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LINEAR STABILITY ANALYSIS OF COUPLE STRESS FLUID-SATURATED ANISOTROPIC POROUS LAYER UNDER THERMAL NONEQUILIBRIUM MODEL IN EXISTENCE OF INTERNAL HEAT SOURCE

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ABSTRACT

The present investigation deals with thermal stability of couple-stress fluid flow through anisotropic porous media with consideration of thermal non equilibrium model assumption of normal mode solutions in view of stress free anisothermal boundary conditions has been carried out. At last the effect of various governing parameters of thermal stability of the system are depicted through graphs of neutral stability curves.

Keywords: *Porous Layer, Darcy-Rayleigh Number, Anisotropic media, couple stress fluid.*

Nomenclature

Latin Symbols	
$\vec{q}$	Velocity
$C_1 = \frac{\mu_1}{\mu'}$	non dimensional couple stress viscosity parameter
μ	viscosity of fluid
S	Concentration
λ'	Coefficient of couple stress parameter
d	Length of the porous layer
$D_a = \frac{K}{d^2}$	Darcy number
$P_r = \frac{\nu\epsilon}{\kappa_f}$	Prandtl Number
$C_2 = \frac{\mu_c}{\mu' d^2}$	non dimensional couple stress parameter
$\vec{g}$	Acceleration due to gravity
c	Specific heat capacity
γ_z	Perturbed vorticity along the z-axis
$\mu' = \mu\left(1 + \frac{\lambda'}{\mu k}\right)$	Modified viscosity
K_x, K_y, K_z	characteristic permeabilities in the x, y , and z directions
K	Permeability of porous layer
ΔT	Temperature difference across the porous layer
p	Reduced pressure
Le	Lewis number, $Le = \frac{\kappa_{fz}}{D_m}$
R	Darcy-Rayleigh number, $R = \frac{\rho_f g \beta_T \Delta T K_z d}{\varepsilon \mu \kappa_{fz}}$
T_l	The temperature of lower surface
R_S	Solutal Rayleigh number, $R_s = \frac{\rho_f g \beta_C \Delta C K_z d}{\epsilon \mu \kappa_{fz}}$
x,y,z	Space Co-ordinates
a	Horizontal wavenumber
l, m	x-component and y-component of wavenumber
T_u	The temperature of upper surface
R_c	Critical Rayleigh number,
T_f	The temperature of fluid,

Greek symbols	
Ψ	The amplitude of fluid concentration
κ_f	Thermal diffusivity for fluid
ρ_0	Reference density
β_C	Coefficient of concentration expansion
χ	Diffusivity ratio, $\chi = \frac{(\rho_s c_s)\kappa_{fz}}{(\rho_f c_f)\kappa_{sz}}$
η_f	Fluid thermal conductivity ratio $(= \frac{\kappa_{fx}}{\kappa_{fz}} = \frac{\kappa_{fy}}{\kappa_{fz}})$
κ	Thermal diffusivity
ρ	Density of fluid
ϵ	Porosity
β_T	Coefficient of thermal expansion
γ	Porosity modified conductivity ratio, $\gamma = \frac{\epsilon\kappa_{fz}}{(1-\epsilon)\kappa_{sz}}$
ξ	Anisotropy ratio $(= \frac{K_x}{K_z}, = \frac{K_y}{K_z})$
Θ	The amplitude of fluid temperature perturbation

1. INTRODUCTION

superscripts	
$*$	Dimensionless quantity
$'$	Perturbed quantity
Other symbols	
D	d/dz
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
Subscripts	
c	Critical
b	basic state
f	Fluid

