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SLIGHTLY αg-OPEN AND SLIGHTLY αg-CLOSED MAPPINGS

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ABSTRACT

The aim of this paper is to introduce and study the concepts of slightly αg-open, slightly αg-closed, almost slightly αg-open and almost slightly αg-closed mappings and the interrelationship between other slightly-open and slightly closed maps.

Keywords: *αg-open set, αg-open map, αg-closed map, slightly-closed map, slightly α-open map, slightly α-closed map, slightly αg-open, slightly αg-closed, almost slightly αg-open and almost slightly αg-closed map.*

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1. INTRODUCTION

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open mappings are one such which are studied for different types of open sets by various mathematicians for the past many years. The first Author of the present paper studied slightly open and slightly closed mappings, slightly semi-open and slightly semi-closed mappings, slightly pre-open and slightly pre-closed mappings in the year 2013. S. Balasubramanian, C. Sandhya and P.A.S. Vyjayanthi studied slightly v-open mappings in the year 2013. S. Balasubramanian and C. Sandhya studied slightly β-open and slightly β-closed mappings in the year 2013. Recently in the year 2014 S. Balasubramanian, P.A.S. Vyjaanthi and C. Sandhya studied slightly v-closed mappings. Inspired with these developments we introduce in this paper a new variety of almost slightly open and closed mappings called slightly αg-open, almost slightly αg-open, slightly αg-closed and almost slightly αg-closed mappings and study its basic properties; interrelation with other type of such mappings available in the literature. Throughout the paper X, Y means topological spaces (X, τ) and (Y, σ) on which no separation axioms are assured.

2. PRELIMINARIES

Definition 2.1: $A\subseteq X$ is said to be

a) regular open[α-open] if $A = int(cl(A))$ $[A\subseteq int(cl(int(A)))]$ and regular closed[α-closed] if $A = cl(int(A))[cl(int(cl(A)))\subseteq A]$
b) g-closed[rg-closed, αg-closed] if $cl(A)\subset U[rcl(A)\subset U, \alpha\text{-}cl(A)\subset U]$ whenever $A\subset U$ and U is open[r-open, α-open] in X and g-open[rg-open, αg-open] if its complement X- A is g-closed[rg-closed, αg-closed].

Remark 1: We have the following implication diagrams for closed sets.

(i)
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        rα.closed                    g. closed
           ↑                            ↓
   r. closed → closed → α.closed →  αg. closed          None is reversible
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The same relation is true for open sets also.

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Definition 2.2: A function *f: X→Y* is said to be

1. continuous [resp: r-continuous, α-continuous] if the inverse image of every open set is open [resp: r-open, α-open].
2. r-irresolute [resp: α-irresolute] if the inverse image of every r-open [resp: α-open] set is r-open [resp: α-open].
3. closed[resp: r-closed, α-closed] if the image of every closed set is closed [resp: r-closed, α-closed].
4. g-continuous [resp: rg-continuous, αg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed, αg-closed].

Definition 2.3: A function *f: X→Y* is said to be

1. slightly closed[resp: slightly *α*-closed; slightly r*α*-closed; slightly r-closed; slightly *g*-closed] if the image of every clopen set in *X* is closed[resp: *α*-closed; r*α*-closed; r-closed; g-closed] in *Y*.
2. almost slightly closed[resp: almost slightly *α*-closed; almost slightly r*α*-closed; almost slightly r-closed; almost slightly g-closed] if the image of every r-clopen set in *X* is closed[resp: *α*-closed; r*α*-closed; r-closed; g-closed] in *Y*.
3. slightly open[resp: slightly *α*-open; slightly r*α*-open; slightly r-open; slightly g-open] if the image of every clopen set in *X* is open[resp: *α*-open; r*α*-open; r-open; g-open] in *Y*.
4. almost slightly open[resp: almost slightly *α*-open; almost slightly r*α*-open; almost slightly r-open; almost slightly *g*-open] if the image of every r-clopen set in *X* is open[resp: *α*-open; r*α*-open; r-open; *g*-open] in *Y*.

Definition 2.4: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

3. SLIGHTLY αg-OPEN MAPPINGS

Definition 3.1: A function *f: X→Y* is said to be

i. slightly αg-open if the image of every clopen set in *X* is αg-open in *Y*.
ii. almost slightly αg-open if the image of every r-clopen set in *X* is αg-open in *Y*.

Theorem 3.1: We have the following interrelation among the following slightly open mappings

(i) al.sl.rα.open   al.sl.g.open
     ↑   ↓
al.sl.r.open → al.sl.open → al.sl.α.open → al.sl.αg.open   None is reversible

(ii) sl.rα.open   sl.g.open
     ↑   ↓
sl.r.open → sl.open → sl.α.open → al.sl.αg.open   None is reversible

(iii) If αGO(Y) = RO(Y), then the reverse relations hold for all almost slightly open maps.

