International Research Journal of Pure Algebra -4(3), 2014, 457-470
Available online through www.rjpa.info ISSN 2248–9037

COMPLETELY PRIME PO IDEALS AND PRIME PO IDEALS IN PO TERNARY SEMIGROUPS

V. Siva Rami Reddy[1], V. Sambasiva Rao[2], A. Anjaneyulu[3] and A. Gangadhara Rao*[4]

[1]Dept. of Mathematics, NRI Engineering College, Guntur, India.

[2]Dept. of Mathematics Acharya Nagarjuna University, Guntur, India.

[3,4]Dept. of Mathematics, V S R & N V R College, Tenali, India.

(Received on: 18-03-14; Revised & Accepted on: 29-03-14)

ABSTRACT

In this paper the terms, completely prime ideal, prime ideal, completely semiprime ideal, semiprime ideal, prime radical and complete prime radical in a po ternary semigroup are introduced. It is proved that in a po ternary semigroup (i) A is a prime ideal of T, (ii) For a, b, c $\in T$; $< a > < b > < c > \subseteq A$ *implies* $a \in A$ *or* $b \in A$ *or* $c \in A$, *(iii) For a; b; c* $\in T$; $T^1T^1aT^1T^1b\, T^1T^1c\, T^1T^1 \subseteq A$ *implies* $a \in A$ *or* $b \in A$ *or* $c \in A$ *are equivalent. It is proved that A A po ternary ideal P of a po ternary semigroup T is (1) completely prime iff T\P is either a po ternary subsemigroup of T or empty (2) prime iff T\P is either an m-system or empty. It is also proved that every completely prime ideal of a po ternary semigroup is prime. In a globally idempotent po ternary semigroup, it is proved that every maximal ideal is prime. It is also proved that a globally idempotent po ternary semigroup having a maximal ideal contains semisimple elements. It is proved that a po ternary ideal A of a po ternary semigroup T is completely semiprime if and only if* $x \in T$, $x^3 \in A$ *implies* $x \in A$. *It is proved that if A is a completely semiprime ideal of a po ternary semigroup T, then x, y, z* $\in T$, $xyz \in A$ *implies that* $xyTTz \subseteq A$, $xTTyz \subseteq A$ *and* $xTyTz \subseteq A$. *It is also proved that every completely semiprime ideal of a po ternary semigroup is semiprime. It is proved that a po ternary ideal A of a po ternary semigroup T is completely semiprime if and only if T\A is a d-system of T or empty. It is also proved that the nonempty intersection of a family of (1) completely prime ideals of a po ternary semigroup is completely semiprime (2) prime ideals of a po ternary semigroup is semiprime. And also proved that a po ternary ideal Q of a semigroup T is (1) semiprime iff T\Q is either an n-system or empty. It is proved that if N is an n-system in a po ternary semigroup T and* $a \in N$, *then there exist an m-system M in T such that* $a \in M$ *and* $M \subseteq N$. *It is proved that to each ideal A of a semigroup T, we associate four types of sets namely* A_1, A_2, A_3, A_4 *and we proved that* $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$. *In a commutative po ternary semigroup, it is proved that* $A_1 = A_2 = A_3 = A_4$ *and in general po ternary semigroups, it is proved that* $A_1 \neq A_2 \neq A_3 \neq A_4$ *by means of examples. It is proved that in a po ternary semigroup T if A, B and C are ideals of T, then i)* $A \subseteq B \Rightarrow \sqrt{A} \subseteq \sqrt{B}$, *ii) if* $A \cap B \cap C \neq \emptyset$ *then* $\sqrt{ABC} = \sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$ *and iii)* $\sqrt{\sqrt{A}} = \sqrt{A}$. *In a po ternary semigroup T if A is a po ternary ideal, then* $\sqrt{A}$ *is a semiprime ideal of T. It is proved that a po ternary ideal Q of a po ternary semigroup T is semiprime iff* $\sqrt{Q} = Q$. *It is proved that in a po ternary semigroup T with identity there is a unique maximal ideal M such that* $\sqrt{M^n} = M$ *for all odd natural numbers n. Further it is proved that if A is a po ideal of a po ternary semigroup T then* $\sqrt{A}$ = *{x* $\in$ *T: every m-system of T containing x meets A} i.e.,* $\sqrt{A} = \{ x \in T : M(x) \cap A \neq \emptyset \}$.

Mathematical subject classification (2010): *20M07; 20M11; 20M12.*

Key Words: *completely prime ideal, prime ideal, completely semiprime ideal, semiprime ideal, prime radical and complete prime radical.*

Corresponding author: A. Gangadhara Rao*[4]
[3,4]Dept. of Mathematics, V S R & N V R College, Tenali, India.
E-mail: raoag1967@gmail.com

1. INTRODUCTION

The algebraic theory of semigroups was widely studied by CLIFFORD [2], [3], PETRICH [11] and LYAPIN [10]. The ideal theory in general semigroups was developed by ANJANEYULU [1]. The theory of ternary algebraic systems was introduced by LEHMER [8] in 1932. LEHMER investigated certain algebraic systems called triplexes which turn out to be commutative ternary groups. Ternary semigroups are universal algebras with one associative ternary operation. The notion of ternary semigroup was known to BANACH who is credited with example of a ternary semigroup which can not reduce to a semigroup. SANTIAGO [12] developed the theory of ternary semigroups. SIOSON [15] introduced the ideal theory in ternary semigroups. He also introduced the notion of regular ternary. In this paper we introduce the notions of completely prime ideal, prime ideal, completely semiprime ideal, semiprime ideal, prime radical and complete prime radical and characterize completely prime ideals, completely semiprime ideals, prime radicals and completely prime radicals in po ternary semigroups.

2. PRELIMINARIES

Definition: 2.1 A ternary semigroup T is said to be a ***partially ordered ternary semigroup*** if T is a partially ordered set such that $a \le b \Rightarrow [aa_1a_2] \le [ba_1a_2]$, $[a_1aa_2] \le [a_1ba_2]$, $[a_1a_2a] \le [a_1a_2b]$ for all $a, b, a_1, a_2 \in T$.

Definition: 2.2 A nonempty subset A of a po-ternary semigroup T is said to be ***po left ternary ideal*** or **po left ideal** of T if i) b, c $\in$ T, a $\in$ A implies bca $\in$ A ii) If a $\in$ A and t $\in$ T such that $t \le a$ then t $\in$ A.

