



**FUZZY VERSION OF SOFT INT G-MODULES**

**<sup>1</sup>G. SUBBIAH\*, <sup>2</sup>M. NAVANEETHAKRISHNAN, <sup>3</sup>D. RADHA AND <sup>4</sup>S. ANITHA**

**<sup>1</sup>Associate Professor in Mathematics,  
Sri K. G. S. Arts College, Srivaikuntam-628 619, (T.N.), India.**

**<sup>2</sup>Associate Professor in Mathematics, Kamaraj College,  
Thoothukudi-628 003, (T.N.), India.**

**<sup>3</sup>Assistant Professor in Mathematics,  
A. P. C. Mahalaxmi College for Women, Thoothukudi-628 002, (T.N.), India.**

**<sup>4</sup>Assistant Professor in Mathematics,  
M. I. E. T Engineering College, Trichy-620007. (T.N.), India.**

*(Received On: 10-07-16; Revised & Accepted On: 28-07-16)*

---

**ABSTRACT**

*In this paper, we introduce fuzzy version of soft int-G-modules of a vector space with respect to soft structures, which are fuzzy soft int-G-modules (IFSG-module). These new concepts show that how a soft set effects on a G-module of a vector space in the mean of intersection, union and inclusion of sets and thus, they can be regarded as a bridge among classical sets, fuzzy soft sets and vector spaces. We then investigate their related properties with respect to soft set operations, soft image, soft pre-image, soft anti image,  $\alpha$ -inclusion of fuzzy soft sets and linear transformations of the vector spaces. Furthermore, we give the applications of these new G-modules on vector spaces.*

*Index terms: Soft set, IFSG-module, fuzzy soft image, fuzzy soft anti image, trivial, whole.*

---

**1. INTRODUCTION**

The concept of soft set theory is introduced by Molodtsov [1] to overcome uncertainties which cannot be dealt with by classical methods in many areas such as engineering, economics, medical science and social science. At present, work on the soft set theory is progressing rapidly. P.K.Maji *et al.* [2] defined basic properties of soft set theory. Aktaş and Çağman [3] compared to soft sets to the related concepts of fuzzy sets and rough sets and introduced soft group and derived their basic properties. Afterward, soft algebraic structures have been studied by some researchers, such as soft ring, soft field and soft modules [5], soft int-groups [4]. Soft linear spaces and soft norm on soft linear spaces are given and some of their properties are studied by Samanta, Das ve P. Majumdar [7]. In [8] Q. Sun, Z. Zang and J. Liu, introduced the definition of soft modules and constructed some basic properties of soft modules, Many important results could be proved only for representations over algebraically closed fields. Module theoretic approach is better suited to deal with deeper results in representation theory. This is the subject matter of representation theory [9, 10, 11]. Soon after the introduction of fuzzy set theory by L.A. Zadeh [12] in 1965, Rosenfield [13] initiated the fuzzification of algebraic structures. Recently, some researchers studied G-modules on fuzzy sets.

As a continuation of these works S. Fernandez [14] introduced fuzzy parallels of the notions of G-modules, group representations, reducibility, irreducibility and completely reducibility and observe, some of their basic properties. In [15] A.K.Sinho and K. Dewangan studied isomorphism theorems for fuzzy submodules of G-modules. Recently, many authors have studied some algebraic structures of soft set theory. [16, 17, 18, 19, 20] Some interesting results in the theory of soft modules are still being explored currently. However the theory of soft modules has not yet been studied. M.Shabir [21] gave some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets along with a new notion of complement of a soft set. The work of this paper is organized as follows. In the second section as preliminaries, we give basic concepts of soft sets and fuzzy soft G-modules. In Section 3, we introduce IFSG-modules and study their characteristic properties. In Section 4, we give the applications of IFSG-modules.

---

**Corresponding Author: <sup>1</sup>G. Subbiah\*, <sup>1</sup>Associate Professor in Mathematics,  
Sri K. G. S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.**

## 2. PRELIMINARIES

In this section as a beginning, the concepts of G-module [22] soft sets introduced by Molodtsov [1] and the notions of fuzzy soft set introduced by Maji *et al.* [23] have been presented.