Due to practical application the importance of internal heat source becomes popular for researchers. Convection in the earth crust occurs due to internal heating of earth after creating a temperature gradient between earth's crust. The application of internal heat source becomes more wide due to its occurrence in oil industry, crystal growth and nuclear industry such as nuclear energy, radioactive decay, nuclear waste disposal e.t.c. Tveitereid [1] had studied earlier work on internal heat source in porous media who found the steady solution of two dimensional rolls of convection in horizontal porous layer with internal heat source in the form of hexagon. The boundary influence convection in presence of internal heat source studied by Bejan [2]. Linear stability analysis with uniform heat source and maximum density in thermal noneqilibrium presence of saturated porous media investigated by Saravanan[3] and saw that internal heat parameters boosting the onset natural convection. Many researchers such as Hill[4], Borujerdi *et al*.[5], Capone *et al*.[6] and many more had studied on internal heat source with various parameters. Recently Bhadauria *et.al* [7, 8, 9, 10] has investigated stability analysis in presence of internal heat source taking different parameters.

Firstly, Stokes [11] was presented theory of couple stress. He studied important features and effect of couple stress for simple polar fluid which is caused by mechanical interaction due to continuum deformations. Effect of couple stress fluid on the onset convection of saturated medium with presence of non uniform temperature gradient is studied by Shivkumara[12]. More recently Naveen kumar *et.al* [13] studied Linear stability analysis of couple stress fluid in exploration of coriolis force induced by double diffusive convection. B.M.Shankar *et.al*.[14] had investigated stability of couple stress fluid flow through a horizontal porous layer.

In recently few decades, the effect of thermal non equilibrium model on fluid flow to study stability analysis has been being one of the interested area among the researchers. Stability analysis of thermal non equilibrium model in presence of vertical porous layer had been studied by Gill[15]. After that Rees had studied the effect of thermal non equilibrium model with Parandental number. Lewis et.al and many other researchers had been done the earlier work for thermal non equilibrium model. More recently Weijie Li [19] , Tahar Tayebi[18] et.al and many other investigated the effect of thermal non equilibrium.

In this present paper thermal non equilibrium model has been considered on couple stress fluid in presence of anisotrpic porous layer. Graphs of various parameters show the effects to study linear stability analysis.

2. GOVERNING EQUATIONS

An anisotrpic porous layer and internal heat source have been taken between two horizontal plate with height d. Solute concentration and Temperature between upper and lower layer taken constant. Thermal non-equilibrium model has been considered. In the above said physical configuration governing equations are modeled as.

$$\nabla.\vec{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q} + \frac{1}{\rho_0}(\mu - \mu_1\nabla^2 + \mu_c\nabla^4)K\cdot\vec{q} = -\nabla P + \rho_f\vec{g} \tag{2}$$

$$\varepsilon(\rho_f c_f)\frac{\partial T_f}{\partial t} + (\vec{q}.\nabla)T_f = \nabla(\kappa_f\nabla T_f) + Q(T_f - T_0) - h_s(T_f - T_s) \tag{3}$$

$$(1-\epsilon)(\rho_s c_s)\frac{\partial T_s}{\partial t} = \nabla\big((1-\epsilon)\kappa_s\nabla T_s\big) - h_s(T_s - T_f), \tag{4}$$

$$\varepsilon\frac{\partial S}{\partial t} + (\vec{q}.\nabla)S = \varepsilon k_s\nabla^2 S + \varepsilon\nabla^2 T_f \tag{5}$$

Where all listed symbols have their physical meanings listed in the nomenclature.

$$\rho_f = \rho = \rho_0\left[1 - \beta_{T_f}(T_{f_u} - T_f) + \beta_C(C_u - C_f)\right]. \tag{6}$$

Equation (6) gives the relation between temperature and the reference density, where $\beta_T > 0$ and $\beta_C > 0$ are expansions coefficients of temperature and solute concentration, respectively.