Definition: 2.3 A nonempty subset A of a po-ternary semigroup T is said to be ***po lateral ternary ideal*** or ***po lateral ideal*** of T if i) $b, c \in$ T, $a \in$ A implies $bac \in$ A. ii) If $a \in$ A and $t \in$ T such that $t \le a$ then $t \in$ A.

Definition: 2.4 A nonempty subset A of a po-ternary semigroup T is said to be ***po right ternary ideal*** or ***po right ideal*** of T if i) $b, c \in$ T, $a \in$ A implies $abc \in$ A. ii) If $a \in$ A and $t \in$ T such that $t \le a$ then $t \in$ A.

Definition: 2.5 A nonempty subset A of a po-ternary semigroup T is said to be ***po two sided ternary ideal*** or ***po two sided ideal*** of T if i) $b, c \in$ T, $a \in$ A implies $bca \in$ A, $abc \in$ A, ii) If $a \in$ A and $t \in$ T such that $t \le a$ then $t \in$ A.

Definition: 2.6 A nonempty subset A of a po-ternary semigroup T is said to be ***po ternary ideal*** or ***po ideal*** of T if
i) $b, c \in$ T, $a \in$ A implies $bca \in$ A, $bac \in$ A, $abc \in$ A,
ii) If $a \in$ A and $t \in$ T such that $t \le a$ then $t \in$ A.

Theorem: 2.7 Let T be a po ternary semigroup and A⊆T, B⊆T. Then
(i) A⊆(A],
(ii) ((A]] = (A],
(iii) (A](B](C] ⊆ (ABC] and
(iv) A ⊆ B ⇒ A ⊆ (B],
(v) A ⊆ B ⇒ (A] ⊆ (B].

Theorem: 2.8 The nonempty intersection of any family of po left ideals (or po lateral ideals or po right ideals or po two sided ideals or po ideals) of a po ternary semigroup T is a po left ideal (or po lateral ideal or po right ideal or po two sided ideal or po ideal) of T.

3. COMPLETELY PRIME PO IDEALS AND PRIME PO IDEALS

Definition: 3.1 A po (left/lateral/right) ideal A of a po ternary semigroup T is said to be a *completely prime* (*left*/*lateral*/*right*) *ideal* of T provided $x, y, z \in$ T and $xyz \in$ A implies either $x \in$ A or $y \in$ A or $z \in$ A.

Theorem: 3.2 A po ideal A of a po ternary semigroup T is completely prime if and only if $x_1, x_2, \ldots, x_n \in$T, n is an odd natural number, $x_1 x_2 \ldots x_n \in$A ⇒ $x_i \in$A for some i = 1, 2, 3, …..n.

Proof: Suppose that A is a completely prime po ideal of T.

Let $x_1, x_2, \ldots, x_n \in$ T where n is an odd natural number and $x_1 x_2 \ldots x_n \in$A.

If n = 1 then clearly $x_1 \in$A.

If n = 3 then $x_1x_2x_3 \in$A ⇒ $x_1 \in$A or $x_2 \in$A or $x_3 \in$A.

If $n = 5$ then $x_1x_2x_3x_4x_5 \in A \Rightarrow x_1x_2x_3 \in A$ or $x_4 \in A$ or $x_5 \in A$.

$\Rightarrow x_1 \in A$ or $x_2 \in A$ or $x_3 \in A$ or $x_4 \in A$ or $x_5 \in A$.

Therefore by induction on n, $x_1 x_2 \ldots x_n \in A \Rightarrow x_i \in A$ for some $i = 1, 2, 3, \ldots n$.

The converse part is trivial.

Theorem: 3.3 A po ideal A of a po ternary semigroup T is completely prime if and only if T\A is either a subsemigroup of T or empty.

Proof: Suppose that A is a completely prime po ideal of T and $T\backslash A \neq \emptyset$.

Let $a, b, c \in T\backslash A$. Then $a \notin A$, $b \notin A$, $c \notin A$. Suppose if possible $abc \notin T\backslash A$.

Then $abc \in A$. Since A is completely prime, either $a \in A$ or $b \in A$ or $c \in A$.

It is a contradiction. Therefore $abc \in T\backslash A$. Hence T\A is a subsemigroup of T.

Conversely suppose that T\A is a subsemigroup of T or T\A is empty.

If T\A is empty then A = T and hence A is completely prime.

Assume that T\A is a subsemigroup of T. Let $a, b, c \in T$ and $abc \in A$.

Suppose if possible $a \notin A$, $b \notin A$, and $c \notin A$.

Then $a \in T\backslash A$, $b \in T\backslash A$ and $c \in T\backslash A$. Since T\A is a subsemigroup, $abc \in T\backslash A$ and hence $abc \notin A$. It is a contradiction.

Hence either $a \in A$ or $b \in A$ or $c \in A$. Therefore A is a completely prime po ideal of T.

Definition: 3.4 A po ideal A of a po ternary semigroup T is said to be a ***prime ideal*** of T provided X, Y, Z are ideals of T and $XYZ \subseteq A \Rightarrow X \subseteq A$ or $Y \subseteq A$ or $Z \subseteq A$.

Theorem: 3.5 In a po ternary semigroup T, the following conditions are equivalent:
(i) A is a prime po ideal of T.
(ii) $a, b, c \in T$; $< a > < b > < c > \subseteq A$ implies $a \in A$ or $b \in A$ or $c \in A$.
(iii) $a, b, c \in T$; $T^1T^1aT^1T^1b\, T^1T^1c\, T^1T^1 \subseteq A$ implies $a \in A$ or $b \in A$ or $c \in A$.

***Proof*:** (i) ⇒ (ii): Suppose that A is a prime po ideal of T. Then (i) ⇒ (ii) is obvious.

(ii) ⇒ (iii): Let $a, b, c \in T$ such that $T^1T^1aT^1T^1b\, T^1T^1c\, T^1T^1 \subseteq A$.

Now $<a> <b> <c> = (T^1T^1aT^1T^1)(T^1T^1bT^1T^1)(T^1T^1cT^1T^1) \subseteq T^1T^1aT^1T^1b\, T^1T^1c\, T^1T^1 \subseteq A \Rightarrow a \in A$ or $b \in A$ or $c \in A$.