(a) al. sl.rα.open   al.sl.g.open
     ↑↓   ↑↓
al.sl.r.open ↔ al.sl.open ↔ al.sl.α.open ↔ al.sl.αg.open

(b) sl.rα.open   sl.g.open
     ↑↓   ↑↓
sl.r.open ↔ sl.open ↔ sl.α.open ↔ sl.αg.open

Example 1: Let *X* = *Y* = {a, b, c}; τ = {φ, {a}, {b, c}, *X*} = σ. Let *f:X→Y* be defined *f*(a) = c, *f*(b) = a and *f*(c) = b. Then *f* is slightly *αg* –open and almost slightly *αg* –open.

Example 2: Let *X* = *Y* = {a, b, c}; τ = {φ, {a}, {b, c}, *X*}; σ = {φ, {a}, {b}, {a, b}, *Y*}. Let *f: X→Y* be defined *f*(a) = c, *f*(b) = a and *f*(c) = b. Then *f* is not slightly *αg*-open, slightly open, slightly α-open, slightly rα-open, almost slightly *αg*-open, almost slightly open, almost slightly α-open, almost slightly rα-open and almost slightly g-open and slightly g-open.

Theorem 3.2:

(i) If (Y, σ) is discrete, then *f* is [almost] slightly open of all types.
(ii) If *f* is [almost]slightly open and *g* is αg-open then *gof* is [almost]slightly αg-open.
(iii) If *f* is [almost]open and *g* is contra αg-open then *gof* is [almost]slightly αg-open.

Corollary 3.1: If *f* is [almost]slightly open and *g* is [*r-; α-*; *rα*-] open then *gof* is [almost] slightly αg-open.

Corollary 3.2: If *f* is [almost]open and *g* is sl-[sl-*r*-; sl-α-; sl-$r\alpha$-] open then *gof* is [almost] slightly αg-open.

Theorem 3.3: If f: X→ Y is [almost] slightly αg-open, then $f(A^o) \subset \alpha g\, (f(A))^o$

Proof: Let A⊆ X be clopen and f: X→ Y is slightly αg-open gives $f(A^o)$ is αg-open in Y and $f(A^o) \subset f(A)$ which in turn gives $\alpha g\, (f(A^o))^o \subset \alpha g\, (f(A))^o$ (1)

Since $f(A^o)$ is αg-open in Y, $\alpha g\, (f(A^o))^o = f(A^o)$ (2)

Combining (1) and (2) we have $f(A^o) \subset \alpha g\, (f(A))^o$ for every subset A of X.

Remark 2: Converse is not true in general

Corollary 3.3:

(i) If f: X→ Y is sl-[sl-*r*-; sl-α-; sl-$r\alpha$-] open, then $f(A^o) \subset \alpha g\, (f(A))^o$
(ii) If f: X→ Y is al-sl-[al-sl-*r*-; al-sl-α-; al-sl-$r\alpha$-] open, then $f(A^o) \subset \alpha g\, (f(A))^o$

Theorem 3.4: If $f: X \to Y$ is [almost]slightly αg-open and $A \subseteq X$ is [r-clopen]clopen, $f(A)$ is $\tau_{\alpha g}$-open in *Y*.

Proo*f:* Let A⊂X be clopen and f:X→Y is slightly αg-open ⇒ $f(A^o) \subset \alpha g\, (f(A))^o$ ⇒ $f(A) \subset \alpha g\, (f(A))^o$, since $f(A) = f(A^o)$. But $\alpha g\, (f(A))^o \subset f(A)$. Combining we get $f(A) = \alpha g\, (f(A))^o$. Hence $f(A)$ is $\tau_{\alpha g}$-open in Y.

Corollary 3.4:

(i) If $f: X \to Y$ is sl-[sl-*r*-; sl-α-; sl-$r\alpha$-] open, then $f(A)$ is $\tau_{\alpha g}$ open in *Y* if A is clopen set in *X*.
(ii) If $f: X \to Y$ is al-sl-[al-sl-*r*-; al-sl-α-; al-sl-$r\alpha$-]open, then $f(A)$ is $\tau_{\alpha g}$ open in *Y* if A is *r*-clopen set in *X*.

Theorem 3.5: If $\alpha g\, (A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:

a) f: X→ Y is [almost]slightly αg-open map
b) $f(A^o) \subset \alpha g\, (f(A))^o$

Proo*f:*

(a) ⇒(b) follows from theorem 3.3.

(b) ⇒ (a) Let A be any clopen set in X, then $f(A) = f(A^o) \subset \alpha g\, (f(A))^o$ by hypothesis. We have $f(A) \subset \alpha g\, (f(A))^o$, which implies $f(A)$ is αg-open. Therefore *f* is [almost] slightly αg-open.

Theorem 3.6: If $\alpha\, (A)^o = r(A)^o$ for every A⊂ Y, then the following are equivalent:

a) f: X→ Y is [almost]slightly αg-open map
b) $f(A^o) \subset \alpha g\, (f(A))^o$

Proo*f:*

(a) ⇒(b) follows from theorem 3.3.

(b) ⇒ (a) Let A be any clopen set in X, then $f(A) = f(A^o) \subset \alpha g\, (f(A))^o$ by hypothesis. We have $f(A) \subset \alpha g\, (f(A))^o$, which implies $f(A)$ is αg-open. Therefore *f* is [almost]slightly αg-open.

