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NEW STRUCTURE PROPERTIES OF FLEXIBLE Q-FUZZY GROUPS AND FLEXIBLE NORMAL Q-FUZZY SUBGROUPS

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ABSTRACT

The study introduces the further concept of flexible fuzzy set and flexible Q-fuzzy subset. Based on this, the further concept of flexible fuzzy group and flexible normal Q-fuzzy subgroups are given, using these definitions some elementary properties are discussed and derived. We shall also extend some results on this paper.

Keywords: *Fuzzy set, flexible fuzzy subset, flexible Q-fuzzy subset, flexible Q-fuzzy group and flexible Q-fuzzy normal subgroups.*

SECTION-1: INTRODUCTION

Zadeh (1965) first introduced the fuzzy set concepts and fuzzy set operations. Rosenfeld (1971) then introduced the elementaryconcepts of fuzzy subgroups, which was the first fuzzification of any algebraic structures. Motivated by this many mathematicians started to review the various concepts and a notion of different fuzzy algebraic structures such as fuzzy ideal in both ring and semi ring etc. Zadeh (1975) introduced the concepts of interval-valued fuzzy set, where the values of member instead of the real points. Motivated by this, Zadeh's definition has been generalized by Anthony and Sherwood (1979). They introduced the concept of fuzzy normal subgroup. Also Mukherjee and Bhattacharya (1986) studied the normal fuzzy groups and fuzzy cosets. On the other hand, the notion of a fuzzy subgroup abelian group was introduced by Murali and Makamba [2006], who counted the number of fuzzy subgroups in an abelian group of order $p^n q$ where p, n, q are positive integers. On the other, Solairaju and Natarajan (2011) have introduced the notion of Q-fuzzy subgroups and upper Q-fuzzy order. In this study, based on the reference of Sarangapani and Muruganantham (2016) the concept of flexible Q-fuzzy groups and flexible normal Q-fuzzy groups are given and some of its elementary properties are discussed and derived. Throughout this paper, G denotes the group and e denotes the identity element of G

SECTION-2: PRELIMINARIES AND DEFINITIONS

The basic definitions and notations are presented now in a flexible Q-fuzzy group.

2.1 Definition: Let X be a set. Then the mapping $X: G \to [0, 1]$ is called a fuzzy subset of X

2.2 Definition: For any Q-fuzzy set A in a group G and $t \in [0, 1]$, the set $U(A: t) = \{x \in G: A(x, q) \geq t$, for all $q \in Q\}$ which is called a cut-set of A.

2.3 Definition: Let G be a group. A mapping $A: G \to [0, 1]$ is a fuzzy group of G if
(1). $A(xy) \leq \max\{A(x), A(y)\}$; (2). $A(x^{-1}) \leq A(x)$ for all $x, y \in G$.

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2.4 Definition: The Q-fuzzy set A is called Q-fuzzy group of G if (QFG1): $A(xy, q) \geq \min\{A(x, q), A(y, q)\}$; (QFG2): $A(x^{-1}, q) = A(x, q)$; (QFG3): $A(x, q) = 1$ for all x, y Є G and q Є G.

2.5 Theorem: If μ is a Q-fuzzy group of a group G having identity e, then **(**i) $\mu(x^{-1}, q) = \mu(x, q)$ and (ii) $\mu(e, q) \leq \mu(x, q)$ for all x Є G.

2.6 Definition: Let μ be any Q-fuzzy group of G. Then μ is called a Q-fuzzy normal group if $\mu(xy, q) = \mu(yx, q)$ for all x, y ЄG.

2.7 Definition: The T-norm is a function that maps in the pair of number in the unit interval into the unit interval, which is given as T: $[0, 1] \times [0, 1] \rightarrow [0, 1]$. The following four conditions are hold hood for all $a_1, a_2, a_3, a_4 \in [0, 1]$.

i. $T(a_1, a_2) = T(a_2, a_1)$
ii. $T(a_1, T(a_2, a_3)) = T(T(a_1, a_2), a_3)$
iii. $T(a_1, 1) = T(1, a_1) = 1$
iv. If $a_1 \leq a_3$ and $a_2 \leq a_4$ then $T(a_1, a_2) \leq T(a_3, a_4)$.

Note: The T-norm is a minimum norm

2.7 Definition: Let X be a set. Then a mapping $\mu: X \times Q \rightarrow Q^*([0, 1])$ is called flexible fuzzy subset of X, where $Q^*([0, 1])$ denotes the set of all non-empty subsets of the interval [0,1].

