

EVALUATION OF A SUMMATION FORMULA IN CONNECTION WITH GAUSS THEOREM

SALAHUDDIN*

P. D. M College of Engineering, Bahadurgarh, Haryana, India

E-mail: sludn@yahoo.com; vsludn@gmail.com

(Received on: 03-12-11; Accepted on: 28-12-11)

ABSTRACT

The main objective of the present paper is to establish a summation formula based on half argument in connection with Hypergeometric function.

A.M.S. Subject Classification (2010): 33C05, 33C20, 33C45, 33D50, 33D60.

Key words and phrases: Contiguous relation, Recurrence relation, Gauss second summation theorem.

A. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable:

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; Z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k}{(b_1)_k (b_2)_k \dots (b_B)_k} \frac{Z^k}{k!} \tag{1}$$

or

$${}_A F_B((a_A); (b_B); Z) \equiv {}_A F_B((a_j)_{j=1}^A; (b_j)_{j=1}^B; Z) = \sum_{k=0}^{\infty} \frac{((a_A))_k}{((b_B))_k} \frac{Z^k}{k!} \tag{2}$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relations:

[Andrews p.363 (9.16), E.D. p.51 (10)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z) \tag{3}$$

Gauss second summation theorem is defined as:

[Prudnikov p.491(7.3.7.3)]

$${}_2F_1(a, b; \frac{a+b+1}{2}; \frac{1}{2}) = 2^{b-1} \frac{\Gamma(\frac{b}{2}) \Gamma(\frac{a+b+1}{2})}{\Gamma(b) \Gamma(\frac{a+1}{2})} \tag{4}$$

Recurrence relation:

$$\Gamma(\zeta+1) = \zeta \Gamma(\zeta) \tag{5}$$

B. MAIN SUMMATION FORMULA:

$${}_2F_1(a, b; \frac{a+b+34}{2}; \frac{1}{2}) = 2^b \frac{\Gamma(\frac{a+b+34}{2})}{(a-b)\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})} \left\{ \frac{65536(-42849873690624000a + 71092846618214400a^2)}{\prod_{\eta=0}^{15} \{a-b-2\eta\}} \prod_{\lambda=1}^{16} \{a-b+2\lambda\} \right\} \right]$$