Theorem 3.7: $f: X \to Y$ is [almost]slightly αg-open iff for each subset S of *Y* and each $U \in RO(X, f^{-1}(S))$, there is an αg-open set V of *Y* such that S⊆V and $f^{-1}(V) \subseteq U$.

Proof: Assume $f: X \to Y$ is slightly αg-open. Let $S \subseteq Y$ and $U \in RO(X, f^{-1}(S))$. Then *X*-U is clopen in *X* and $f(X\text{-}U)$ is αg-open in *Y* as *f* is slightly αg-open and $V = Y\text{-}\, f(X\text{-}U)$ is αg-open in *Y*. $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$ and $f^{-1}(V) = f^{-1}(Y\text{-}\, f(X\text{-}U)) = f^{-1}(Y)\text{-}\, f^{-1}(f(X\text{-}U)) = f^{-1}(Y)\text{-}(X\text{-}U) = X\text{-}(X\text{-}U) = U$

Conversely Let F be clopen in $X \Rightarrow F^c$ is clopen. Then $f^{-1}(f(F^c)) \subseteq F^c$. By hypothesis there exists an αg-open set V of *Y*, such that $f(F^c) \subseteq V$ and $f^{-1}(V) \supset F^c$ and so $F \subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$. Thus $f(F)$ is αg-open in *Y*. Therefore *f* is slightly αg-open.

Remark 3: Composition of two [almost] slightly αg-open maps is not [almost] slightly αg-open in general

Theorem 3.8: Let *X*, *Y*, *Z* be topological spaces and every αg-open set is [r-clopen] clopen in *Y*. Then the composition of two [almost] slightly αg-open maps is [almost] slightly αg-open.

Proof: (a) Let *f* and g be slightly αg-open maps. Let A be any clopen set in *X* ⇒ *f*(A) is clopen in *Y* (by assumption) ⇒ *g*(*f* (A)) = *g o f*(A) is αg-open in *Z*. Therefore *gof* is slightly αg-open.

Corollary 3.5: Let *X*, *Y*, *Z* be topological spaces and

(i) every [*r*-; *α*-; *rα*-] open set is [r-clopen]clopen in *Y*. Then the composition of two sl-[sl-*r*-; sl-*α*-; sl-*rα*-] open maps is [almost] slightly αg-open.

(ii) every [*r*-; *α*-; *rα*-] open set is *r*-clopen in *Y*. Then the composition of two al-sl-[al-sl-*r*-; al-sl-*α*-; al-sl-*rα*-] open maps is almost slightly αg-open.

Example 3: Let *X* = *Y* = Z = {a, b, c}; τ = {φ, {a}, {a, b}, *X*}; σ = {φ, {a, c}, *Y*} and η = {φ, {a}, {b, c}, *Z*}. *f:X→Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a and *g:Y→Z* be defined *g*(a) = b, *g*(b) = a and *g*(c) = c, then *g, f* and *g o f* are [almost] slightly αg-open.

Theorem 3.9: If *f: X→Y* is [almost] slightly g-open [[almost] slightly rg-open], g: *Y→Z* is αg-open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost]slightly αg-open.

Proof: (a) Let A be clopen in *X*. Then *f*(A) is *g*-open and so open in *Y* as *Y* is $T_{½}$ ⇒ *g*(*f* (A)) = *gof* (A) is αg-open in *Z* (since *g* is αg-open). Hence *gof* is slightly αg-open.

Corollary 3.6: If *f: X→Y* is [almost] slightly *g*-open[[almost]slightly *rg*-open], *g*:*Y→Z* is [*r*-; *α*-; *rα*-] open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost]slightly αg-open.

Theorem 3.10: If *f:X→Y* is [almost]g-open[[almost]rg-open], g:*Y→Z* is [almost]slightly αg-open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost]slightly αg-open.

Proof: (a) Let A be clopen in *X*. Then *f*(A) is *g*-open and so open in *Y* as *Y* is $T_{½}$ ⇒ *g*(*f* (A)) = *gof* (A) is αg-open in *Z* (since *g* is slightly αg-open). Hence *gof* is slightly αg-open.

Corollary 3.7: If *f:X→Y* is [almost]g-open[[almost]rg-open], g:*Y→Z* is sl-[sl-*r*-; sl-*α*-; sl-*rα*-] open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost]slightly αg-open.

Theorem 3.11: If *f:X→Y* is [almost]sl-*g*-open[[almost]sl-*rg*-open], *g*:*Y→Z* is [*r*-; *α*-; *rα*-] open and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is [almost]slightly αg-open.

Proof: Let A be clopen set in X, then *f*(A) is *g*-open in Y and so open in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *gs*-open in *Z*. Hence *gof* is slightly αg-open [since every *gs*-open set is αg-open].

Theorem 3.12: If *f: X→Y*, *g*:*Y→Z* be two mappings such that *gof* is [almost]slightly αg-open [[almost]slightly clopen] then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is [almost] slightly αg-open.

b) If *f* is g-continuous [resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is [almost] slightly αg-open.