The required boundary conditions are as

$$T_f = T_l, C = C_l \text{atz} = 0 \text{and} T_f = T_u, C = C_u \text{atz} = d. \tag{7}$$

3. LINEAR STABILITY ANALYSIS

The perturbation of basic state is given by

$$[\vec{q}, T_f, T_s, S] = [\vec{q}_b, T_{f_b}, T_{sb}, s_b] + [\vec{q}', T', T_s', S']. \tag{8}$$

where the basic state is assumed to be quiescent as given by

$$\bar{q}_b = (0,0,0), T_{fb} = C_b = 1 - z,$$

Using equation on to eq (1-6) we linearized them by eliminating the pressure on applying curl once,twice of momentum equation thereafter non-dimentionalization using following transformation distance by d,velocity by $\frac{\varepsilon k_{T_f}}{d}, T_f by\Delta T$ We get following form of linearized equation.

$$\{\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 + \left(\nabla_1^2 + \frac{1}{\xi} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{1}{Da} - C_1\nabla^2 + \nabla^4\right)\}w - R_T\nabla_1^2\theta + R_s\nabla_1^2\psi = 0 \tag{9}$$

$$w\frac{\partial T}{\partial z} + \left[\frac{\partial}{\partial t} - \left(\eta_f\nabla_1^2 + D^2 - R_i\right)\right]\theta - H(\phi - \theta) = 0 \tag{10}$$

$$-\gamma H\theta + \left(\frac{\partial}{\partial t} - n_s - D^2 + \gamma H\right)\phi = 0 \tag{11}$$

and

$$-W + \left(-\frac{1}{Le}(-\nabla_1^2 + D^2) + K\right)\Psi - Sr\frac{Ra_T}{Ra_s}\nabla^2\Theta = 0 \tag{12}$$

where $\nabla_1^2 = l^2 + m^2 = a^2$ where a is horizontal wave number and $D = \frac{d}{dz}$

Boundary conditions is as

$$W = D^2W = D^4W = DZ = D\Phi = 0, \Theta Or D\Theta = 0 \tag{13}$$

and

$$W = DW = D^3W = Z = \Phi = 0, \Theta = 0, D\Theta 0 \tag{14}$$

$$\left(w, T_f, T_s, S\right) = (W, Z, \Theta, \Psi, \Phi)e^{i(lx+my)} \quad (15)$$

$$\psi = A_1 sin(ax)sin(\pi z), T_f = A_2 cos(ax)sin(\pi z), T_s = A_3 cos(ax)sin(\pi z), S = A_4 cos(ax)sin(\pi z) \quad (16)$$

$$\begin{vmatrix} \delta_1^2\left(\frac{1}{D_a} + C_1\delta^2 + C_2\delta^4\right) & -a^2R_{a_T} & a^2R_{a_S} & 0 \\ 2a^2F & \left(R_i - {\delta_2}^2\right) & 0 & -H \\ 0 & -\gamma + 1 & 0 & \eta a^2 + \pi^2 + \gamma H \\ -a^2 & -Sr\frac{Ra_T}{Ra_s}\delta^2 & -\frac{\delta^2}{L_e} & 0 \end{vmatrix} = 0. \quad (17)$$

Where; $\delta_1^2 = \frac{\pi^2}{\xi} + a^2,\ F = \int_0^1 \uparrow\left(\frac{dT_b}{dz} sin^2(\pi z)\right),\ \ \delta = \sqrt{a^2 + \pi^2},\ \ \delta_2 = \sqrt{a^2\eta + \pi^2}$

4. RESULT AND DISCUSSION

The Impact of various parameters to study linear stability on onset convection has been analyzes in this section. Graphs for various parameters have been depicted between thermal Rayleigh number and wave number which shows nature of neutral stability curve.