**2.1 Definition (Molodtsov<sup>1</sup>):** Let  $U$  be an initial universe,  $P(U)$  be the power set of  $U$ ,  $E$  be the set of all parameters and  $A \subseteq E$ . A soft set  $(f_A, E)$  on the universe  $U$  is defined by the set of order pairs

$$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A \in P(U)\} \text{ where } f_A: E \rightarrow P(U) \text{ such that } f_A(e) = \phi \text{ if } e \notin A.$$

Here  $f_A$  is called an approximate function of the soft set.

**Example:** Let  $U = \{u_1, u_2, u_3, u_4\}$  be a set of four shirts and  $E = \{\text{yellow}(p_1), \text{green}(p_2), \text{black}(p_3)\}$  be a set of parameters.

If  $A = \{p_1, p_2\} \subseteq E$ ,  $f_A(p_1) = \{u_1, u_2, u_3, u_4\}$  and  $f_A(p_2) = \{u_1, u_2, u_3\}$ , then we write the soft set  $(f_A, E) = \{(p_1, \{u_1, u_2, u_3, u_4\}), (p_2, \{u_1, u_2, u_3\})\}$  over  $U$  which describe the "colour of the shirts" which Mr. X is going to buy.

**2.2 Definition (P.K.Maji<sup>23</sup>):** Let  $U$  be an initial universe,  $E$  be the set of all parameters and  $A \subseteq E$ . A pair  $(F, A)$  is called a fuzzy soft set over  $U$  where  $F: A \rightarrow P(U)$  is a mapping from  $A$  into  $P(U)$ , where  $P(U)$  denotes the collection of all fuzzy subsets of  $U$ .

**Example:** Consider the above example, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval  $[0,1]$ . Then  $(f_A, E) = \{(f_A(p_1) = \{(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)\}, f_A(p_2) = \{(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)\})\}$  is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

**2.3 Definition (Curties<sup>9</sup>):** Fuzzy soft class, Let  $U$  be an initial Universe set and  $E$  be the set of attributes. Then the pair  $(U, E)$  denotes the collection of all fuzzy soft sets on  $U$  with attributes from  $E$  and is called a fuzzy soft class.

**Definition 2.4 (Ali al<sup>21</sup>):** Let  $F_A$  and  $G_B$  be two soft sets over  $U$  such that  $A \cap B \neq \phi$ . The restricted intersection of  $F_A$  and  $G_B$  is denoted by  $F_A \cup G_B$ , and is defined as  $F_A \cup G_B = (H, C)$ , where  $C = A \cap B$  and for all  $c \in C$ ,  $H(c) = F(c) \cap G(c)$ .

**2.5 Definition (Shery Fernandez<sup>22</sup>):** Let  $M$  and  $M^*$  be G-modules. A mapping  $\phi: M \rightarrow M^*$  is a G-module homomorphism if

1.  $\phi(k_1 m_1 + k_2 m_2) = k_1 \phi(m_1) + k_2 \phi(m_2)$
2.  $\phi(gm) = g \phi(m)$ ,  $k_1, k_2 \in K$ ,  $m, m_1, m_2 \in M$  &  $g \in G$ .

**2.6 Definition (A'gman al<sup>26</sup>):** Let  $F_A$  and  $G_B$  be soft sets over the common universe  $U$  and  $\psi$  be a function from  $A$  to  $B$ . Then we can define the soft set  $\psi(F_A)$  over  $U$ , where  $\psi(F_A): B \rightarrow P(U)$  is a set valued function defined by

$$\psi(F_A)(b) = \cup \{F(a) \mid a \in A \text{ and } \psi(a) = b\},$$

If  $\psi^{-1}(b) \neq \phi, = 0$  otherwise for all  $b \in B$ . Here,  $\psi(F_A)$  is called the soft image of  $F_A$  under  $\psi$ . Moreover we can define a soft set  $\psi^{-1}(G_B)$  over  $U$ , where  $\psi^{-1}(G_B): A \rightarrow P(U)$  is a set-valued function defined by  $\psi^{-1}(G_B)(a) = G(\psi(a))$  for all  $a \in A$ . Then,  $\psi^{-1}(G_B)$  is called the soft pre image (or inverse image) of  $G_B$  under  $\psi$ .