Corresponding author: SALAHUDDIN, *E-mail: vsludn@gmail.com

$$\begin{aligned}
 & + \frac{65536(-50510377899786240a^3 + 20713454402863104a^4 - 5543553731788800a^5 + 1033904237649920a^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-139675243868160a^7 + 13985569165568a^8 - 1050576384000a^9 + 59398127360a^{10} - 2513871360a^{11})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(78393952a^{12} - 1747200a^{13} + 26320a^{14} - 240a^{15} + a^{16} + 42849873690624000b + 204349877570764800ab)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-73098173806018560a^2b + 168862864276979712a^3b - 32173231185985536a^4b)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(16624053726560256a^5b - 1763428292096000a^6b + 392442369636352a^7b - 24299638755328a^8b)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(2797381257728a^9b - 100356672000a^{10}b + 6328583040a^{11}b - 121428736a^{12}b + 4090016a^{13}b)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-32240a^{14}b + 496a^{15}b + 71092846618214400b^2 + 73098173806018560ab^2 + 314212975802449920a^2b^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-29343138531508224a^3b^2 + 70732204950478848a^4b^2 - 5665613912485888a^5b^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(3067648334289920a^6b^2 - 159783963901952a^7b^2 + 36873469383424a^8b^2 - 1169137503744a^9b^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(137450339520a^{10}b^2 - 2399481344a^{11}b^2 + 151276528a^{12}b^2 - 1107568a^{13}b^2 + 35960a^{14}b^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(50510377899786240b^3 + 168862864276979712ab^3 + 29343138531508224a^2b^3)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(111793974123528192a^3b^3 - 4166767625613312a^4b^3 + 9685423051917312a^5b^3)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-381032960901120a^6b^3 + 202174524192768a^7b^3 - 5473329255936a^8b^3 + 1236553700736a^9b^3)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-19477086720a^{10}b^3 + 2219221056a^{11}b^3 - 15146352a^{12}b^3 + 906192a^{13}b^3 + 20713454402863104b^4)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(32173231185985536ab^4 + 70732204950478848a^2b^4 + 4166767625613312a^3b^4)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(14046774740067840a^4b^4 - 247159742668800a^5b^4 + 537085526945280a^6b^4 - 10876913832960a^7b^4)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(5478695724960a^8b^4 - 73789920000a^9b^4 + 15833547600a^{10}b^4 - 98170800a^{11}b^4 + 10518300a^{12}b^4)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(5543553731788800b^5 + 16624053726560256ab^5 + 5665613912485888a^2b^5 + 9685423051917312a^3b^5)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(247159742668800a^4b^5 + 739053809433600a^5b^5 - 6502833792000a^6b^5 + 13020674123520a^7b^5)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(-129818476800a^8b^5 + 60988879200a^9b^5 - 322561200a^{10}b^5 + 64512240a^{11}b^5 + 1033904237649920b^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(1763428292096000ab^6 + 3067648334289920a^2b^6 + 381032960901120a^3b^6 + 537085526945280a^4b^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(6502833792000a^5b^6 + 17303373763200a^6b^6 - 73365304320a^7b^6 + 134329371120a^8b^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{65536(-521060400a^9b^6 + 225792840a^{10}b^6 + 139675243868160b^7 + 392442369636352ab^7)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(159783963901952a^2b^7 + 202174524192768a^3b^7 + 10876913832960a^4b^7 + 13020674123520a^5b^7)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(73365304320a^6b^7 + 174242597760a^7b^7 - 282861360a^8b^7 + 471435600a^9b^7 + 13985569165568b^8)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(24299638755328ab^8 + 36873469383424a^2b^8 + 5473329255936a^3b^8 + 5478695724960a^4b^8)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(129818476800a^5b^8 + 134329371120a^6b^8 + 282861360a^7b^8 + 601080390a^8b^8 + 1050576384000b^9)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(2797381257728ab^9 + 1169137503744a^2b^9 + 1236553700736a^3b^9 + 73789920000a^4b^9)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(60988879200a^5b^9 + 521060400a^6b^9 + 471435600a^7b^9 + 59398127360b^{10} + 100356672000ab^{10})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(137450339520a^2b^{10} + 19477086720a^3b^{10} + 15833547600a^4b^{10} + 322561200a^5b^{10} + 225792840a^6b^{10})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(2513871360b^{11} + 6328583040ab^{11} + 2399481344a^2b^{11} + 2219221056a^3b^{11} + 98170800a^4b^{11})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(64512240a^5b^{11} + 78393952b^{12} + 121428736ab^{12} + 151276528a^2b^{12} + 15146352a^3b^{12} + 10518300a^4b^{12})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(1747200b^{13} + 