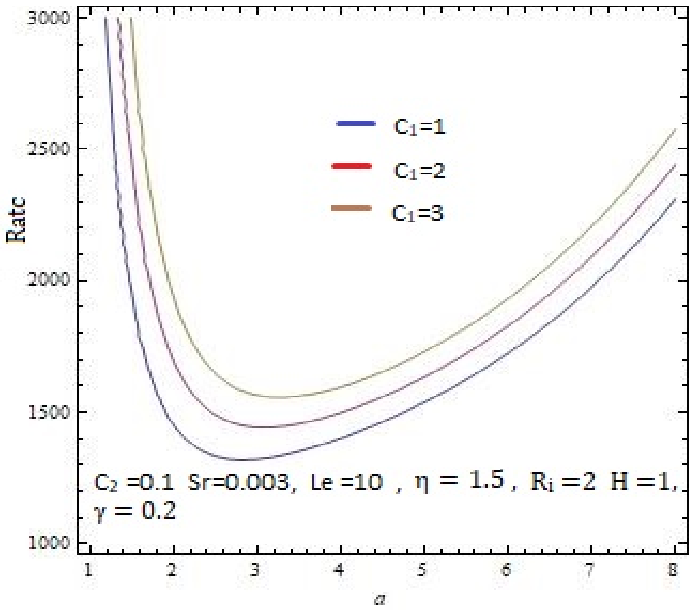

Figure-1: Impact of C_1

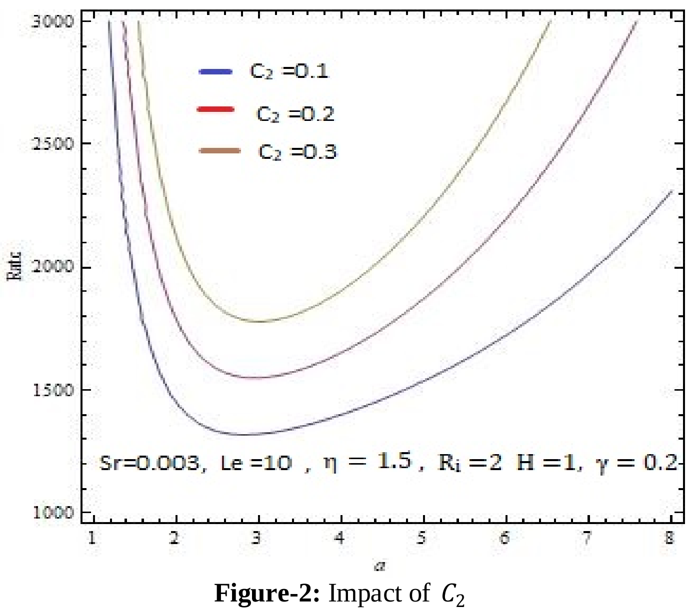

Figure-2: Impact of C_2

Figure 1 and Figure 2 show the effect of couple-stress parameter of second and fourth order respectively. In Figure 1 neutral stability curves for $C_1 = 1$, $C_1 = 2$ and $C_1 = 3$ have been drawn taking other parameters are in rest while $C_2 = 0.1$, $C_2 = 0.2$ and $C_2 = 0.3$ have been taken for natural stability curve in figure 2. It is clear from the figure enhancement in values of C_1 and C_2 boosting the values of Rayleigh number and shifted graph upward hence couple stress parameter has stabilizing effect.