**2.1. Theorem (A'gman al<sup>25</sup>):** Let  $F_H$  and  $T_K$  be soft sets over  $U$ ,  $F_H^r, T_K^r$  be their relative soft sets, respectively and  $\psi$  be a function from  $H$  to  $K$ . then,

- i)  $\psi^{-1}(T_K^r) = (\psi^{-1}(T_K))^r$ ,
- ii)  $\psi(F_H^r) = (\psi^*(F_H))^r$  and  $\psi^*(F_H^r) = (\psi(F_H))^r$ .

## 3. IFSG-MODULES

In this section, we first define intersection fuzzy soft G-modules of a vector space, abbreviated as IFSG-modules.

We then investigate its related properties with respect to soft set operations.

Let  $G$  be a non-empty set. A fuzzy subset  $\mu$  on  $G$  is defined by  $\mu: G \rightarrow [0,1]$  for all  $x \in G$ .

**3.1. Definition:** Let  $G$  be a group. Let  $M$  be a G-module of  $V$  and  $A_M$  be a fuzzy soft set over  $V$ . Then  $A_M$  is called Intersection Fuzzy Soft G-module of  $V$  (IFSG-m), denoted by  $A_M \lesssim_i V$  if the following properties are satisfied

$$(IFSG-m_1) A(ax + by) \geq A(x) \cap A(y)$$

$$(IFSG-m_2) A(\alpha x) \geq A(x), \text{ for all } x, y \in M, a, b, \alpha \in F.$$

**Example:** Let  $G = \{1, -1\}$ ,  $M = \mathbb{R}^4$  over  $\mathbb{R}$ . Then  $M$  is a  $G$ -module.

Define  $A$  on  $M$  by,

$$A(x) = \begin{cases} 1, & \text{if } x_i = 0 \forall i. \\ 0.5, & \text{if atleast } x_i \neq 0. \end{cases}$$

Where  $x = \{x_1, x_2, x_3, x_4\}$ ;  $x_i \in \mathbb{R}$ . Then  $A$  is a fuzzy soft  $G$ -Module.

**3.1. Proposition:** If  $A_M \lesssim_i V$ , then  $A(0_V) \supseteq A(x)$  for all  $x \in M$ .

**Proof:** Since  $A_M$  is an IFSG-module of  $V$ , then  $A(ax+by) \supseteq A(x) \cap A(y)$  for all  $x, y \in M$  and since  $(M, +)$  is a group, if we take  $ay = -ax$  then, for all  $x \in M$ ,  $A(ax-ax) = A(0_V) \supseteq A(x) \cap A(x) = A(x)$ .

**3.2. Proposition:** If  $A_{M_1} \lesssim_i V$  and  $B_{M_2} \lesssim_i V$ , then  $A_{M_1} \cap B_{M_2} \lesssim_i V$ .

**Proof:** Since  $M_1$  and  $M_2$  are  $G$ -modules of  $V$ , then  $M_1 \cap M_2$  is a  $G$ -module of  $V$ . By definition 2.6, let

$$A_{M_1} \cap B_{M_2} = (A, M_1) \cap (B, M_2) = (T, M_1 \cap M_2),$$

Where,  $T(x) = A(x) \cap B(x)$  for all  $x \in M_1 \cap M_2 \neq \emptyset$ . Then for all  $x, y \in M_1 \cap M_2$  and  $\alpha \in F$ .

$$\begin{aligned} \text{(IFSG-m}_1\text{)} \quad T(ax+by) &= A(ax+by) \cap B(ax+by) \supseteq (A(x) \cap A(y)) \cap (B(x) \cap B(y)) \\ &= (A(x) \cap B(x)) \cap (A(y) \cap B(y)) = T(x) \cap T(y), \end{aligned}$$

$$\text{(IFSG-m}_2\text{)} \quad T(\alpha x) = A(\alpha x) \cap B(\alpha x) \supseteq A(x) \cap B(x) = T(x).$$

There fore  $A_{M_1} \cap B_{M_2} = T_{M_1 \cap M_2} \lesssim_i V$ .