4090016ab^{13} + 1107568a^2b^{13} + 906192a^3b^{13} + 26320b^{14} + 32240ab^{14} + 35960a^2b^{14})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{65536(240b^{15} + 496ab^{15} + b^{16})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} + \frac{2097152b(5414618293862400a + 1239306479861760a^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(2154746702462976a^3 + 271225105022976a^4 + 139645398196224a^5 + 10510582036480a^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(2413316114944a^7 + 111044152448a^8 + 13192757312a^9 + 361614240a^{10} + 23257104a^{11})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(345176a^{12} + 11564a^{13} + 70a^{14} + a^{15} + 5414618293862400b + 8679091003195392a^2b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(800747346984960a^3b + 1209092630388736a^4b + 78587208572928a^5b + 37494327717376a^6b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(1582993562880a^7b + 342709269568a^8b + 8914977024a^9b + 997563632a^{10}b + 14411280a^{11}b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(859196a^{12}b + 5208a^{13}b + 155a^{14}b - 1239306479861760b^2 + 8679091003195392ab^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(3160235408179200a^3b^2 + 141614461491200a^4b^2 + 187929492044288a^5b^2 + 6711397879552a^6b^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(2932365276928a^7b^2 + 69517440480a^8b^2 + 13942713264a^9b^2 + 191105824a^{10}b^2 + 19727656a^{11}b^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(116870a^{12}b^2 + 6293a^{13}b^2 + 2154746702462976b^3 - 800747346984960ab^3)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(3160235408179200a^2b^3 + 402878073343488a^4b^3 + 9531817662720a^5b^3 + 11198193245952a^6b^3)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{2097152b(220854007296a^7b^3 + 87791751504a^8b^3 + 1088940720a^9b^3 + 199987944a^{10}b^3 + 1121952a^{11}b^3)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(105183a^{12}b^3 - 271225105022976b^4 + 1209092630388736ab^4 - 141614461491200a^2b^4)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(402878073343488a^3b^4 + 21404702094720a^5b^4 + 272020714560a^6b^4 + 284258190240a^7b^4)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(2894959800a^8b^4 + 1042390500a^9b^4 + 5259150a^{10}b^4 + 876525a^{11}b^4 + 139645398196224b^5)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(-78587208572928ab^5 + 187929492044288a^2b^5 - 9531817662720a^3b^5 + 21404702094720a^4b^5)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(504594891360a^6b^5 + 3245461920a^7b^5 + 3025872180a^8b^5 + 12406200a^9b^5 + 4032015a^{10}b^5)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(-10510582036480b^6 + 37494327717376ab^6 - 6711397879552a^2b^6 + 11198193245952a^3b^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(-272020714560a^4b^6 + 504594891360a^5b^6 + 5106391920a^7b^6 + 13026510a^8b^6 + 10855425a^9b^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(2413316114944b^7 - 1582993562880ab^7 + 2932365276928a^2b^7 - 220854007296a^3b^7)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(284258190240a^4b^7 - 3245461920a^5b^7 + 5106391920a^6b^7 + 17678835a^8b^7 - 111044152448b^8)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(342709269568ab^8 - 69517440480a^2b^8 + 87791751504a^3b^8 - 2894959800a^4b^8 + 3025872180a^5b^8)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(-13026510a^6b^8 + 17678835a^7b^8 + 13192757312b^9 - 8914977024ab^9 + 13942713264a^2b^9)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(-1088940720a^3b^9 + 1042390500a^4b^9 - 12406200a^5b^9 + 10855425a^6b^9 - 361614240b^{10})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(997563632ab^{10} - 191105824a^2b^{10} + 199987944a^3b^{10} - 5259150a^4b^{10} + 4032015a^5b^{10})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(23257104b^{11} - 14411280ab^{11} + 19727656a^2b^{11} - 1121952a^3b^{11} + 876525a^4b^{11} - 345176b^{12})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(859196ab^{12} - 116870a^2b^{12} + 105183a^3b^{12} + 11564b^{13} - 5208ab^{13} + 6293a^2b^{13})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{2097152b(-70b^{14} + 155ab^{14} + b^{15})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} - \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})} \left\{ \frac{2097152a(5414618293862400a - 1239306479861760a^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \right\} \\