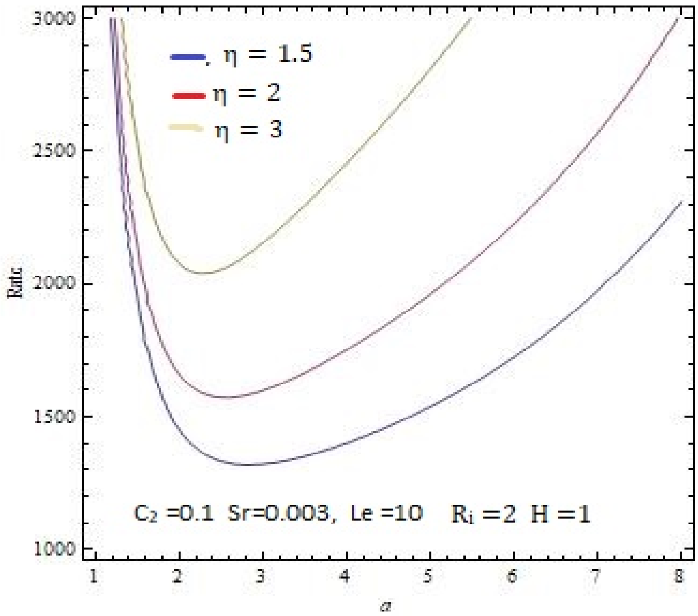

Figure-3: Impact of η

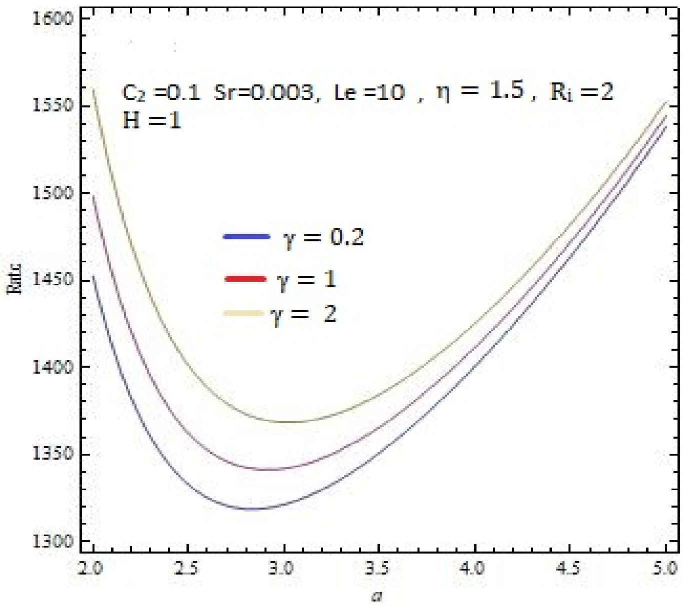

Figure-4: Impact of gamma

In figure 3, neutral stability curves for $\eta = 1.5$, $\eta = 2$, $\eta = 3$ has been drawn. It is noticed that increment in the values of η enhancing the value of Rayleigh number and shifted graph upward which shows η has stabilizing effect. In figure 3, neutral stability curve for $\gamma = 0.2$, $\gamma = 1$, $\gamma = 2$ has been drawn. It is noticed that increment in the values of γ enhancing the value of Rayleigh number and shifted graph upward which shows γ has stabilizing effect.

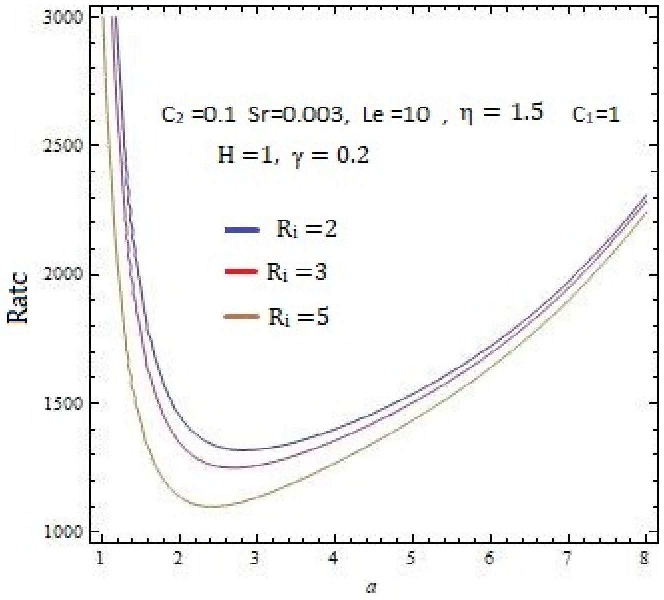

Figure-5: Impact of internal heat

In Figure 5, We can easily find out the effect of internal heat. Neutral stability curves for $R_i = 2, R_i = 3, R_i = 5$ have been depicted. It is observed that Increment in R_i decline the Rayleigh number and shifted curve downward which revels that internal heat parameter has destabilizing effect on onset convection.

5. CONCLUSION

- Internal heat parameter destabilizing the onset convection.
- Couple stress parameter stabilizing the onset convection.
- η stabilizing the onset convection.
- γ stabilizing the onset convection.