**3.2. Definition:** Let  $(A, M_1)$  and  $(B, M_2)$  be two IFSG-modules of  $V_1$  and  $V_2$  respectively, the product of IFSG-modules  $(A, M_1)$  and  $(B, M_2)$  is defined as  $(A, M_1) \times (B, M_2) = (Q, M_1 \times M_2)$ , where  $Q(x,y) = A(x) \times B(y)$  for all  $(x, y) \in M_1 \times M_2$ .

**3.1. Theorem:** If  $A_{M_1} \lesssim_i V$  and  $B_{M_2} \lesssim_i V$ , then  $A_{M_1} \times B_{M_2} \lesssim_i V_1 \times V_2$ .

**Proof:** Since  $M_1$  and  $M_2$  are  $G$ -modules of  $V_1$  and  $V_2$  respectively, then  $M_1 \times M_2$  is a  $G$ -module of  $V_1 \times V_2$ . By definition 3.2, let

$$\begin{aligned} A_{M_1} \times B_{M_2} &= (A, M_1) \times (B, M_2) \\ &= (Q, M_1 \times M_2), \end{aligned}$$

where  $Q(x, y) = A(x) \times B(y)$  for all  $(x,y) \in M_1 \times M_2$ .

Then for all  $(x_1, y_1), (x_2, y_2) \in M_1 \times M_2$  and  $(\alpha_1, \alpha_2) \in F \times F$ ,

$$\begin{aligned} \text{(IFSG-m}_1\text{)} \quad Q \{(ax_1, by_1) + (ax_2, by_2)\} &= Q(ax_1 + ax_2, by_1 + by_2) \\ &= A(ax_1 + ax_2) \times B(by_1 + by_2) \\ &\supseteq (A(x_1) \cap A(x_2)) \times (B(y_1) \cap B(y_2)) \\ &= Q(x_1, y_1) \cap Q(x_2, y_2) \end{aligned}$$

$$\begin{aligned} \text{(IFSG-m}_2\text{)} \quad Q((\alpha_1, \alpha_2)(x_1, y_1)) &= Q(\alpha_1 x_1 + \alpha_2 y_1) \\ &= A(\alpha_1 x_1) \times B(\alpha_2 y_2) \supseteq A(x_1) \cap B(y_2) = Q(x_1, y_1). \end{aligned}$$

Hence  $A_{M_1} \times B_{M_2} = Q_{M_1 \times M_2} \lesssim_i V_1 \times V_2$ .

**3.3. Definition:** Let  $A_{M_1}$  and  $B_{M_2}$  be two IFSG-module's of  $V$ . If  $M_1 \cap M_2 = \{0_V\}$ , then the sum of IFSG-module's  $A_{M_1}$  and  $B_{M_2}$  is defined as  $A_{M_1} + B_{M_2} = T_{M_1 + M_2}$  where  $T(ax+by) = A(x)+B(y)$  for all  $ax+by \in M_1 + M_2$ .

**3.2. Theorem:** If  $A_{M_1} \lesssim_i V$  and  $B_{M_2} \lesssim_i V$  where  $M_1 \cap M_2 = \{0_V\}$ , then  $A_{M_1} + B_{M_2} \lesssim_i V$ .

**Proof:** Since  $M_1$  &  $M_2$  are  $G$ -modules of  $V$ , then  $M_1 + M_2$  is a  $G$ -modules of  $V$ . By definition: 3.3,