(iii) ⇒ (i): Suppose that $a, b, c \in T$; $T^1T^1aT^1T^1bT^1T^1cT^1T^1 \subseteq A \Rightarrow a \in A$ or $b \in A$ or $c \in A$.

Let X, Y, Z be the three ideals of T and $XYZ \subseteq A$.

Suppose if possible $X \nsubseteq A$, $Y \nsubseteq A$, $Z \nsubseteq A$.

Since $X \nsubseteq A$, $Y \nsubseteq A$, $Z \nsubseteq A$, there exists $a, b, c \in T$ such that

$a \in X$ and $a \notin A$, $b \in Y$ and $b \notin A$ and $c \in Z$ and $c \notin A$.

Now $T^1T^1aT^1T^1bT^1T^1cT^1T^1 \subseteq XYZ \subseteq A \Rightarrow a \in A$ or $b \in A$ or $c \in A$. It is a contradiction.

Therefore $X \subseteq A$ or $Y \subseteq A$ or $Z \subseteq A$ and hence A is a prime po ideal of T.

Theorem: 3.6 A po ternary ideal A of a po ternary semigroup T is prime if and only if $X_1, X_2, \ldots, X_n$ are ideals of T, n is an odd natural number, $X_1 X_2 \ldots X_n \subseteq A \Rightarrow X_i \subseteq A$ for some $i = 1, 2, 3 \ldots n$.

***Proof*:** Suppose that A is a prime ideal of T.

Let $X_1, X_2, \ldots, X_n$ are ideals of T, n is an odd natural number and $X_1 X_2 \ldots X_n \subseteq A$

If $n = 1$ then clearly $X_1 \subseteq A$.

If $n = 3$ then $X_1X_2X_3 \subseteq A \Rightarrow X_1 \subseteq A$ or $X_2 \subseteq A$ or $X_3 \subseteq A$.

If $n = 5$ then $X_1X_2X_3X_4X_5 \subseteq A \Rightarrow X_1X_2X_3 \subseteq A$ or $X_4 \subseteq A$ or $X_5 \subseteq A$

$\Rightarrow X_1 \subseteq A$ or $X_2 \subseteq A$ or $X_3 \subseteq A$ or $X_4 \subseteq A$ or $X_5 \subseteq A$.

Therefore by induction on n, $X_1 X_2 \ldots X_n \subseteq A \Rightarrow X_i \subseteq A$ for some $i = 1, 2, 3, \ldots n$.

The converse part is trivial.

Theorem: 3.7 Every completely prime po ideal of a po ternary semigroup T is a prime po ideal of T.

Proof: Suppose that A is a completely prime po ideal of a po ternary semigroup T.

Let $a, b, c \in T$ and $<a><b><c> \subseteq A$. Then $abc \in A$. Since A is a completely prime po ideal of T, either $a \in A$ or $b \in A$ or $c \in A$. Therefore A is a prime po ideal of T.

Theorem: 3.8 Let T be a commutative po ternary semigroup. A po ideal A of T is a prime po ideal if and only if A is a completely prime po ideal.

Definition: 3.9 A nonempty subset A of a po ternary semigroup T is said to be an ***m-system*** provided for any $a, b, c \in A$ implies that $(T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \cap A \neq \emptyset$.

Theorem: 3.10 A po ideal A of a po ternary semigroup T is a prime po ideal of T if and only if T\A is an m-system of T or empty.

Proof: Suppose that A is a prime po ideal of a po ternary semigroup T and $T \backslash A \neq \emptyset$.

Let $a, b, c \in T \backslash A$. Then $a \notin A$, $b \notin A$ and $c \notin A$.

Suppose if possible $(T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \cap T\backslash A = \emptyset$.

$(T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \cap T\backslash A = \emptyset \Rightarrow (T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \subseteq A$.

Since A is prime, either $a \in A$ or $b \in A$ or $c \in A$.

It is a contradiction. Therefore $(T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \cap T\backslash A \neq \emptyset$.

Hence T\A is an m-system.

Conversely suppose that T\A is either an m-system of T or $T\backslash A = \emptyset$.

If $T\backslash A = \emptyset$, then T = A and hence A is a prime po ideal of T.

Assume that T\A is an m-system of T. Let $a, b, c \in T$ and $<a><b><c> \subseteq A$.

Suppose if possible $a \notin A$, $b \notin A$ and $c \notin A$. Then $a, b, c \in T\backslash A$. Sine T\A is an m-system,

$\Rightarrow (T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \cap T\backslash A \neq \emptyset \Rightarrow (T^1T^1aT^1T^1bT^1T^1c\, T^1T^1) \nsubseteq A$

$\Rightarrow <a><b><c> \nsubseteq A$. It is a contradiction.

Therefore $a \in A$ or $b \in A$ or $c \in A$. Hence A is a prime po ideal of T.

Theorem: 3.11 If T is a po ternary semigroup such that $T = T^3$ then every maximal po ideal of T is a prime po ideal of T.

Proof**:** Let M be a maximal ideal of T. Let A, B, C be three ideals of T such that

$ABC \subseteq M$. Suppose if possible $A \nsubseteq M$, $B \nsubseteq M$, $C \nsubseteq M$.

Now $A \nsubseteq M \Rightarrow M \bigcup A$ is a po ternary ideal of T and $M \subset M \bigcup A \subseteq T$.

Since M is a maximal, $M \bigcup A = T$.

Similarly $B \nsubseteq M \Rightarrow M \bigcup B = T$ and $C \nsubseteq M \Rightarrow M \bigcup C = T$.

Now $T = TTT = (M \bigcup A)(M \bigcup B)(M \bigcup C) \subseteq M \Rightarrow T \subseteq M$. Thus M = T.

It is a contradiction. Therefore either $A \subseteq M$ or $B \subseteq M$ or $C \subseteq M$. Hence M is a prime po ideal of T.

Theorem: 3.12 If T is a po ternary semigroup having maximal ideals and $T = T^3$ then T contains semisimple elements.

Proof**:** Suppose that T is a globally idempotent po ternary semigroup having maximal ideals. Let M be a maximal ideal of T. Then by theorem 3.11., M is prime.

Now if $a \in T \setminus M$ then $< a > \nsubseteq M$ and $< a >^3 \nsubseteq M$. Now $T = M \cup < a > = M \cup < a >^3$.

Therefore $a \in < a >^3$ and hence $< a > = < a >^3$. Thus a is a semisimple element.