Proof: (a) For A clopen in *Y*, f^{-1}(A) open in *X* ⇒ (*g o f*)(f^{-1}(A)) = *g*(A) αg-open in *Z*. Hence *g* is slightly αg-open. Similarly one can prove the remaining parts and hence omitted.

Corollary 3.8: If *f: X→Y*, g:*Y→Z* be two mappings such that *gof* is sl-[sl-*r*-; sl-*α*-; sl-*rα*-] open then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is [almost]slightly αg-open.

b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is [almost]slightly αg-open.

Theorem 3.13: If *f: X→Y*, *g*:*Y→Z* be two mappings such that *gof* is [almost]αg-open then the following statements are true.

a) If *f* is contra-continuous [contra-*r*-continuous] and surjective then *g* is [almost] slightly αg-open.

b) If *f* is contra-g-continuous[contra-rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is [almost] slightly αg-open.

Proof: (a) For A clopen in *Y*, f^{-1}(A) open in *X* ⇒ (*g o f*) (f^{-1}(A)) = *g*(A) αg-open in *Z*. Hence *g* is slightly αg-open.

Corollary 3.9: If *f: X→Y*, g: *Y→Z* be two mappings such that *gof* is [*r*-; *α*-; *rα*-] open then the following statements are true.

a) If *f* is contra-continuous [contra-*r*-continuous] and surjective then *g* is [almost] slightly αg-open.
b) If *f* is contra-g-continuous [contra-rg-continuous], surjective and *X* is $T_{½}$ [r-$T_{½}$] then *g* is [almost] slightly αg-open.

Theorem 3.14: If *X* is αg-regular, *f: X→Y* is r-clopen, *r*-continuous, slightly αg-open surjective and $A^o = A$ for every αg-open set in *Y* then *Y* is αg-regular.

Proof: Let p∈U∈αGO(Y), ∃ a point x∈X ∍ *f*(x) = p by surjection. Since X is αg-regular and *f* is nearly-continuous, ∃ V∈RC(X) ∍ x∈V^o⊂ V⊂ f^{-1}(U) which implies p∈ *f*(V^o)⊂ *f*(V) ⊂ U……………………………(1)
for *f* is αg-open, *f*(V^o)⊂ U is αg-open. By hypothesis $f(V^o)^o = f(V^o)$ and $f(V^o)^o = \{f(V)\}^o$…………(2)

Combining (1) and (2) p∈ $f(V)^o$⊂ *f*(V) ⊂ U and *f*(V) is r-clopen. Hence Y is αg-regular.

Corollary 3.10: If *X* is αg-regular, *f: X→Y* is r-clopen, *r*-continuous, slightly αg-open, surjective and $A^o = A$ for every r-clopen set in *Y* then *Y* is αg-regular.

Theorem 3.15: If *f: X→Y* is [almost] slightly αg-open and *A∈RC(X)*, then f_A:(*X*, τ(A))→(*Y*,σ) is [almost] slightly αg-open.

Proof: Let F be an clopen set in *A*. Then F = A∩E for some clopen set E of *X* and so F is clopen in *X* ⇒ *f*(A) is αg-open in *Y*. But *f*(F) = f_A(F). Therefore f_A is slightly αg-open.

Theorem 3.16: If *f:X→Y* is [almost]slightly αg-open, *X* is $rT_{½}$ and A is rg-open set of *X* then f_A:(*X*, τ(A))→(*Y*, σ) is [almost] slightly αg-open.

Proof: Let F be a clopen set in *A*. Then F = A∩E for some clopen set E of *X* and so F is clopen in *X* ⇒ *f*(A) is αg-open in *Y*. But *f*(F) = f_A(F). Therefore f_A is slightly αg-open.

Corollary 3.11:

(i) If *f:X→Y* is sl-[sl-*r*-; sl-*α*-; sl-*rα*-] open and *A∈RC(X)*, then f_A:(*X*, τ(A))→(*Y*, σ) is [almost]slightly αg-open.
(ii) If *f:X→Y* is al-sl-[al-sl-*r*-; al-sl-*α*-; al-sl-*rα*-] open and *A∈RC(X)*, then f_A:(*X*,τ(A))→(*Y*,σ) is almost slightly αg-open.

Theorem 3.17: If f_i: $X_i→Y_i$ be [almost] slightly αg-open for i =1, 2. Let *f*: X_1xX_2→Y_1xY_2 be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then *f*: X_1xX_2→Y_1xY_2 is [almost] slightly αg-open.

Proof: Let U_1xU_2 ⊆ X_1xX_2 where U_i is clopen in X_i for i = 1, 2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is αg-open set in Y_1xY_2. Hence *f* is slightly αg-open.

Corollary 3.12:

(i) If f_i: $X_i→Y_i$ be sl-[sl-*r*-; sl-*α*-; sl-*rα*-] open for i =1, 2. Let *f*: X_1xX_2→Y_1xY_2 be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then *f*:$X_1xX_2$→$Y_1xY_2$ is [almost] slightly αg-open.
(ii) If f_i: $X_i→Y_i$ be al-sl-[al-sl-*r*-; al-sl-*α*-; al-sl-*rα*-]open for i =1, 2. Let *f*: X_1xX_2→Y_1xY_2 be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then *f*:$X_1xX_2$→$Y_1xY_2$ is almost slightly αg-open.