Let  $A_{M_1} + B_{M_2} = (A, M_1) + (B, M_2) = (T, M_1 + M_2)$ , where  $T(ax+by) = A(x)+B(y)$  for all  $ax+by \in M_1 + M_2$ . It is obvious that since  $M_1 \cap M_2 = \{0_V\}$ , then the sum is well defined. Then for all  $ax_1 + by_1, ax_2 + by_2 \in M_1 + M_2$  and  $\alpha \in F$ ,

$$\begin{aligned} T((ax_1 + by_1) + (ax_2 + by_2)) &= T((ax_1 + ax_2) + (by_1 + by_2)) \\ &= A(ax_1 + ax_2) + B(by_1 + by_2) \\ &\supseteq (A(x_1) \cap A(x_2)) + (B(y_1) \cap B(y_2)) \\ &= (A(x_1) + B(y_1)) \cap (A(x_2) + B(y_2)) \\ &= T(ax_1 + by_1) \cap T(ax_2 + by_2) \end{aligned}$$

$$\begin{aligned} T(\alpha(x_1 + y_1)) &= T(\alpha x_1 + \alpha y_1) \\ &= A(\alpha x_1) + B(\alpha y_1) \supseteq A(x_1) + B(y_1) \\ &= T(x_1 + y_1) \end{aligned}$$

Thus,  $A_{M_1} + B_{M_2} \lesssim_i V$ .

**3.4. Definition:** Let  $A_M$  be an IFSG-module of  $V$ . Then,

- (i)  $A_M$  is said to be trivial if  $A(x) = \{0_V\}$  for all  $x \in M$ .
- (ii)  $A_M$  is said to be whole if  $A(x) = V$  for all  $x \in M$ .

**3.3. Proposition:** Let  $A_{M_1}$  and  $B_{M_2}$  be two IFSG-modules of  $V$ , then

- (i) If  $A_{M_1}$  and  $B_{M_2}$  are trivial IFSG-modules of  $V$ , then  $A_{M_1} \cap B_{M_2}$  is a trivial IFSG-module of  $V$ .
- (ii) If  $A_{M_1}$  and  $B_{M_2}$  are whole IFSG-modules of  $V$ , then  $A_{M_1} \cap B_{M_2}$  is a whole IFSG-module of  $V$ .
- (iii) If  $A_{M_1}$  is a trivial IFSG-module of  $V$  and  $B_{M_2}$  is a whole IFSG-modules of  $V$ , then  $A_{M_1} \cap B_{M_2}$  is a trivial IFSG-module of  $V$ .
- (iv) If  $A_{M_1}$  and  $B_{M_2}$  are trivial IFSG-modules of  $V$  where  $M_1 \cap M_2 = \{0_V\}$ , then  $A_{M_1} + B_{M_2}$  is a trivial IFSG-module of  $V$ .
- (v) If  $A_{M_1}$  and  $B_{M_2}$  are whole IFSG-modules of  $V$  where  $M_1 \cap M_2 = \{0_V\}$ , then  $A_{M_1} + B_{M_2}$  is a whole IFSG-module of  $V$ .
- (vi) If  $A_{M_1}$  is a trivial IFSG-module of  $V$  and  $B_{M_2}$  is a whole IFSG-modules of  $V$  where  $M_1 \cap M_2 = \{0_V\}$ , then  $A_{M_1} + B_{M_2}$  is a whole IFSG-module of  $V$ .

**Proof:** The proof is easily seen by definition 2.4, definition3.3, definition3.4 and theorem 3.1.

**3.4. Proposition:** Let  $A_{M_1}$  and  $B_{M_2}$  be two IFSG-modules of  $V_1$  and  $V_2$  respectively. Then

- (i) If  $A_{M_1}$  and  $B_{M_2}$  are trivial IFSG-modules of  $V_1$  and  $V_2$  respectively, then  $A_{M_1} \times B_{M_2}$  is a trivial IFSG-module of  $V_1 \times V_2$ .
- (ii) If  $A_{M_1}$  and  $B_{M_2}$  are whole IFSG-modules of  $V_1$  and  $V_2$  respectively, then  $A_{M_1} \times B_{M_2}$  is a whole IFSG-module of  $V_1 \times V_2$ .

**Proof:** The proof is easily seen by definition 3.2 and definition 3.4

**Applications of IFSG modules:** In this section, we give the applications of soft image, soft pre image, upper  $\alpha$ -inclusion of fuzzy soft sets and linear transformation of vector spaces on vector space with respect to IFSG-modules.