Therefore T contains semisimple elements.

4. COMPLETELY SEMIPRIME PO IDEALS AND SEMIPRIME PO IDEALS

Definition: 4.1 A po ideal A of a po ternary semigroup T is said to be a *completely semiprime po ideal* provided $x \in T$, $x^n \in A$ for some odd natural number $n > 1$ implies $x \in A$.

Theorem: 4.2 A po ideal A of a po ternary semigroup T is completely semiprime if and only if $x \in T$, $x^3 \in A$ implies $x \in A$.

Proof: Suppose that A is a completely semiprime po ideal of T.

Then clearly $x \in T$, $x^3 \in A \Rightarrow x \in A$.

Conversely suppose that $x \in T$, $x^3 \in A \Rightarrow x \in A$.

We prove that $x \in T$, $x^n \in A$, for some odd natural number $n > 1 \Rightarrow x \in A \longrightarrow (1)$, by induction on ***n*****.** Clearly (1) is true for $n = 3$.

Assume that (1) is true for $n = k$. i.e., $x^k \in A \Rightarrow x \in A$ for some odd natural number $k > 3$.

Suppose that $x^{k+2} \in A$. Then $x^{k+2} \in A \Rightarrow x^{k+2}.x^{k+2}.x^{k-4} \in A \Rightarrow x^{3k} \in A \Rightarrow (x^k)^3 \in A \Rightarrow x^k \in A \Rightarrow x \in A$.

Therefore $x^k \in A \Rightarrow x \in A$.

By induction, $x^n \in A$ for some natural number n, $n > 1$ implies $x \in A$.

Therefore A is completely semiprime.

Theorem: 4.3 If A is a completely semiprime po ideal of a po ternary semigroup T, then $x, y, z \in T$, $xyz \in A$ implies that $xyTTz \subseteq A$, $xTTyz \subseteq A$ and $xTyTz \subseteq A$.

Proof**:** Let A be a completely semiprime po ideal of a semigroup T. Let $x, y, z \in T$, $xyz \in A$.

Now $xyz \in A \Longrightarrow (zxy)^3 = (zxy)(zxy)(zxy) = z(xyz)(xyz)\, xy \in A$.

$(zxy)^3 \in A$, A is completely semiprime implies $zxy \in A$.

Let $s, t \in T$. Consider $(xystz)^3 = (xystz)(xystz)(xystz) = xyst(zxy)st(zxy)sty \in A$.

$(xystz)^3 \in A$, A is completely semiprime implies $xystz \in A$.

Therefore $x, y, z \in T$, $xyz \in A \Longrightarrow xystz \in A$ for all $s, t \in T \Longrightarrow xyTTz \subseteq A$.

Now $xyz \in A \Longrightarrow (yzx)^3 = (yzx)(yzx)(yzx) = yz(xyz)(xyz)x \in A$.

$(yzx)^3 \in A$, A is completely semiprime $\Rightarrow yzx \in A$.

Let $s, t \in T$. Consider $(xstyz)^3 = (xstyz)(xstyz)(xstyz) = xst(yzx)st(yzx)styz \in A$.

$(xstyz)^3 \in A$, A is completely semiprime implies $xstyz \in A$.

Therefore $x, y, z \in T$, $xstyz \in A$ for all $s, t \in T \Rightarrow xTTyz \subseteq A$.

If $s, t \in T$, then $(xsytz)^3 = (xsytz)(xsytz)(xsytz) = xsyt[zx(syt)(zxs)y]tz \in A$.

$(xsytz)^3 \in A$, A is completely semiprime $\Rightarrow xsytz \in A$.

Therefore $x, y, z \in T$, $xsytz \in A$ for all $s, t \in T \Rightarrow xTyTz \subseteq A$.

Corollary: 4.4 If a po ideal A of a po ternary semigroup T is completely semiprime then $x, y, z \in T$, $xyz \in A$

$\Longrightarrow \langle x \rangle \langle y \rangle \langle z \rangle \subseteq A$.

Theorem: 4.5 Every completely prime po ideal of a po ternary semigroup T is a completely semiprime po ideal of T.

Proof: Let A be a completely prime po ideal of a po ternary semigroup T. Suppose that $x \in T$ and $x^3 \in A$. Since A is a completely prime po ideal of T, $x \in A$.

Therefore A is a completely semiprime po ideal.

Theorem: 4.6 Let A be a prime po ideal of a po ternary semigroup T. If A is completely semiprime po ideal of T then A is completely prime.

Proof: Let $x, y, z \in T$ and $xyz \in A$. Since A is completely semiprime, by corollary 4.4., $xyz \in A \Rightarrow \langle x \rangle\langle y \rangle\langle z \rangle \subseteq A$ $\Rightarrow x \in A$ or $y \in A$ or $z \in A$ and hence A is completely prime.

Theorem: 4.7 The nonempty intersection of any family of a completely prime po ideal of a po ternary semigroup T is a completely semiprime po ideal of T.

Proof: Let $\{A_\alpha\}_{\alpha \in \Delta}$ be a family of a completely prime po ideals of T such that $\bigcap_{\alpha \in \Delta} A_\alpha \neq \emptyset$.

It is clear that $\bigcap_{\alpha \in \Delta} A_\alpha$ is a po ideal. Let $a \in T$ and $a^3 \in \bigcap_{\alpha \in \Delta} A_\alpha$. Then $a^3 \in A_\alpha$ for all $\alpha \in \Delta$.

Since A_α is completely prime, $a \in A_\alpha$ for all $\alpha \in \Delta$ and hence $a \in \bigcap_{\alpha \in \Delta} A_\alpha$.

Therefore $\bigcap_{\alpha \in \Delta} A_\alpha$ is a completely semiprime po ideal of T.

Definition: 4.8 Let T be a po ternary semigroup. A non-empty subset A of T is said to be a ***d*-system** of T if $a \in A$ $\Rightarrow a^n \in A$ for all odd natural number n.

Theorem: 4.9 A po ternary ideal A of a po ternary semigroup T is completely semiprime if and only if T\A is a *d*-system of T or empty.

Proof: Suppose that A is a completely semiprime po ideal of T and $T \backslash A \neq \emptyset$.

Let $a \in T \backslash A$. Then $a \notin A$. Suppose if possible $a^n \notin T \backslash A$ for some odd natural number *n*.

Then $a^n \in A$. Since A is a completely semiprime po ideal then $a \in A$.