Theorem 3.18: Every [almost]α*g*-open and [almost]contra α*g*-closed is [almost] slightly α*g*-open map but not conversely.

Proof: Let *A* be any clopen set in X, then *A* is both open and closed in *X*. For, *f* is α*g*-open and contra α*g*-closed, *f* (*A*) is α*g*-open. Hence *f* is slightly α*g*-open.

Theorem 3.19: Every slightly α*g*-open map is almost slightly α*g*-open map but not conversely.

Theorem 3.20: Every [almost]α*g*-open and [almost] contra α*g*-closed map is [almost] slightly α*g*-open map but not conversely.

Proof: Let *A* be any r-clopen set in X, then *A* is both r-open and r-closed in *X*. For, *f* is almost α*g*-open and almost contra α*g*-closed, *f* (*A*) is α*g*-open. Hence *f* is almost slightly α*g*-open.

Example 4: Let $X = Y = Z = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$, $f:X\to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$, then f is [almost]slightly αg-open but not αg-open and contra αg-closed.

Note 1:

(ii) αg.open → sl.αg.open ← c. αg.closed
↑ ↓
al.αg.open → al.sl.αg.open ← al.c. αg.closed None is reversible

Corollary 3.13:

(i) If f is [r-; α-; $r\alpha$-]open and [c-r-;c-α-; c-$r\alpha$-] closed then f is [almost] slightly αg-open.
(ii) If f is [al-; al-r-; al-α-; al-$r\alpha$-]open and [c--; c-r-; c-α-; c-$r\alpha$-]closed then f is slightly αg-open.

Corollary 3.14:

(i) If f is open and g is [sl-r-; *sl*-α-; *sl*-$r\alpha$-] open then *gof* is [almost] slightly αg-open.
(ii) If f is open and g is al-sl-[al-sl-r-; al-*sl*-α-; al-*sl*-$r\alpha$-] open then *gof* is almost slightly αg-open.

4. SLIGHTLY αg-CLOSED MAPPINGS

Definition 4.1: A function $f: X\to Y$ is said to be

i. slightly αg-closed if the image of every clopen set in X is αg-closed in Y.
ii. almost slightly αg-closed if the image of every r-clopen set in X is αg-closed in Y.

Theorem 4.1: We have the following interrelation among the following slightly closed mappings

(i) al.sl.rα.closed al.sl.g.closed
↑ ↓
al.sl.r.closed → al.sl.closed → al.sl.α.closed → al.sl.αg.closed None is reversible

(ii) sl.rα.closed sl.g.closed
↑ ↓
sl.r.closed → sl.closed → sl.α.closed → al.sl.αg.closed None is reversible

(iii) If αGC(Y) = RC(Y), then the reverse relations hold for all almost slightly closed maps.

(c) al. sl.rα.closed al.sl.g.closed
↑↓ ↑↓
al.sl.r.closed ↔ al.sl.closed ↔ al.sl.α.closed ↔ al.sl.αg.closed

(d) sl.rα.closed sl.g.closed
↑↓ ↑↓
sl.r.closed ↔ sl.closed ↔ sl.α.closed ↔ sl.αg.closed

Example 3: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\} = \sigma$. Let $f:X\to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is slightly αg–closed and almost slightly αg–closed.

Example 4: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X\to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not slightly αg-closed, slightly closed, slightly α-closed, slightly rα-closed, almost slightly αg-closed, almost slightly closed, almost slightly α-closed, almost slightly rα-closed and almost slightly g-closed and slightly g-closed.

Theorem 4.2:

(i) If (Y, σ) is discrete, then f is [almost]slightly closed of all types.
(ii) If f is [almost] slightly closed and g is αg-closed then *gof* is [almost] slightly αg-closed.
(iii) If f is [almost] closed and g is contra αg-closed then *gof* is [almost] slightly αg-closed.

Corollary 4.1:

(i) If f is [almost] slightly closed and g is [r-; α-; $r\alpha$-] closed then *gof* is [almost] slightly αg-closed.
(ii) If f is closed[r-closed] and g is sl-[sl-r-; sl-α-; sl-$r\alpha$-] closed then *gof* is slightly αg-closed.
(iii) If f is almost closed [almost r-closed] and g is sl-[sl-r-; sl-α-; sl-$r\alpha$-] closed then *gof* is almost slightly αg-closed.

Theorem 4.3: If f: X→ Y is [almost] slightly αg-closed, then αg(cl(f(A))) ⊂ f(cl(A))

Proof: Let A⊂X be r-clopen and f: X→ Y is slightly αg-closed gives f(cl(A)) is αg-closed in Y and f(A) ⊂ f(cl(A)) which in turn gives αg(cl(f(A))) ⊂ αgcl(f(cl(A)))…………………..(1)

Since f(cl(A)) is αg-closed in Y, αgcl(f(cl(A))) = f(cl(A))………….(2)

From (1) and (2) we have αg(cl(f(A))) ⊂ f(cl(A)) for every subset A of X.