**4.1. Theorem:** If  $A_M \lesssim_i V$ , then  $M_G = \{x \in M / A(x) = A(0_V)\}$  is a G-module of  $M$ .

**Proof:** It is obvious that  $0_V \in M_G$  and  $\emptyset \neq M_G \subseteq M$ . We need to show that  $ax+by \in M_G$  and  $\alpha x \in M_G$  for all  $x, y \in M_G$  and  $\alpha \in F$ , which means that  $A(ax+by) = A(0_V)$  and  $A(\alpha x) = A(0_V)$  have to be satisfied. Since  $x, y \in M_G$  and  $A_M$  is an IFSG-Module of  $V$ , then  $A(x) = A(y) = A(0_V)$ ,  $A(ax+by) \supseteq A(x) \cap A(y) = A(0_V)$ ,  $A(\alpha x) \supseteq A(x) = A(0_V)$  for all  $x, y \in M_G$  and  $\alpha \in F$ . Moreover, by Proposition3.1,  $A(0_V) \supseteq A(ax+by)$  and  $A(0_V) \supseteq A(\alpha x)$  which completes the proof.

**4.2. Theorem:** Let  $A_M$  be a fuzzy soft set over  $V$  and  $\alpha$  be a subset of  $V$  such that  $A(0_V) \supseteq \alpha$ . If  $A_M$  is an IFSG-module of  $V$ , then  $A_M \supseteq \alpha$  is a G-module of  $V$ .

**Proof:** Since  $A(0_V) \supseteq \alpha$ , then  $0_V \in A_M \supseteq \alpha$  and  $\emptyset \neq A_M \supseteq \alpha \subseteq V$ . Let  $x, y \in A_M \supseteq \alpha$ , then  $A(x) \supseteq \alpha$  and  $A(y) \supseteq \alpha$ . We need to show that  $x + y \in A_M \supseteq \alpha$  and  $nx \in A_M \supseteq \alpha$  for all  $x, y \in A_M \supseteq \alpha$  and  $n \in F$ . Since  $A_M$  is an IFSG-module of  $V$ , it follows that  $A(ax + by) \supseteq A(x) \cap A(y) \supseteq \alpha \cap \alpha = \alpha$ .

Furthermore,  $A(nx) \supseteq A(x) \supseteq \alpha$ , which completes the proof.

**4.3. Theorem:** Let  $A_M$  and  $T_W$  be fuzzy soft sets over  $V$ , where  $M$  and  $W$  are G-modules of  $\gamma$  and  $\Psi$  be a linear isomorphism from  $M$  to  $W$ . If  $A_M$  is an IFSG-Module of  $V$ , then so is  $\Psi(A_M)$ .

**Proof:** Let  $w_1, w_2 \in W$ . Since  $\Psi$  is a subjective linear transformation. Then there exists  $m_1, m_2 \in M$  such that  $\Psi(m_1) = w_1, \Psi(m_2) = w_2$ . Then

$$\begin{aligned} (\Psi(A_M))(aw_1 + bw_2) &= \cup\{A(m) : m \in M, \Psi(m) = aw_1 + bw_2\} \\ &= \cup\{A(m) : m \in M, m = \Psi^{-1}(aw_1 + bw_2)\} \\ &= \cup\{A(m) : m \in M, m = \Psi^{-1}(\Psi(aw_1 + bw_2)) = am_1 + bm_2\} \\ &= \cup\{A(am_1 + bm_2) : m_i \in M, \Psi(m_i) = w_i, i = 1,2\} \\ &\supseteq \cup\{A(m_1) \cap A(m_2) : m_i \in M, \Psi(m_i) = w_i, i = 1,2\} \\ &= (\cup A(m_1) : m_1 \in M, \Psi(m_1) = w_1) \cap (\cup A(m_2) : m_2 \in M, \Psi(m_2) = w_2) \\ &= (\Psi(A_M))(w_1) \cap (\Psi(A_M))(w_2) \end{aligned}$$