It is a contradiction. Therefore $a^n \in T \backslash A$ and hence T\A is a *d*-system.

Conversely suppose that T\A is a *d*-system of T or T\A is empty.

If T\A is empty then T = A and hence A is completely semiprime.

Assume that T\A is a *d*-system of T. Let $a \in T$ and $a^n \in A$.

Suppose if possible $a \notin A$. Then $a \in T \backslash A$.

Since T\A is a *d*-system, $a^n \in T \backslash A$. It is a contradiction. Hence $a \in A$.

Thus A is a completely semiprime po ideal of T.

Definition: 4.10 A po ideal A of a po ternary semigroup T is said to be ***semiprime po ideal*** provided X is po ideal of T and $X^n \subseteq A$ for some odd natural number *n* implies $X \subseteq A$.

Theorem: 4.11 A po ideal A of a po ternary semigroup T is semiprime if and only if X is po ideal of T, $X^3 \subseteq A$ implies $X \subseteq A$.

***Proof*:** Suppose that A is a semiprime po ideal. Then clearly $X^3 \subseteq A \Rightarrow X \subseteq A$.

Conversely suppose that X is a po ideal of T, $X^3 \subseteq A \Rightarrow X \subseteq A$.

We prove that $X^n \subseteq A$, for some odd natural number $n \Rightarrow X \subseteq A \longrightarrow (1)$, by induction on *n*. Since $X^3 \subseteq A \Rightarrow X \subseteq A$, (1) is true for $n = 3$.

Assume that $X^k \subseteq A$ for some odd natural number k, $1 \leq k < n \Rightarrow X \subseteq A$.

Now $X^{k+2} \subseteq A \Rightarrow X^{k+2}.X^{k+2}.X^{k-} \subseteq A \Rightarrow X^{3k} \subseteq A \Rightarrow (X^k)^3 \subseteq A \Rightarrow X^k \subseteq A \Rightarrow X \subseteq A$ by assumption. By induction $X^n \subseteq A$ for some odd natural number $n \Rightarrow X \subseteq A$.

Therefore A is semiprime.

Theorem: 4.12 Every prime po ideal of a po ternary semigroup T is semiprime.

***Proof*:** Suppose that A is a prime po ideal of a po ternary semigroup T. Let X be a po ideal of T such that $X^3 \subseteq A$.

Since A is prime, $X \subseteq A$. Hence A is semiprime.

Theorem: 4.14 If A is a po ideal of a po ternary semigroup T then the following are equivalent.

1. A is a semiprime ideal.
2. For $a \in T$; $< a >^3 \subseteq A$ implies $a \in A$.
3. For $a \in T$; $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \subseteq A$ implies $a \in A$.

Proof:
(i) ⇒ (ii): Suppose that A is a semiprime ideal of T. Then (i) ⇒ (ii) is obvious.

(ii) ⇒ (iii): Let $a \in T$ such that $T^1T^1aT^1T^1a\, T^1T^1a\, T^1T^1 \subseteq A$.

Now $< a >^3 = (T^1T^1aT^1T^1)(T^1T^1aT^1T^1)(T^1T^1aT^1T^1) \subseteq T^1T^1aT^1T^1aT^1T^1aT^1T^1 \subseteq A$

$\Rightarrow a \in A$.

(iii) ⇒ (i): Suppose that $a \in T$; $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \subseteq A \Rightarrow a \in A$.

Let X be a po ideal of T and $X^3 \subseteq A$.

Suppose if possible $X \nsubseteq A$.

Suppose $X \nsubseteq A$ there exists a such that $a \in X$ and $a \notin A$. $a \in X \Rightarrow a^3 \in X^3 \subseteq A$.

Now $T^1T^1aT^1T^1aT^1T^1aT^1T^1 \subseteq X^3 \subseteq A \Rightarrow a \in A$. It is a contradiction.

Therefore $X \subseteq A$ and hence A is a semiprime po ideal of T.

Theorem: 4.14 Every completely semiprime po ideal of a po ternary semigroup T is a semiprime po ideal of T.

***Proof*:** Suppose that A is a completely semiprime po ideal of a po ternary semigroup T.

Let $a \in T$ and $<a>^n \subseteq A$ for some odd natural number n.

Now $aaa\ldots..a$(n odd terms) $\in < a^n > \subseteq <a>^n \subseteq A \Rightarrow a^n \in A \Rightarrow a \in A \Rightarrow < a > \subseteq A$.

Therefore A is a semiprime po ideal of T.

Theorem: 4.15 Let T be a commutative po ternary semigroup. A po ideal A of T is completely semiprime if and only if it is semiprime.

Proof: Suppose that A is a completely semiprime po ideal of T. By theorem 4.14, A is a semiprime po ideal of T.

Conversely suppose that A is a semiprime po ideal of T.

Let $x \in T$ and $x^n \in A$ for some odd natural number n.

Now $x^n \in A \Rightarrow < x >^n \subseteq A \Rightarrow < x > \subseteq A \Rightarrow x \in A$. Since A is semiprime.

Therefore A is a completely semiprime po ideal of T.

Theorem: 4.16 The nonempty intersection of any family of prime po ideals of a po ternary semigroup T is a semiprime po ideal of T.

Proof: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of prime ideals of T such that $\bigcap_{\alpha\in\Delta} A_\alpha \neq \emptyset$. It is clear that $\bigcap_{\alpha\in\Delta} A_\alpha$ is a po ideal. Let

$a \in T$, $< a >^3 \subseteq \bigcap_{\alpha\in\Delta} A_\alpha$ then $< a >^3 \subseteq A_\alpha$ for all $\alpha \in \Delta$.

Since A_α is a prime, $< a > \subseteq A_\alpha$ for all $\alpha \in \Delta$. So $< a > \in \bigcap_{\alpha\in\Delta} A_\alpha$.

Therefore $\bigcap_{\alpha\in\Delta} A_\alpha$ is a semiprime po ideal of T.

Definition: 4.17 A non-empty subset A of a po ternary semigroup T is said to be an ***n*-system** provided $a \in A$ implies that $(T^1T^1aT^1T^1aT^1T^1aT^1T^1) \cap A \neq \emptyset$.

Theorem: 4.18 Every *m*-system in a po ternary semigroup T is an *n*-system.

Proof: Let A be an *m*-system of a po ternary semigroup T. Let $a \in$ A.

