |V_3| 6^a$$

$$= [n(n+1)]^a + n(n+7)^a + n6^a.$$

Corollary 7.1: Let G_n be a gear graph with 2n+1 vertices and 3n edges. Then

(i) $TKB(G_n) = 2n^2 + 14n$. (ii) ${}^m KB_1^*(G_n) = \dfrac{1}{n^2(n+1)^2} + \dfrac{n}{(n+7)^2} + \dfrac{n}{36}$

(iii) $KBID(G_n) = \dfrac{1}{n(n+1)} + \dfrac{n}{n+7} + \dfrac{n}{6}$. (iv) $KBZ(G_n) = \dfrac{1}{\sqrt{n(n+2)}} + \dfrac{n}{\sqrt{n+7}} + \dfrac{n}{\sqrt{6}}$

(v) $FKB(G_n) = n^3(n+1)^3 + n(n+7)^3 + 216n.$

Proof: Put a = 1, –2, –1, $-\dfrac{1}{2}$, 3 in equation (6) we get the desired results.

Theorem 8: Let G_n be *a* gear graph with $2n+1$ vertices and $3n$ edges. Then

(i) $TKB(G_n, x) = x^{n(n+1)} + nx^{n+7} + nx^6.$ (ii) $FKB(G_n, x) = x^{n^3(n+1)^3} + nx^{(n+7)^3} + nx^{216}.$

Proof:

(i) By using equation (2) and Lemma 6, we derive

$$TKB(G_n, x) = \sum_{u \in V(G_n)} x^{S_e(u)} = |V_1| \times x^{n(n+1)} + |V_2| \times x^{n+7} + |V_3| x^6$$

$$= x^{n(n+1)} + nx^{n+7} + nx^6.$$

(ii) By using equation (3) and Lemma 6, we deduce

$$FKB(G_n, x) = \sum_{u \in V(G_n)} x^{S_e(u)^3} = |V_1| \times x^{n^3(n+1)^3} + |V_2| \times x^{(n+7)^3} + |V_3| x^{6^3}$$

$$= x^{n^3(n+1)^3} + nx^{(n+7)^3} + nx^{216}.$$

5. RESULTS FOR HELM GRAPHS

A helm graph, denoted by H_n, is *a* graph obtained from W_n by attaching an end edge to each rim vertex Clearly H_n has m +1 vertices and $3n$ edges. A graph H_n is depicted in Figure 3.

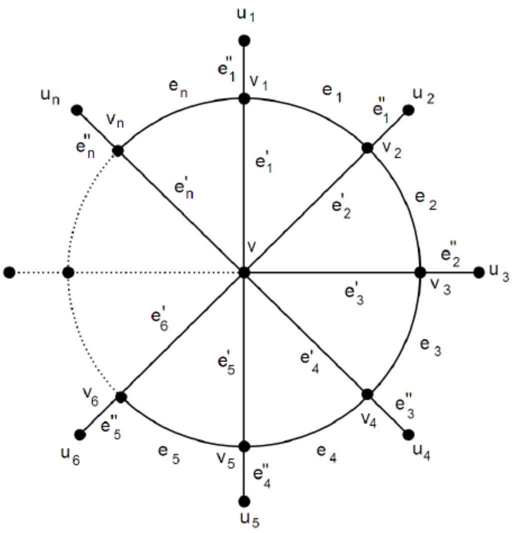

Figure-3: Helm graph H_n

Lemma 9: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then H_n has three types of vertices as follows:

$V_1 = \{u \in V(H_n) \mid S_e(u) = n(n+2)\}$, $|V_1| = 1$.

$V_2 = \{u \in V(H_n) \mid S_e(u) = n+17\}$, $|V_2| = n$.

$V_3 = \{u \in V(H_n) \mid S_e(u) = 3\}$, $|V_3| = n$.

Theorem 10: The general Kulli-Basava index of a helm graph H_n is given by

$$KB^a(H_n) = [n(n+2)]^a + n(n+17)^a + 3^a n. \tag{7}$$

Proof: Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges From equation (1) and by using Lemma 9, we derive

$$KB^a(H_n) = \sum_{u \in V(H_n)} S_e(u)^a = |V_1| \times [n(n+2)]^a + |V_2| \times (n+17)^a + |V_3| 3^a$$

$$= [n(n+2)]^a + n(n+17)^a + n3^a.$$

Corollary 10.1: Let H_n be *a* helm graph with 2n+1 vertices and 3n edges. Then

(i) $TKB(H_n) = 2n^2 + 22n.$ (ii) $^{m}KB_1^*(H_n) = \dfrac{1}{n^2(n+2)^2} + \dfrac{n}{(n+17)^2} + \dfrac{n}{9}$

(iii) $KBID(H_n) = \dfrac{1}{n(n+2)} + \dfrac{n}{n+17} + \dfrac{n}{3}.$ (iv) $KBZ(H_n) = \dfrac{1}{\sqrt{n(n+2)}} + \dfrac{n}{\sqrt{n+17}} + \dfrac{n}{\sqrt{3}}$

(v) $FKBZ(H_n) = n^3(n+2)^3 + n(n+17)^3 + 27n.$

Proof: Put a = 1, –2, –1, $-\frac{1}{2}$, 3 in equation (7) we obtain desired results.

Theorem 11: Let H_n be a helm graph with $2n$+1 vertices and $3n$ edges. Then

(i) $TKB\left(H_n, x\right) = x^{n(n+2)} + nx^{n+17} + nx^3.$

(ii) $FKB\left(H_n, x\right) = x^{n^3(n+2)^3} + nx^{(n+17)^3} + nx^{27}.$

Proof: (i) From equation (2) and using Lemma 9, we obtain

$$TKB\left(H_n, x\right) = \sum_{u \in V(H_n)} x^{S_e(u)} = |V_1| \times x^{n(n+2)} + |V_2| \times x^{n+17} + |V_3| x^3$$

$$= x^{n(n+2)} + nx^{n+17} + nx^3.$$

(ii) By using equation (3) and Lemma 9, we have

$$FKB\left(H_n, x\right) = \sum_{u \in V(H_n)} x^{S_e(u)^3} = |V_1| \times x^{n^3(n+1)^3} + |V_2| \times x^{(n+17)^3} + |V_3| x^{3^3}$$

$$= x^{n^3(n+2)^3} + nx^{(n+17)^3} + nx^{27}.$$

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