Now let  $\alpha \in F$  and  $w \in W$ . Since  $\Psi$  is a surjective linear transformation, there exists  $\tilde{m} \in M$  such that  $\Psi(\tilde{m}) = w$ . Then

$$\begin{aligned} (\Psi(A_M))(\alpha w) &= \cup\{A(m) : m \in M, \Psi(m) = \alpha w\} \\ &= \cup\{A(m) : m \in M, m = \Psi^{-1}(\alpha w)\} \\ &= \cup\{A(m) : m \in M, m = \Psi^{-1}(\Psi(\alpha \tilde{m})) = \alpha \tilde{m}\} \\ &= \cup\{A(\alpha \tilde{m}) : \alpha \tilde{m} \in M, \Psi(\tilde{m}) = w\} \\ &= (\Psi(A_M))(w) \end{aligned}$$

Hence,  $\Psi(A_M)$  is an IFSG –module of  $V$ .

**4.4. Theorem:** Let  $A_M$  and  $T_W$  be fuzzy soft sets over  $V$ , where  $M$  and  $W$  are  $G$ -modules of  $\gamma$  and  $\Psi$  be a linear isomorphism from  $M$  to  $W$ . If  $T_W$  is an IFSG-Module of  $V$ , then so is  $\Psi^{-1}(T_W)$ .

**Proof:** Let  $m_1, m_2 \in M$ . Then

$$\begin{aligned} \Psi^{-1}(T_W)(am_1 + bm_2) &= T(\Psi(am_1 + bm_2)) \\ &= T(\Psi(am_1) + \Psi(bm_2)) \\ &\supseteq T(\Psi(m_1)) \cap T(\Psi(m_2)) \\ &= (\Psi^{-1}(T_W))(m_1) \cap (\Psi^{-1}(T_W))(m_2) \end{aligned}$$

Now let  $\alpha \in F$  and  $m \in M$ . Then,

$$\begin{aligned} \Psi^{-1}(T_W)(\alpha m) &= T(\Psi(\alpha m)) \\ &= T(\alpha \Psi(m)) \\ &\supseteq T(\Psi(m)) = \Psi^{-1}(T_W)(m) \end{aligned}$$

Hence  $\Psi^{-1}(T_W)$  is an IFSG –module of  $V$ .

**4.5. Theorem:** Let  $V_1$  and  $V_2$  be two vector spaces and  $(A_1, M_1) \prec_i V_1, (A_2, M_2) \prec_i V_2$ . If  $f: M_1 \rightarrow M_2$  is a linear transformation of vector spaces, then

- (i)  $f$  is surjective, then  $(A_1, f^{-1}(M_2)) \prec_i V_1$ ,
- (ii)  $(A_2, f(M_1)) \prec_i V_2$ ,
- (iii)  $(A_1, \ker f) \prec_i V_1$ .

**Proof:**

- (i) Since  $M_1 < V_1, M_2 < V_2$  and  $f: M_1 \rightarrow M_2$  is a surjective linear transformation, then it is clear that  $f^{-1}(M_2) < V_1$ . Since  $(A_1, M_1) \prec_i V_1$  and  $f^{-1}(M_2) < M_1, A_1(ax+by) \supseteq A_1(x) \cap A_1(y)$  and  $A_1(\alpha x) \supseteq A_1(x)$  for all  $x, y \in f^{-1}(M_2)$  and  $\alpha \in F$ . Hence  $(A_1, f^{-1}(M_2)) \prec_i V_1$ .
- (ii) Since  $M_1 < V_1, M_2 < V_2$  and  $f: M_1 \rightarrow M_2$  is a vector space linear transformation, then  $f(M_1) < V_2$ . Since  $f(M_1) \subseteq M_2$ , the result is obvious by definition 3.1.
- (iii) Since  $\ker f < V_1$  and  $\ker f \subseteq M_1$ , the rest of the proof is clear by definition 3.1.

**4.1. Corollary:** Let  $(A_1, M_1) \prec_i V_1, (A_2, M_2) \prec_i V_2$ . If  $f: M_1 \rightarrow M_2$  is a linear transformation, then  $(A_2, \{0M_2\}) \prec_i V_2$ .