Remark 2: Converse is not true in general

Corollary 4.2:

(i) If f: X→ Y is sl-[sl-r-; sl-α-; sl-$r\alpha$-] closed, then αg(cl(f(A))) ⊂ f(cl(A))
(ii) If f: X→ Y is al-sl-[al-sl-r-; al-sl-α-; al-sl-$r\alpha$-] closed, then αg(cl(f(A))) ⊂ f(cl(A))

Theorem 4.4: If $f: X\to Y$ is [almost] slightly αg-closed and A⊆X is [r-]clopen, f(A) is $\tau_{\alpha g}$-closed in Y.

Proof: Let A⊂X be clopen and f: X→ Y is slightly αg-closed implies αg(cl(f(A))) ⊂ f(cl(A)) which in turn implies αg(cl(f(A))) ⊂ f(A), since f(A) = f(cl(A)). But f(A) ⊂ αg(cl(f(A))). Combaining we get f(A) = αg(cl(f(A))). Hence f(A) is $\tau_{\alpha g}$-closed in Y.

Corollary 4.3:

(i) If $f: X\to Y$ is sl-[sl-r-; sl-α-; sl-$r\alpha$-] closed, then f(A) is $\tau_{\alpha g}$ closed in Y if A is clopen set in X.
(ii) If $f{:}X\to Y$ is al-sl-[al-sl-r-; al-sl-α-; al-sl-$r\alpha$-]closed, then f(A) is $\tau_{\alpha g}$ closed in Y if A is r-clopen set in X.

Theorem 4.5: If αg(cl(f(A))) = rcl(A) for every A⊂Y and X is discrete space, then the following are equivalent:

a) f: X→ Y is [almost]slightly αg-closed map
b) αg(cl(f(A))) ⊂ f(cl(A))

Proof:
(a) ⇒ (b) follows from theorem 4.3

(b) ⇒ (a) Let A be any clopen set in X, then f(A) = f(cl(A)) ⊃ αg(cl(f(A))) by hypothesis. We have f(A) ⊂ αg(cl(f(A))).

Combining we get f(A) = αg(cl(f(A))) = r(cl(f(A)))[by given condition] which implies f(A) is clopen and hence αg-closed. Thus f is slightly αg-closed.

Theorem 4.6: If α(cl(A)) = rcl(A) for every A⊂Y and X is discrete space, then the following are equivalent:

a) f: X→ Y is [almost] slightly αg-closed map
b) αg(cl(f(A))) ⊂ f(cl(A))

Proof:
(a) ⇒ (b) follows from theorem 4.3

(b) ⇒ (a) Let A be any clopen set in X, then f(A) = f(cl(A)) ⊃ αg(cl(f(A))) by hypothesis. We have f(A) ⊂ αg(cl(f(A))). Combining we get f(A) = αg(cl(f(A))) = rcl(f(A))[by given condition] which implies f(A) is clopen and hence αg-closed. Thus f is slightly αg-closed.

Theorem 4.7: $f{:}X\to Y$ is [almost]slightly αg-closed iff for each subset S of Y and each U∈RC(X, f^{-1}(S)), there is an αg-closed set V of Y such that S⊆V and f^{1}(V) ⊆ U.

Proof: Assume $f{:}X\to Y$ is slightly αg-closed. Let S ⊆ Y and U∈RC(X, f^{-1}(S)). Then X-U is clopen in X and f(X-U) is αg-closed in Y as f is slightly αg-closed and V=Y- f(X-U) is αg-closed in Y. f^{-1}(S) ⊆ U ⇒ S⊆ f(U) ⇒ S ⊆ V and f^{-1}(V) = f^{-1}(Y- f(X-U)) = f^{-1}(Y)- f^{-1}(f (X-U)) = f^{-1}(Y)-(X-U) = X-(X-U) = U

Conversely Let F be clopen in X ⇒ F^c is clopen. Then $f^{-1}(f(F^c)) \subseteq F^{c.}$ By hypothesis there exists an αg-closed set V of Y, such that $f(F^c) \subseteq$ V and $f^{-1}(V) \supset F^c$ and so F ⊆ $[f^{-1}(V)]^{c.}$ Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^{c.}$ Thus f (F) is αg-closed in Y. Therefore f is slightly αg-closed.

Remark 3: Composition of two [almost] slightly αg-closed maps is not [almost] slightly αg-closed in general

Theorem 4.8: Let *X*, *Y*, *Z* be topological spaces and every αg-closed set is [r-clopen]clopen in *Y*. Then the composition of two [almost] slightly αg-closed maps is [almost] slightly αg-closed.

Proof: (a) Let *f* and g be slightly αg-closed maps. Let A be any clopen set in *X* ⇒ *f*(A) is *r*-closed in *Y* (by assumption) ⇒ *g*(*f* (A)) = *g o f*(A) is αg-closed in *Z*. Therefore *gof* is slightly αg-closed.