REFERENCES

1. Tveitereid, M. (1977). Thermal convection in a horizontal porous layer with internal heat sources. International Journal of Heat and Mass Transfer, 20(10), 1045-1050.
2. Bejan, A. (1978). Natural convection in an infinite porous medium with a concentrated heat source. Journal of Fluid Mechanics, 89(1), 97-107.
3. Saravanan, S. (2009). Thermal non-equilibrium porous convection with heat generation and density maximum. Transport in porous media, 76(1), 35-43.
4. Hill, A. A. (2005). Double–diffusive convection in a porous medium with a concentration based internal heat source. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 461(2054), 561-574.
5. Nouri-Borujerdi, A., Noghrehabadi, A. R.,& Rees, D. A. S. (2007). Onset of convection in a horizontal porous channel with uniform heat generation using a thermal nonequilibrium model. Transport in porous media, 69(3), 343-357.
6. Israel-Cookey, C., Omubo-Pepple, V. B., Obi, B. I.,& Eze, L. C. (2010). Onset of thermal instability in a low Prandtl number fluid with internal heat source in a porous medium. Am. J. Sci. Res, 1.
7. Bhadauria, B. S., Anoj Kumar, Jogendra Kumar, Nirmal C. Sacheti, and Pallath Chandran. "Natural convection in a rotating anisotropic porous layer with internal heat generation." Transport in Porous Media 90, no. 2 (2011): 687-705.
8. Bhadauria, B. S. (2012). Double-diffusive convection in a saturated anisotropic porous layer with internal heat source. Transport in porous media, 92(2), 299-320.
9. Bhadauria, B. S., Hashim, I., & Siddheshwar, P. G. (2013). Study of heat transport in a porous medium under G-jitter and internal heating effects. Transport in porous media, 96(1), 21-37.
10. Bhadauria, B. S., Hashim, I.,&Siddheshwar, P. G. (2013). Effects of time-periodic thermal boundary conditions and internal heating on heat transport in a porous medium. Transport in porous media, 97(2), 185-200.
11. Stokes, V. K. (1984). Couple stresses in fluids. In Theories of Fluids with Microstructure (pp. 34-80). Springer, Berlin, Heidelberg.
12. Shivakumara, I. S. (2010). Onset of convection in a couple-stress fluid-saturated porous medium: effects of non-uniform temperature gradients. Archive of Applied Mechanics, 80(8), 949-957.

13. Naveen Kumar, S. B., Shivakumara, I. S.,&Shankar, B. M. (2019). Exploration of Coriolis force on the linear stability of couple stress fluid flow induced by double diffusive convection. Journal of Heat Transfer, 141(12).
14. Shankar, B. M., Shivakumara, I. S.,& Ng, C. O. (2016). Stability of couple stress fluid flow through a horizontal porous layer. Journal of Porous Media, 19(5).
15. Gill, A. E. (1969). A proof that convection in a porous vertical slab is stable. Journal of Fluid Mechanics, 35(3), 545-547.
16. Rees, D. A. S. (2011). The effect of local thermal nonequilibrium on the stability of convection in a vertical porous channel. Transport in porous media, 87(2), 459-464.
17. Lewis, S., Bassom, A. P.,& Rees, D. A. S. (1995). The stability of vertical thermal boundary-layer flow in a porous medium. EUROPEAN JOURNAL OF MECHANICS SERIES B FLUIDS, 14, 395-395.
18. Tayebi, T., & Chamkha, A. J. (2021). Analysis of the effects of local thermal non-equilibrium (LTNE) on thermo-natural convection in an elliptical annular space separated by a nanofluid-saturated porous sleeve. International Communications in Heat and Mass Transfer, 129, 105725.
19. Li, W., Huang, J., Zhang, Z., Wang, L., Huang, H., & Liang, J. (2021). A model for thermal protection ablative material with local thermal non-equilibrium and thermal radiation mechanisms. Acta Astronautica, 183, 101-111.

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