2.8 Definition: Let X be a non-empty set and M, N be two flexible Q-fuzzy subsets of X. Then the intersection of M and N denoted by $(M \cap N)$ and is defined by $M \cap N = \{\min \{a, b\}/ a \in M(x), b \in N(x)\}$ for all $x \in X$. The union of M and N denoted by $(M \cup N)$ and is defined by $M \cup N = \{\max \{a, b\}/ a \in M(x), b \in N(x)\}$ for all $x \in X$.

2.9 Definition: Let X be a groupoid (it is a set which is closed under a binary relation, multiplicatively). A mapping is called a flexible Q-fuzzy groupoid if the following conditions are hold hood:$\inf \mu(xy, q) \geq T\{\inf \mu(x, q), \inf \mu(y,q)\}$ and $\sup \mu(xy, q) \geq T\{\sup \mu(x, q), \sup \mu(y, q)\}$ for all $x, y \in X$.

2.10 Definition: Let G be any group. A mapping $\mu: G \times Q \rightarrow Q^*([0, 1])$ is called a flexible Q-fuzzy group of G if the following conditions are hold hood: (1). $\inf \mu(xy, q) \geq T\{\inf \mu(x, q), \inf \mu(y,q)\}$; (2). $\sup \mu(xy, q) \geq T\{\sup \mu(x, q), \sup \mu(y, q)\}$; (3). $\inf \mu(x^{-1}, q) \geq \inf \mu(x, q)$; (4).$\sup \mu(x^{-1}, q) \geq \sup \mu(x, q)$ for all $x, y \in G$.

2.11 Example: Let G = {e, p, q, r} be the Klein's four group. We define the multiplication in a group Gas follows:

•	e	p	q	r
e	e	e	e	e
p	p	p	p	p
q	e	e	e	q
r	p	p	p	e

Then (G, •) is a group. Define a flexible of fuzzy subset $\mu: G \rightarrow Q^*([0, 1])$ by μ (e) = 0.75, μ(p) = 0.25, μ(q) = 0.0.25, μ(r) = 0.75. Then μ is a flexible of fuzzy subgroup of G.

2.12 Note: In definition * if $\mu: G \times Q \rightarrow [0, 1]$, then $\mu(x, q)$ for all $x \in G$ are real points in [0, 1] and also $\inf \mu(x, q) = \sup \mu(x, q) = \mu(x, q)$, $x \in G$ and q in Q. Thus definition * reduces to definition of Rosenfeld's fuzzy group. So a flexible fuzzy subgroup is a generalization of Rosenfeld's fuzzy group.

SECTION-3: PROPERTIES OF FLEXIBLE Q-FUZZY GROUPS

Some properties and the basic results are presented now in a flexible Q-fuzzy group

3.1 Theorem: A flexible Q-fuzzy subset μ of a group G is a flexible Q-fuzzy group if and only if the following conditions are hold.

(1). $\inf \mu(xy^{-1}, q) \geq T\{\inf \mu(x, q), \inf \mu(y, q)\}$ and

(2). $\sup \mu(xy^{-1}, q) \geq T\{\sup \mu(x, q), \sup \mu(y, q)\}$ for all $x, y \in G$.

Proof: Let μ be flexible Q-fuzzy group of G and x, y $\in$G. Then it follows that

$$\inf\mu(xy^{-1},q) \geq T\{\inf \mu(x, q), \inf \mu(y^{-1},q)\}$$
$$= T\{\inf \mu(x, q), \inf \mu(y, q)\} \text{ and}$$
$$\sup \mu(xy^{-1}, q) \geq T\{\sup \mu(x, q), \sup \mu(y, q)\}$$
$$= T\{\sup \mu(x, q), \sup \mu(y, q)\}.$$

Conversely: let μ be a flexible Q-fuzzy subset of G and given conditions hold. Then it follows that

$$\inf \mu(e, q) = \inf \mu (xx^{-1}, q)$$
$$\geq T\{\inf \mu(x, q), \inf \mu(x, q)\}$$
$$= \inf \mu(x, q) \text{ ----- (1).}$$
$$\sup \mu(e, q) = \sup \mu (xx^{-1}, q)$$
$$\geq T\{\sup \mu(x, q), \sup \mu(x, q)\}$$
$$= \sup \mu(x, q) \text{ --- (2) for all } x \in G.$$

It implies that $\inf \mu(x^{-1}, q) = \inf \mu(ex^{-1}, q)$
$$\geq T\{\inf \mu(e, q), \inf \mu(x, q)\}$$
$$= \inf \mu(x, q) \text{ by (1)}$$
and $\sup \mu(x^{-1}, q) = \sup \mu(ex^{-1}, q) \geq T\{ \sup \mu(e, q), \sup \mu(x, q)\}= \sup \mu(x, q)$ by (2).