Corollary 4.4: Let *X*, *Y*, *Z* be topological spaces and

(i) every [*r*-; α-; *r*α-] closed set is [r-clopen]clopen in *Y*. Then the composition of two sl-[sl-*r*-; sl-α-; sl-*r*α-] closed maps is [almost] slightly αg-closed.
(ii) every [*r*-; α-; *r*α-] closed set is r-clopen in *Y*. Then the composition of two al-sl-[al-sl-*r*-; al-sl-α-; al-sl-*r*α-] closed maps is almost slightly αg-closed.

Example 5: Let *X* = *Y* = Z = {a, b, c}; τ = {φ, {a}, {a, b}, *X*}; σ = {φ, {a, c}, *Y*} and η = {φ, {a}, {b, c}, *Z*}. *f:X*→*Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a and *g:Y*→*Z* be defined *g*(a) = b, *g*(b) = a and *g*(c) = c, then *g, f* and *g o f* are slightly αg-closed.

Theorem 4.9: If *f: X*→*Y* is [almost] slightly g-closed[[almost] slightly rg-closed], g:*Y*→*Z* is αg-closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost]slightly αg-closed.

Proof: (a) Let A be clopen in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½}$ ⇒ *g*(*f* (A)) = *gof (*A) is αg-closed in *Z* (since *g* is αg-closed). Hence *gof* is slightly αg-closed.

Corollary 4.5: If *f:X*→*Y* is [almost]slightly *g*-closed[[almost]slightly *rg*-closed], *g*:*Y*→*Z* is [*r*-; α-; *r*α-] closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost] slightly αg-closed.

Theorem 4.10: If *f: X*→*Y* is [almost]g-closed[[almost]rg-closed], g:*Y*→*Z* is αg-closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost] slightly αg-closed.

Proof: (a) Let A be clopen in *X*. Then *f*(A) is *g*-closed and so closed in *Y* as *Y* is $T_{½}$ ⇒ *g*(*f* (A)) = *gof (*A) is αg-closed in *Z* (since *g* is contra αg-closed). Hence *gof* is slightly αg-closed.

Corollary 4.6: If *f: X*→*Y* is [almost]g-closed[[almost]rg-closed], g:*Y*→*Z* is sl-[sl-*r*-; sl-α-; sl-*r*α-] closed and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is [almost]slightly αg-closed.

Theorem 4.11: If *f: X*→*Y* is [almost]sl-*g*-closed[[almost]sl-*rg*-closed], *g*:*Y*→*Z* is [*r*-; α-; *r*α-] closed and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is [almost]slightly αg-closed.

Proof: Let A be clopen set in X, then *f*(A) is *g*-closed in Y and so closed in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *gs*-closed in *Z*. Hence *gof* is slightly αg-closed [since every *gs*-closed set is αg-closed].

Theorem 4.12: If *f: X*→*Y*, *g*: *Y*→*Z* be two mappings such that *gof* is [almost]slightly αg-closed [[almost] slightly *r*-closed] then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is [almost]slightly αg-closed.
b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is [almost]slightly αg-closed.

Proof: (a) For A clopen in *Y*, f^{-1}(A) closed in *X* ⇒ (*g o f*)(f^{-1}(A)) = *g*(A) αg-closed in *Z*. Hence *g* is slightly αg-closed. Similarly one can prove the remaining parts and hence omitted.

Corollary 4.7: If *f:X*→*Y*, g:*Y*→*Z* be two mappings such that *gof* is sl-[sl-*r*-; sl-α-; sl-*r*α-] closed then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is [almost] slightly αg-closed.
b) If *f* is g-continuous [rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is [almost] slightly αg-closed.

Theorem 4.13: If *f: X*→*Y*, *g*: *Y*→*Z* be two mappings such that *gof* is [almost]αg-closed then the following statements are true.

a) If *f* is contra-continuous [contra-*r*-continuous] and surjective then *g* is [almost] slightly αg-closed.
b) If *f* is contra-g-continuous [contra-rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is [almost] slightly αg-closed.

Proof: (a) For A clopen in *Y*, f^{-1}(A) closed in *X* ⇒ (*g o f*)(f^{-1}(A)) = *g*(A) αg-closed in *Z*. Hence *g* is slightly αg-closed.

Corollary 4.8: If $f: X\to Y$, g: $Y\to Z$ be two mappings such that gof is [r-; α-; $r\alpha$-] closed then the following statements are true.

a) If f is contra-continuous [contra-r-continuous] and surjective then g is [almost] slightly αg-closed.
b) If f is contra-g-continuous [contra-rg-continuous], surjective and X is $T_{½}$ [r-$T_{½}$] then g is [almost] slightly αg-closed.

Theorem 4.14: If X is αg-regular, f: X→ Y is r-closed, nearly-continuous, [almost] slightly αg-closed surjection and $\bar{A} = A$ for every αg-closed set in Y, then Y is αg-regular.