**Proof:** By theorem: 4.5, (iii)  $(A_1, \ker f) \prec_i V_1$ , then  $(A_2, f(\ker f)) = (A_2, \{0M_2\}) \prec_i V_2$ , By theorem 4.5 (ii).

## CONCLUSION

Throughout this paper, we have dealt with IFSG-modules of a vector space. We have investigated their related properties with respect to soft set operations Furthermore; we have derived some applications of IFSG-modules with respect to soft image, soft pre image, soft anti image, Further study could be done for fuzzy soft sub structures of different algebras.

## REFERENCES

1. D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37 (1999) 19 - 31.
2. P.K. Maji, R. Bismas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555 - 562.
4. H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007), 2226 - 2735.
5. N. C, a ğman, F. C, itak, H. Akta, s, Soft int-group and its applications to group theory, Neural Comput. Appl. 21 (2012) 151-158.
6. A. O. Atag`un, A. Sezgin, Soft substructures of rings, fields and modules, Comput. Math. Appl. 61 (3) (2011) 592-601.
7. S. Das and S.K. Samanta, Soft metric, Annas of fuzzy mathematics and informatics, 6 (1) (2013) 77 -94
8. S. Das, P. Majumdar and S. K. Majumdar, On Soft Linear Space and Soft Normed linear space, Math. GM, (2013) orXiv 1308.1016
9. Q. Sun, Z. Zang and J. Liu, Soft sets and soft modules, Lecture Notes in Computer, Sci, 5009 (2008) 403 – 409.
10. C. W. Curties, Representation theory of finite group and associative algebra. Inc, (1962)
11. H. Keneth and K. Ray, Linear algebra, Eastern Economy Second Edition (1990), 20, K. Kaygısız, Normal soft int-groups, arXiv: 1209.3157.
12. John. B. Fraleigh, A First Course in Abstract Algebra, Third Edition, Addition-Wesley / Narosa (1986).
13. L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338 - 353
14. A. Rosenfield. Fuzzy groups, J. Math. Anal. Appl., 35 (1971), 512-517.
15. S. Fernandez, Ph.D. Thesis “A study of fuzzy G-modules” Mahatma Gandhi University, April 2004.
16. A. K. Sinho and K. Dewangan, Isomorphism Theory for Fuzzy Submodules of G-modules, International Journal of Engineering, 3 (2013) 852 – 854
17. S.R.Lopez-Permouth, D.S.Malik, On categories of fuzzy modules, Information Sciences 52 (1990), 211-220.
18. Çağman and Enginoğlu, Soft Matrix Theory and its decision making, Computer and Mathematic with Applications, 59 (2010) 3308-3314
19. F. Feng, Y. B. Jun and X. Zhao, soft semi rings, Comput. Math. Appl. 56 (2008) 2621 - 2628.
20. E.Türkmen, A.Pancar, On some new operations in soft module Theory, Neural Comp and Applic (2012).
21. F. Feng, Y. B. Jun, X. Zhao, Soft Semi rings, Journal Comp Math with Applic, V 56, issue 10, November,(2008), 2621-2628.
22. Ali M.I, Feng F, Liu XY, Min WK, Shabir M (2009) “On some new operations in soft set theory”. Computers and Mathematics with Applications 57:1547–1553
23. Shery Fernandez, Ph.D. thesis “A study of fuzzy g-modules” April 2004.
24. P.K.Maji, R. Biswas and A.R. Roy, Soft set theory, Comput.Math. Appl. 45, 555-562 (2003).
25. N. C, a ğman, F. C, itak and H. Aktas, Soft int-groups and its applications to group theory, Neural Comput. Appl. 21, 151-158 (2012).
26. N. C, a ğman, A. Sezgin and A.O. Atag`un, Soft uni-groups and its applications to group theory, (submitted).
27. N. C, a ğman, F. C, itak and H. Aktas, Soft int-groups and its applications to group theory, Neural Comput. Appl. 21, 151-158 (2012).

**Source of Support: Nil, Conflict of interest: None Declared**

**[Copy right © 2016, RJPA. All Rights Reserved. This is an Open Access article distributed under the terms of the International Research Journal of Pure Algebra (IRJPA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**