Since A is an *m*-system, $a \in$ A, $(T^1T^1aT^1T^1aT^1T^1aT^1T^1) \cap A \neq \emptyset$.

Therefore *A is* an *n*-system of T.

Theorem: 4.19 A po ideal Q of a po ternary semigroup T is a semiprime po ideal if and only if T\Q is an *n*-system of T (or) empty.

Proof: Suppose that A is a semiprime po ideal of a po ternary semigroup T and T\A $\neq \emptyset$.

Let $a \in$ T\A. Then $a \notin$ A.

Suppose if possible $(T^1T^1aT^1T^1aT^1T^1aT^1T^1) \cap T \backslash A = \emptyset$.

$(T^1T^1aT^1T^1aT^1T^1aT^1T^1) \cap T\backslash A = \emptyset \Rightarrow (T^1T^1aT^1T^1aT^1T^1aT^1T^1) \subseteq A$.

Since A is semiprime, either $a \in$ A.

It is a contradiction. Therefore $(T^1T^1aT^1T^1aT^1T^1aT^1T^1) \cap T\backslash A \neq \emptyset$.

Hence T\A is an *n*-system.

Conversely suppose that T\A is either an *n*-system or T\A $= \emptyset$.

If T\A = $\emptyset$ then T = A and hence A is a semiprime ideal.

Assume that T\A is an *n*-system of T. Let $a \in$ T and $< a > \subseteq$A.

Let $a \in$ T\A, T\A is an *n*-system of T $\Rightarrow (T^1T^1aT^1T^1aT^1T^1aT^1T^1) \cap T\backslash A \neq \emptyset$.

Suppose if possible $a \notin$ A. Then $a \in$ T\A. Since T\A is an *m*-system.

Then $(T^1T^1aT^1T^1aT^1T^1aT^1T^1) \subseteq T\backslash A \Rightarrow (T^1T^1aT^1T^1aT^1T^1aT^1T^1) \nsubseteq A \Rightarrow < a > \nsubseteq A$.

It is a contradiction. Therefore $a \in$ A. Hence A is a semiprime po ideal of T.

Theorem: 4.20 If N is an *n*-system in a po ternary semigroup T and $a \in$ N, then there exist an *m*-system M in T such that $a \in$ M and M $\subseteq$ N.

***Proof*:** We construct a subset M of N as follows:

Define $a_1 = a$, Since $a_1 \in$ N and N is an *n*-system, $(T^1T^1a_1T^1T^1a_1T^1T^1a_1T^1T^1) \cap N \neq \emptyset$.

Let $a_2 \in (T^1T^1a_1T^1T^1a_1T^1T^1a_1T^1T^1) \cap N$.

Since $a_2 \in$ N and N is an *n*-system, $(T^1T^1a_2T^1T^1a_2T^1T^1a_2T^1T^1) \cap N \neq \emptyset$ and so on.

In general, if a_i has been defined with $a_i \in$ N, choose a_{i+1} as an element of $(T^1T^1a_2T^1T^1a_2T^1T^1a_2T^1T^1] \cap N$. Let M = $\left\{a_1, a_2 .....a_{i,} a_{i+1}....\right\}$. Now $a \in$ M and M $\subseteq$ N.

We now show that M is an *m*-system.

Let a_i, a_j, $a_k \in$ M (for $i \leq j \leq k$).

Then $a_{k+1} \in (T^1T^1a_kT^1T^1a_kT^1T^1a_kT^1T^1) \subseteq (T^1T^1a_jT^1T^1a_jT^1T^1a_kT^1T^1)$
$\subseteq (T^1T^1a_iT^1T^1a_jT^1T^1a_kT^1T^1)$

$\Rightarrow a_{k+1} = T^1T^1a_iT^1T^1a_jT^1T^1a_kT^1T^1$. But $a_{k+1} \in M$, so $a_{k+1} \in (T^1T^1a_iT^1T^1a_jT^1T^1a_kT^1T^1) \cap M$,

Therefore M is an *m*-system.

5. PRIME PO RADICAL AND COMPLETELY PRIME PO RADICAL

Notation: 5.1 If A is a po ideal of a po ternary semigroup T, then we associate the following four types of sets.
A_1 = The intersection of all completely prime po ideals of T containing A.
$A_2 = \{x \in T: x^n \in A$ for some odd natural numbers $n\}$
A_3 = The intersection of all prime po ideals of T containing A.
$A_4 = \{x \in T: \langle x \rangle^n \subseteq A$ for some odd natural number $n\}$

Theorem: 5.2 If A is a po ideal of a po ternary semigroup T, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

Proof:
i) $A \subseteq A_4$: Let $x \in A$. Then $\langle x \rangle \subseteq A$ and hence $x \in A_4$

Therefore $A \subseteq A_4$
ii) $A_4 \subseteq A_3$: Let $x \in A_4$. Then $\langle x \rangle^n \subseteq A$ for some odd natural number *n.*

Let P be any prime po ideal of T containing A.

Then $\langle x \rangle^n \subseteq A$ for some odd natural number $n \Rightarrow \langle x \rangle^n \subseteq P$.

Since P is prime, $\langle x \rangle \subseteq P$ and hence $x \in P$.

Since this is true for all prime ideals of P containing A, $x \in A_3$. Therefore $A_4 \subseteq A_3$
iii) $A_3 \subseteq A_2$: Let $x \in A_3$. Suppose if possible $x \notin A_2$.
Then $x^n \notin A$ for all odd natural number *n*.

Consider $Q = \bigcup x^n$ for all odd natural number *n*, and $x \in T$.

Let *a, b, c* $\in Q$. Then $a = (x)^r$, $b = (x)^s$, $c = (x)^t$ for some odd natural numbers *r, s, t*.

Therefore $abc = (x)^r\ (x)^s\ (x)^t = x^{r+s+t} \in Q$ and hence Q is a subsemigroup of T.

By theorem 3.3, P = T\Q is a completely prime po ideal of T and $x \notin P$.

By theorem 3.8, P is a prime po ideal of T and $x \notin P$. Therefore $x \notin A_3$.

It is a contradiction. Therefore $x \in A_2$ and hence $A_3 \subseteq A_2$.
iv) $A_2 \subseteq A_1$: Let $x \in A_2$. Now $x \in A_2 \Rightarrow x^n \in A$ for some odd natural number *n*.