 & + \frac{2097152a(2154746702462976a^3 - 271225105022976a^4 + 139645398196224a^5 - 10510582036480a^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(2413316114944a^7 - 111044152448a^8 + 13192757312a^9 - 361614240a^{10} + 23257104a^{11})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(-345176a^{12} + 11564a^{13} - 70a^{14} + a^{15} + 5414618293862400b + 8679091003195392a^2b)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(-800747346984960a^3b + 1209092630388736a^4b - 78587208572928a^5b + 37494327717376a^6b)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{2097152a(-1582993562880a^7b + 342709269568a^8b - 8914977024a^9b + 997563632a^{10}b - 14411280a^{11}b)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(859196a^{12}b - 5208a^{13}b + 155a^{14}b + 1239306479861760b^2 + 8679091003195392ab^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(3160235408179200a^3b^2 - 141614461491200a^4b^2 + 187929492044288a^5b^2 - 6711397879552a^6b^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(2932365276928a^7b^2 - 69517440480a^8b^2 + 13942713264a^9b^2 - 191105824a^{10}b^2 + 19727656a^{11}b^2)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(-116870a^{12}b^2 + 6293a^{13}b^2 + 2154746702462976b^3 + 800747346984960ab^3)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(3160235408179200a^2b^3 + 402878073343488a^4b^3 - 9531817662720a^5b^3 + 11198193245952a^6b^3)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(-220854007296a^7b^3 + 87791751504a^8b^3 - 1088940720a^9b^3 + 199987944a^{10}b^3 - 1121952a^{11}b^3)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(105183a^{12}b^3 + 271225105022976b^4 + 1209092630388736ab^4 + 141614461491200a^2b^4)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(402878073343488a^3b^4 + 21404702094720a^5b^4 - 272020714560a^6b^4 + 284258190240a^7b^4)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(-2894959800a^8b^4 + 1042390500a^9b^4 - 5259150a^{10}b^4 + 876525a^{11}b^4 + 139645398196224b^5)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(78587208572928ab^5 + 187929492044288a^2b^5 + 9531817662720a^3b^5 + 21404702094720a^4b^5)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(504594891360a^6b^5 - 3245461920a^7b^5 + 3025872180a^8b^5 - 12406200a^9b^5 + 4032015a^{10}b^5)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(10510582036480b^6 + 37494327717376ab^6 + 6711397879552a^2b^6 + 11198193245952a^3b^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(272020714560a^4b^6 + 504594891360a^5b^6 + 5106391920a^7b^6 - 13026510a^8b^6 + 10855425a^9b^6)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(2413316114944b^7 + 1582993562880ab^7 + 2932365276928a^2b^7 + 220854007296a^3b^7)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(284258190240a^4b^7 + 3245461920a^5b^7 + 5106391920a^6b^7 + 17678835a^8b^7 + 111044152448b^8)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(342709269568ab^8 + 69517440480a^2b^8 + 87791751504a^3b^8 + 2894959800a^4b^8 + 3025872180a^5b^8)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(13026510a^6b^8 + 17678835a^7b^8 + 13192757312b^9 + 8914977024ab^9 + 13942713264a^2b^9)}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(1088940720a^3b^9 + 1042390500a^4b^9 + 12406200a^5b^9 + 10855425a^6b^9 + 361614240b^{10})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(997563632ab^{10} + 191105824a^2b^{10} + 199987944a^3b^{10} + 5259150a^4b^{10} + 4032015a^5b^{10})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(23257104b^{11} + 14411280ab^{11} + 19727656a^2b^{11} + 1121952a^3b^{11} + 876525a^4b^{11} + 345176b^{12})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} \\
 & + \frac{2097152a(859196ab^{12} + 116870a^2b^{12} + 105183a^3b^{12} + 11564b^{13} + 5208ab^{13} + 6293a^2b^{13} + 70b^{14})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{2097152a(155ab^{14}+b^{15})}{[\prod_{\eta=0}^{15}\{a-b-2\eta\}][\prod_{\lambda=1}^{16}\{a-b+2\lambda\}]} + \frac{65536(42849873690624000a + 71092846618214400a^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(50510377899786240a^3+20713454402863104a^4 + 5543553731788800a^5 + 1033904237649920a^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(139675243868160a^7 + 13985569165568a^8 + 1050576384000a^9 + 59398127360a^{10}+2513871360a^{11})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(78393952a^{12}+ 1747200a^{13} + 26320a^{14} + 240a^{15} + a^{16}-42849873690624000b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(204349877570764800ab+ 73098173806018560a^2b+168862864276979712a^3b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(32173231185985536a^4b + 16624053726560256a^5b+1763428292096000a^6b + 392442369636352a^7b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(24299638755328a^8b+2797381257728a^9b+100356672000a^{10}b + 6328583040a^{11}b + 121428736a^{12}b)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(4090016a^{13}b + 32240a^{14}b + 496a^{15}b + 71092846618214400b^2 - 73098173806018560ab^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(314212975802449920a^2b^2+ 29343138531508224a^3b^2 + 70732204950478848a^4b^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(5665613912485888a^5b^2 + 3067648334289920a^6b^2 + 