So $\inf \mu(xy, q) \geq T \{\inf \mu(x, q), \inf \mu(y^{-1},q)\}$
$$\geq T \{\inf \mu(x, q), \inf \mu(y, q)\}.$$
$$\sup \mu(xy, q) \geq T \{\sup \mu(x, q), \sup \mu(y, q)\}$$
$$\geq T \{\sup \mu(x, q), \sup \mu(y, q)\}.$$
Thus μ is a flexible Q-fuzzy group of G.

3.2 Theorem: If μ is a flexible Q-fuzzy groupoid of an infinite group G, μ is a flexible Q-fuzzy group of G.

Proof: Let x be any element in a group G. Since G is finite, x has finite order, say p. then $x^p = e$, where 'e' is the identity element of G.
Thus $x^{-1} = \mu^{p-1}$ using the definition of flexible Q-fuzzy groupoid, it follows that
$\inf \mu(x^{-1}, q) = \inf \mu(x^{p-1}, q) = \inf \mu(x^{p-2}, q) \geq T\{\inf \mu(x^{p-2}, q), \mu(x, q)\}$

Again $\inf \mu(x^{p-2}, q) = \inf \mu(x^{p-3},x, q) \geq T\{ \inf \mu(x^{p-3}, q), \mu(x, q)\}$
Then we have $\inf \mu(x^{-1}, q) \geq T\{\inf \mu(x^{p-3}, q), \inf \mu(x, q)\}$.

So applying the definition of flexible of Q-fuzzy groupoid repeatedly, $\inf \mu(x^{-1}, q) \leq \inf \mu(x, q)$.

Similarly it gives that $\sup \mu(x^{-1}, q) \leq \sup \mu(x, q)$.Therefore μ is a flexible Q-fuzzy group.

3.3 Theorem: The intersection of any two flexible Q-fuzzy group is also a flexible Q-fuzzy group of G.

Proof: Let A and B be any two flexible Q-fuzzy groups of G and x, y $\in$ G. Then

$$\inf (A \cap B) (xy^{-1}, q) = T \{\inf A(xy^{-1}, q), \inf B(xy^{-1}, q)\}$$
$$\geq T \{T\{\{\inf A(x, q), \inf A(x, q)\}, \{\inf B(x, q), \inf B(y, q)\}\}\}$$
$$= T \{T\{\inf A(x, q), \inf B(x, q)\}, T\{\inf A(x, q), \inf B(y, q)\}\}\}$$
$$= T \{\inf A\cap B(x, q), \inf A\cap B(y, q)\} \text{ …… (1).}$$
$$\sup (A\cap B) (xy^{-1}, q) = T \{\sup A(xy^{-1}, q), \sup B(xy^{-1}, q)\} \text{ by definition}$$
$$\geq T \{T \{\{\sup A(x, q), \sup A(x, q)\}, \{\sup B(x, q), \sup B(y,q)\}\}\}$$
$$= T \{T \{\sup A(x, q), \sup B(x, q)\}, T\{\sup A(x, q), \sup B(y,q)\}\}\}$$
$$= T \{\sup A\cap B(x, q), \sup A\cap B(y, q)\} \text{….. (2).}$$

Therefore by (1) and (2) and using theorem 3.1

It gets that $(A \cap B)$ is a flexible Q- fuzzy group of G.

3.5 Theorem: If A is a flexible Q-fuzzy group of a group G having identity e, then for all x $\in$X we have
(1). inf $A(x^{-1}, q)$ = inf A(x, q), (2). sup $A(x^{-1}, q)$ = sup A(x, q).
(3). inf A(e, q) = inf A(x, q) ; (4). sup A(e, q) = sup A(x, q)

Proof: (i) If A is a Q-fuzzy group of a group G, then it gets that inf $A(x^{-1}, q) \leq$ inf A(x, q).