Let P be any completely prime po ideal of T containing A.

Then $x^n \in A \subseteq P \Rightarrow x^n \in P \Rightarrow x \in P$. Therefore $x \in A_1$. Therefore $A_2 \subseteq A_1$.
Hence $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

Theorem: 5.3 If A is a po ideal of a commutative po ternary semigroup T, then $A_1 = A_2 = A_3 = A_4$

Proof: By theorem 5.2, A $\subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$. By theorem 3.8, in a commutative po ternary semigroup T, A po ideal A is a prime po ideal if A is completely prime po ideal.

So $A_1 = A_3$. By theorem 4.15, in a commutative po ternary semigroup T A po ideal A is semiprime if and only if A is completely semiprime po ideal.

So $A_4 = A_2$ and hence $A_1 = A_2 = A_3 = A_4$.

Note: 5.4 In an arbitrary po ternary semigroup $A_1 \neq A_2 \neq A_3 \neq A_4$.

Definition: 5.5 If A is a po ideal of a po ternary semigroup T, then the intersection of all prime po ideals of T containing A is called ***prime po radical*** or simply ***po radical*** of A and it is denoted by $\sqrt{A}$ or *rad* A.

Definition: 5.6 If A is a po ideal of a po ternary semigroup T , then the intersection of all completely prime po ideals of T containing A is called *completely prime po radical* or simply *complete po radical* of A and it is denoted by *c.rad* A.

Note: 5.7 If A is a po ideal of a po ternary semigroup T, then *rad* A = A_3, *c.rad* A = A_1 and *rad* A ⊆*c.rad* A**.**

Corollary: 5.8 If $a \in \sqrt{A}$, then there exist a positive integer *n* such that $a^n \in$ A for some odd natural number $n \in$ N.

Proof: By theorem 5. 2, $A_3 \subseteq A_2$ and hence $a \in \sqrt{A} = A_3 \subseteq A_2$.

Therefore $a^n \in$ **A** for some odd natural number $n \in$ N.

Corollary: 5.9 If A is a po ideal of a commutative po ternary semigroup T, then *rad* A = *c.rad* A.

Proof: By theorem 5.3, *rad* A = *c.rad* A.

Corollary: 5.10 If A is a po ideal of a po ternary semigroup T then *c.rad* A is a completely semiprime po ideal of T.

Proof: By theorem 4.5, *c.rad* A is a completely semiprime po ideal of T.

Theorem: 5.11 If A, B and C are any three ideals of a po ternary semigroup T, then

i) A $\subseteq$ B $\Rightarrow \sqrt{A} \subseteq \sqrt{B}$

ii) if A ∩ B∩ C ≠ ∅ then $\sqrt{ABC} = \sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$

iii) $\sqrt{\sqrt{A}} = \sqrt{A}$ **.**

Proof:

i) Suppose that A $\subseteq$ B. If P is a prime po ideal containing B then P is a prime po ideal containing A. Therefore $\sqrt{A} \subseteq \sqrt{B}$.

ii) Let P be a prime po ideal containing ABC. Then ABC ⊆ P $\Rightarrow$ A⊆P or B⊆P or C⊆P $\Rightarrow A \cap B \cap C$ ⊆P.

Therefore P is a prime po ideal containing A $\cap$ B $\cap$ C.

Therefore *rad*($A \cap B \cap C$) ⊆ *rad*(ABC).

Now let P be a prime po ideal containing $A \cap B \cap C$.

Then $A \cap B \cap C$ ⊆ P $\Rightarrow$ ABC ⊆ $A \cap B \cap C$ ⊆ P $\Rightarrow$ ABC ⊆ P.

Hence P is a prime po ideal containing ABC. Therefore *rad* (ABC) ⊆ *rad*($A \cap B \cap C$).

Therefore *rad*(ABC) = *rad*($A \cap B \cap C$).

Since $A \cap B \cap C \neq \emptyset$, it is clear that A ∩ B ∩ C is a po ideal in T. Let $x \in \sqrt{A \cap B \cap C}$.

Then there exists an odd natural number n ∈N such that $x^n \in A \cap B \cap C$.

Therefore $x^n \in$ A, $x^n \in$B and $x^n \in$C. It follows that $x \in \sqrt{A}$, $x \in \sqrt{B}$ and $x \in \sqrt{C}$.

Therefore $x \in \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$.

Consequently, $x \in \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$ implies that there exists odd natural numbers n, m, p ∈N such that $x^n \in$ A, $x^m \in$B and $x^p \in$ C. Clearly, $x^{nmp} \in$A∩B∩C.

Thus $x \in \sqrt{A \cap B \cap C}$. Therefore if A ∩ B ∩ C ≠ ∅ then $\sqrt{A \cap B \cap C} = \sqrt{A} \cap \sqrt{B} \cap \sqrt{C}$.

iii) $\sqrt{A}$ = The intersection of all prime po ideals of T containing A.

Now $\sqrt{\sqrt{A}}$ = The intersection of all prime po ideals of T containing $\sqrt{A}$.
= The intersection of all prime po ideals of T containing A = $\sqrt{A}$

Therefore $\sqrt{\sqrt{A}} = \sqrt{A}$.

Theorem: 5.12 If A is a po ideal of a po ternary semigroup T then $\sqrt{A}$ is a semiprime po ideal of T.

Proof: By theorem 4.16, $\sqrt{A}$ is a semiprime po ideal of T.

Theorem: 5.13 A po ideal Q of po ternary semigroup T is a semiprime po ideal of T if and only if $\sqrt{Q}$ =Q.

***Proof*:** Suppose that Q is a semiprime po ideal. Clearly Q⊆√Q.

Suppose if possible √Q ⊈ Q.

Let a ∈ √Q and a ∉ Q. Now a ∉ Q ⇒ a ∈T\Q and Q is semiprime. By theorem 4.19,

T\Q is an n-system. By theorem 4.20, there exists an m-system M such that a∈M ⊆T\Q.

Q ⊆ T\M and now T\M is a prime po ideal of T, a ∉ T\M. It is a contradiction.

Therefore √Q ⊆Q. Hence √Q = Q.

Conversely suppose that Q is a po ideal of T such that √Q = Q.

By corollary 5.12, √Q is a semiprime po ideal of T. Therefore Q is semiprime.

Corollary: 5.14 A po ideal Q of a po ternary semigroup T is a semiprime po ideal if and only if Q is the intersection of all prime po ideal of T contains Q.