Proof: Let $p\in U\in \alpha gO(Y)$. Then there exists a point $x\in X$ such that $f(x) = p$ as f is surjective. Since X is αg-regular and f is r-continuous there exists $V\in RO(X)$ such that $x\in V\subseteq \overline{V}\subseteq f^{-1}(U)$ which implies $p\in f(V) \subseteq f(\overline{V}) \subseteq f(f^{-1}(U)) = U \to$ (1)

Since f is αg-closed, $f(\overline{V}) \subseteq$ U, By hypothesis $\overline{f(\overline{V})} = f(\overline{V})$ *and* $\overline{f(\overline{V})} = \overline{f(V)} \to$(2)

By (1) & (2) we have $p\in f(V) \subseteq f(\overline{V}) \subseteq U$ and $f(V)$ is αg-closed. Hence Y is αg-regular.

Corollary 4.9: If X is αg-regular, f: X→ Y is r-closed, nearly-continuous, [almost] slightly αg-closed surjection and $\bar{A} = A$ for every r-closed set in Y then Y is αg-regular.

Theorem 4.15: If $f: X\to Y$ is [almost] slightly αg-closed and $A\in RC(X)$, then $f_A:(X,\tau(A))\to(Y,\sigma)$ is [almost] slightly αg-closed.

Proof: Let F be an clopen set in A. Then F = $A\cap E$ for some clopen set E of X and so F is clopen in $X \Rightarrow f(A)$ is αg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is slightly αg-closed.

Theorem 4.16: If $f: X\to Y$ is [almost] slightly αg-closed, X is $rT_{½}$ and A is rg-closed set of X then $f_A:(X, \tau(A))\to(Y, \sigma)$ is [almost]slightly αg-closed.

Proof: Let F be a clopen set in A. Then F = $A\cap E$ for some clopen set E of X and so F is clopen in $X \Rightarrow f(A)$ is αg-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is slightly αg-closed.

Corollary 4.10:

(i) If $f:X\to Y$ is sl-[sl-r-; sl-α-; sl-$r\alpha$-] closed and $A\in RC(X)$, then $f_A:(X, \tau(A))\to(Y, \sigma)$ is [almost] slightly αg-closed.
(ii) If $f:X\to Y$ is al-sl-[al-sl-r-; al-sl-α-; al-sl-$r\alpha$-] closed and $A\in RC(X)$, then $f_A:(X, \tau(A))\to(Y, \sigma)$ is almost slightly αg-closed.

Theorem 4.17: If f_i: $X_i\to Y_i$ be [almost] slightly αg-closed for i =1, 2. Let f: $X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f: $X_1 x X_2\to Y_1 x Y_2$ is [almost]slightly αg-closed.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is clopen in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)$ x $f_2(U_2)$ is αg-closed set in $Y_1 x Y_2$. Hence f is slightly αg-closed.

Corollary 4.11:

(i) If f_i: $X_i\to Y_i$ be sl-[sl-r-; sl-α-; sl-$r\alpha$-] closed for i =1, 2. Let $f:X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is [almost] slightly αg-closed.
(ii) If f_i: $X_i\to Y_i$ be al-sl-[al-sl-r-; al-sl-α-; al-sl-$r\alpha$-] closed for i =1, 2. Let f: $X_1 x X_2\to Y_1 x Y_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1 x X_2\to Y_1 x Y_2$ is almost slightly αg-closed.

Theorem 4.18: Every [almost]αg-closed and [almost]contra αg-open is [almost] slightly αg-closed map but not conversely.

Proof: Let A be any clopen set in X, then A is both open and closed in X. For, f is αg-closed and contra αg-open, $f(A)$ is αg-closed. Hence f is slightly αg-closed.

Example 6: Let $X = Y = Z$ = {a, b, c}; τ = {φ, {a}, {a, b}, X}; σ = {φ, {a}, {b}, {a, b}, Y}, $f:X\to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$, then f is [almost]slightly αg-closed but not αg-closed and contra αg-open.

Note-2:

(ii) αg.closed → sl.αg.closed ← c.αg.open
↑ ↓
al.αg.closed → al.sl.αg.closed ← al.c.αg.open None is reversible

Theorem 4.19: Every slightly α*g*-closed map is almost slightly α*g*-closed map but not conversely.

Corollary 4.12:

(i) If *f* is [*r*-; *α*-; *rα*-]closed and [c-*r*-; c-*α*-; c-*rα*-]open then *f* is [almost]slightly α*g*-closed.
(ii) If *f* is [al-; al-*r*-;al-*α*-; al-*rα*-]closed and [c-; c-*r*-; c-*α*-; c-*rα*-]open then *f* is slightly α*g*-closed.

Corollary 4.13:

(i) If *f* is closed and *g* is [sl-*r*-; *sl*-*α*-; *sl*-*rα*-] closed then *gof* is [almost] slightly α*g*-closed.
(ii) If *f* is closed and *g* is al-sl-[al-sl-*r*-; al-*sl*-*α*-; al-*sl*-*rα*-] closed then *gof* is almost slightly α*g*-closed.

CONCLUSION

In this paper we introduced the concept of slightly α*g*-open, slightly α*g*-closed, almost slightly α*g*-open and almost slightly α*g*-closed mappings, studied their basic properties and the interrelationship between other slightly open and slightly closed maps.

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