Again inf A(x, q) = inf $A((x^{-1})^{-1}, q) \leq$ inf $A(x^{-1}, q)$. Therefore inf $A(x^{-1}, q)$ = inf A(x, q).

Similarly it can prove that sup $A(x^{-1}, q)$ = sup A(x, q)

inf A(e, q) = inf $A(xx^{-1}, q) \geq T\{$ inf A(x, q), inf $A(x^{-1}, q)\}$

and sup A(e, q) = sup $A(xx^{-1}, q) \geq T\{$sup A(x, q), sup $A(x^{-1}, q)\}$.

3.6 Theorem: Let μ and λ be two flexible Q-fuzzy group of G_1 and G_2 respectively, and f be a homomorphism from G_1 to G_2. Then f (μ, q) is a flexible Q-fuzzy group of G_2 and f(λ, q) is a flexible of Q-fuzzy group of G_1.

Proof: It is straight forward.

3.7 Remark: If μ is flexible Q-fuzzy group of G and K is subgroup of G, then the restriction of μ to K(μ/K) is a flexible Q-fuzzy group of K.

SECTION-4: NORMAL FLEXIBLE OF Q-FUZZY GROUP

The basic results and properties are presented now in a normal flexible Q-fuzzy group.

4.1 Definition: If μ is a flexible Q-fuzzy group of a group G, then μ is called a normal flexible Q-fuzzy group of G if inf μ(xy, q) = inf μ(yx, q) and sup μ(xy, q) = sup μ(yx, q)} for all x, y $\in$ G.

4.2 Definition: Let μ be a flexible of fuzzy subgroup of G. For any x in G and the smallest positive integer n such that $\mu(x^n) = \mu$ (e) is called a flexible fuzzy order of x. If there does not exist such n, then x is said to have an infinite flexible fuzzy order. We shall denote flexible fuzzy order of x by O(μ(x)).

4.3 Example: Let G = {e, p, q, pq} be the Klein's four group and let = μ\{(e , 0.25), (p, 0.75), (q, 0.75), (pq, 0.25)\} be a flexible of fuzzy group. Then O (μ (pq)) = 1 and O (μ (p)) =2.

4.4 Theorem: The intersection of any two normal flexible Q-fuzzy groups of G is also a normal flexible Q-fuzzy group of G.

Proof: Let A and B be any two normal flexible Q-fuzzy groups of G. Then $A\cap B$ is a flexible Q-fuzzy group of G.
Let x, y be any two elements in a group G. Then, by definition

inf $(A \cap B)$ (xy, q) = T {inf A(xy, q), inf B(xy, q)} by definition
= T {inf A(yx, q), inf B(yx, q)}
= inf$(A \cap B)$ (yx, q).

Similarly sup $(A \cap B)$ (xy, q) = sup $(A\cap B)$ (yx, q).

This shows that $A\cap B$ is normal flexible Q-fuzzy group of G.

4.5 Theorem: The intersection of any arbitrary collection of normal flexible Q-fuzzy groups of a group G is also a normal flexible Q-fuzzy group of G.

Proof: Let x, y be any two elements in a group G and α in G. Then,

inf $A(xy^{-1}, q)$ = inf $A(\alpha^{-1}xy^{-1}\alpha, q)$ by definition
= inf $A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha, q)$
= inf $(A(\alpha^{-1}x\alpha, q), A((\alpha^{-1}y\alpha)^{-1}, q))$
$\geq$ T {inf $(A(\alpha^{-1}x\alpha, q)$, inf $A(\alpha^{-1}y\alpha, q))$}
= T {inf (A(x, q),A((y, q))}.

$$
\begin{aligned}
\sup A(xy^{-1}, q) &= \sup A(\alpha^{-1}xy^{-1}\alpha, q) \text{ by definition} \\
&= \sup A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha, q) \\
&= \sup (A(\alpha^{-1}x\alpha, q), A((\alpha^{-1}y\alpha)^{-1}, q)) \\
&\geq T\{\sup (A(\alpha^{-1}x\alpha, q), \sup A(\alpha^{-1}y\alpha), q))\} \\
&= T\{\sup (A(x, q), A((y, q))\}.
\end{aligned}
$$

It follows that A is normal flexible Q-fuzzy group of G.

CONCLUSION

In this study we introduced, the concept of flexible fuzzy set and a flexible Q-fuzzy group. Based on this flexible Q-fuzzy normal groups are given and its some elementary properties are discussed. Some properties are derived.

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