***Proof*:** By theorem 5.13, Q is semiprime iff Q is the intersection of all prime po ideals of T contains Q.

Corollary: 5.15 If A is a po ideal of a po ternary semigroup T, then √A is the smallest semiprime po ideal of T containing A.

***Proof*:** We have that √A is the intersection of all prime po ideals containing A in T.

Since intersection of prime po ideals is semiprime, we have √A is semiprime.

Further, let Q be any semiprime po ideal containing A, i.e. A ⊆Q. So √A ⊆√Q.

Since Q is semiprime, By theorem 5.13, √Q = Q. Therefore √A ⊆Q.

Hence √A is the smallest semiprime po ideal of T containing A.

Theorem: 5.16 If P is a prime ideal of a po ternary semigroup T, then $\sqrt{(P)^n} = P$ for all odd natural numbers $n \in$ N.

Proof: We use induction on *n* to prove $\sqrt{P^n} = P$.

First we prove that $\sqrt{P} = P$. Since P is a prime ideal, $P \subseteq \sqrt{P} \subseteq P \Rightarrow \sqrt{P} = P$.

Assume that $\sqrt{P^k} = P$ for odd natural number k such that $1 \leq k < n$.

Now $\sqrt{P^{k+2}} = \sqrt{P^k.P.P} = \sqrt{P^k} \cap \sqrt{P} \cap \sqrt{P} = \sqrt{P} \cap \sqrt{P} \cap \sqrt{P} = \sqrt{P} = P$.

Therefore $\sqrt{P^{k+2}} = P$. By induction $\sqrt{P^n}$ = P for all odd natural number $n \in$ N.

Theorem: 5.17 In a po ternary semigroup T with identity there is a unique maximal ideal M such that $\sqrt{(M)^n}$ = M for all odd natural numbers $n \in$ N.

Proof: Since T contains identity, T is a globally idempotent po ternary semigroup.

Since M is a maximal ideal of T, by theorem 3.11, M is prime.

By theorem 5.16, $\sqrt{(M)^n}$ = M for all odd natural numbers *n*.

Theorem: 5.18 If A is a po ideal of a po ternary semigroup T then $\sqrt{A}$ = {$x \in$T: every *m*-system of T containing *x* meets A} i.e., $\sqrt{A} = \{ x \in T : M(x) \cap A \neq \emptyset \}$.

Proof: Suppose that $x \in \sqrt{A}$. Let M be an m-system containing *x*.

Then T\M is a prime po ideal of T and $x \notin$ T\M. If M $\cap$ A = $\emptyset$ then A $\subseteq$ T\M.

Since T\M is a prime po ideal containing A, $\sqrt{A} \subseteq$ T\M and hence $x \in$ T\M.

It is a contradiction. Therefore M(x) $\cap$ A $\neq \emptyset$. Hence $x \in \{ x \in T : M(x) \cap A \neq \emptyset \}$. Conversely suppose that $x \in \{ x \in T : M(x) \cap A \neq \emptyset \}$.

Suppose if possible $x \notin \sqrt{A}$. Then there exists a prime po ideal P containing A such that $x \notin$ P. Now T\P is an *m*-system and $x \in$ T\P.

A $\subseteq$ P $\Rightarrow$ T\P $\cap$ A = $\emptyset \Rightarrow x \notin \{ x \in T : M(x) \cap A \neq \emptyset \}$.

It is a contradiction. Therefore $x \in \sqrt{A}$. Thus $\sqrt{A} = \{ x \in T : M(x) \cap A \neq \emptyset \}$.

REFERENCES

[1] Anjaneyulu. A., Structure and ideal theory of Duo semigroups, Semigroup forum, vol22 (1981), 237-276.

[2] Clifford A.H and Preston G.B., The algebroic theory of semigroups, volI American Math. Society, Province (1961).

[3] Clifford A.H and Preston G.B., The algebroic theory of semigroups, vol – II American Math. Society, Province (1967).

[4] Hewitt. E. and Zuckerman H.S., Ternary opertions and semigroups, semigroups, Proc. Sympos. Wayne State Univ., Detroit, 1968, 33-83.

[5] Iampan. A., Lateral ideals of ternary semigroups , Ukrainian Math, Bull., 4 (2007), 323-334.

[6] Kar.S ., On ideals in ternary semigroups . Int. J. Math. Gen. Sci., 18 (2003)3013-3023.

[7] Kar.S and Maity.B.K., *Some ideals of ternary semigroups*. Analele Stintifice Ale Universitath "ALI CUZA" DIN IASI (S.N) Mathematica, Tumul LVII. 2011-12.

[8] Lehmer. D.H., A ternary analave of abelian groups, Amer. J. Math., 39 (1932), 329- 338.

[9] Los. J., *On the extending of models I*, Fundamenta Math. 42(1955), 38 – 54.

[10]Lyapin.E.S., Realisation of ternary semigroup, Russian Modern Algebra, Leningrad University, Leningrad, 1981, Pp.43-48.

[11] Petrch.M., Introduction to semigroups, Merril publishing company, Columbus, Ohio (1973).

[12] Santiago. M. L. and Bala S.S., "Ternary semigroups" Semigroups Forum, Vol 81,no.2,pp.380-388,2010.

[13] Sarala. Y, Anjaneyulu. A and Madhusudhana Rao. D., *On ternary semigroups*, International eJournal of Mathematics, Engineering and Technology accepted for publication.

14] Sarala. Y, Anjaneyulu. A and Madhusudhana Rao. D., *Ideals in ternary semigroups*, submitted for publication in International eJournal of Mathematics, Engineering and Technology accepted for publication.

[15] Sioson. F. M., *Ideal theory in ternary semigroups*, Math. Japon.10 (1965), 63-84.

[16] Sivarami Reddy. V, Sambasiva Rao. V, Anjaneyulu. A and Gangadhara Rao. A, *Ideals in partially ordered ternary semigroups*, International Journal of Mathematical Sciences, Technology and Humanities 110 (2014). 1177-1194.

[17] Sivarami Reddy. V, Anjaneyulu. A and Gangadhara Rao. A, *Simple partially ordered ternary semigroups*, IOSR Journal of Mathematics Volume 9, Issue 5 (Jan.2014), PP 25-29.

Source of Support: Nil, Conflict of interest: None Declared