159783963901952a^7b^2+36873469383424a^8b^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(1169137503744a^9b^2 + 137450339520a^{10}b^2 + 2399481344a^{11}b^2+151276528a^{12}b^2 + 1107568a^{13}b^2)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(35960a^{14}b^2 - 50510377899786240b^3+168862864276979712ab^3 - 29343138531508224a^2b^3)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(111793974123528192a^3b^3+4166767625613312a^4b^3 + 9685423051917312a^5b^3)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(381032960901120a^6b^3+202174524192768a^7b^3 + 5473329255936a^8b^3 + 1236553700736a^9b^3)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(19477086720a^{10}b^3+2219221056a^{11}b^3 + 15146352a^{12}b^3 + 906192a^{13}b^3 + 20713454402863104b^4)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-32173231185985536ab^4 + 70732204950478848a^2b^4 - 4166767625613312a^3b^4)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(14046774740067840a^4b^4 + 247159742668800a^5b^4 + 537085526945280a^6b^4+10876913832960a^7b^4)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(5478695724960a^8b^4 + 73789920000a^9b^4 + 15833547600a^{10}b^4+98170800a^{11}b^4 + 10518300a^{12}b^4)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-5543553731788800b^5+16624053726560256ab^5-5665613912485888a^2b^5+9685423051917312a^3b^5)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-247159742668800a^4b^5+739053809433600a^5b^5 + 6502833792000a^6b^5 + 13020674123520a^7b^5)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(129818476800a^8b^5+60988879200a^9b^5+ 322561200a^{10}b^5 + 64512240a^{11}b^5 + 1033904237649920b^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-1763428292096000ab^6 + 3067648334289920a^2b^6 - 381032960901120a^3b^6+537085526945280a^4b^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{65536(-6502833792000a^5b^6 + 17303373763200a^6b^6 + 73365304320a^7b^6 + 134329371120a^8b^6)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(521060400a^9b^6 + 225792840a^{10}b^6 - 139675243868160b^7 + 392442369636352ab^7)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-159783963901952a^2b^7 + 202174524192768a^3b^7 - 10876913832960a^4b^7 + 13020674123520a^5b^7)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-73365304320a^6b^7 + 174242597760a^7b^7 + 282861360a^8b^7 + 471435600a^9b^7 + 13985569165568b^8)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-24299638755328ab^8 + 36873469383424a^2b^8 - 5473329255936a^3b^8 + 5478695724960a^4b^8)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-129818476800a^5b^8 + 134329371120a^6b^8 - 282861360a^7b^8 + 601080390a^8b^8 - 1050576384000b^9)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(2797381257728ab^9 - 1169137503744a^2b^9 + 1236553700736a^3b^9 - 73789920000a^4b^9)}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(60988879200a^5b^9 - 521060400a^6b^9 + 471435600a^7b^9 + 59398127360b^{10} - 100356672000ab^{10})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(137450339520a^2b^{10} - 19477086720a^3b^{10} + 15833547600a^4b^{10} - 322561200a^5b^{10} + 225792840a^6b^{10})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(-2513871360b^{11} + 6328583040ab^{11} - 2399481344a^2b^{11} + 2219221056a^3b^{11} - 98170800a^4b^{11})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(64512240a^5b^{11} + 78393952b^{12} - 121428736ab^{12} + 151276528a^2b^{12} - 15146352a^3b^{12})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(10518300a^4b^{12} - 1747200b^{13} + 4090016ab^{13} - 1107568a^2b^{13} + 906192a^3b^{13} + 26320b^{14} - 32240ab^{14})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \\
 & + \frac{65536(35960a^2b^{14} - 240b^{15} + 496ab^{15} + b^{16})}{[\prod_{\mu=0}^{16}\{a-b-2\mu\}][\prod_{\xi=1}^{15}\{a-b+2\xi\}]} \tag{6}
 \end{aligned}$$

C. DERIVATION OF MAIN FORMULA:

Replacing $c = \frac{a+b+34}{2}$ and $z = \frac{1}{2}$ in equation (3), we get

$$(a-b) {}_2F_1(a, b; \frac{a+b+34}{2}; \frac{1}{2}) = a {}_2F_1(a+1, b; \frac{a+b+34}{2}; \frac{1}{2}) - b {}_2F_1(a, b+1; \frac{a+b+34}{2}; \frac{1}{2})$$

Now involving the same parallel method of Ref [4], the main formula is derived.

REFERENCES:

[1] Andrews, L.C.(1992); *Special Function of mathematics for Engineers, second Edition*, McGraw-Hill Co Inc., New York.

[2] Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; *Integrals and Series Vol.3: More Special Functions*. Nauka, Moscow, 1986. Translated from the Russian by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.

[3] Erdélyi, A., Magnus, W., Oberhettinger, F. and Tricomi, F.G.(1953) ; *Higher Transcendental Functions Vol. 1* (Bateman Manuscript Project) McGraw-Hill Book Co. Inc, New York, Toronto and London.

[4] Salahuddin.; Evaluation of a Summation Formula Involving Recurrence Relation, *Gen. Math. Notes.*, 2(2010), 42-59.
