International Research Journal of Pure Algebra-6(10), 2016, 417-421
Available online through www.rjpa.info ISSN 2248–9037

ON NANO (1, 2)* GENERALIZED-REGULAR CLOSED SETS IN NANO BITOPOLOGICAL SPACES

K. BHUVANESWARI[1], K. SHEELA[2]

[1]Associate Professor, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal, Tamil Nadu, India.

[2]Research Scholar, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal, Tamil Nadu, India.

(Received On: 19-09-16; Revised & Accepted On: 22-10-16)

ABSTRACT

The purpose of this paper is to define and study a new class of set called Nano (1, 2) generalized-regular closed sets in nano bitopological spaces. Basic properties of nano (1, 2)* generalized regular closed sets are analyzed. The new notion of nano (1, 2)* generalized-regular closure and their relation with already existing well known sets are also investigated.*

Keywords: *Nano (1, 2)* Generalized-Regular Closed sets, Nano (1, 2)* Regular-Closure, Nano (1, 2)* Regular-Interior, Nano (1, 2)* regular closed sets.*

1. INTRODUCTION

In 1970, Levine [5] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Later on N.Palaniappan [7] studied the concept of regular generalized closed set in a topological space. In 2011, Sharmistha Bhattacharya [8] have introduced the notion of generalized regular closed sets in topological space. The notion of nano topology was introduced by Lellis Thivagar [6]. In 1963, J.C.Kelly[3] initiated the study of bitopological spaces. In 2014 K.Bhuvaneswari *et al*., [1, 2] have introduced the notion of nano regular generalized and generalized regular closed sets in nano topological space and Nano bitopological spaces. In this paper, we have introduced a new class of sets on nano bitopological spaces called nano (1, 2)* generalized regular closed sets and the relation of these new sets with the existing sets.

2. PRELIMINARIES

Definition 2.1[7]: A subset A of a topological space (X,τ) is called a regular open set if $A = Int[cl(A)]$. The complement of a regular open set of a space X is called regular closed set in X.

Definition 2.2 [7]: A regular-closure of a subset A of X is the intersection of all regular closed sets that contains A and it is denoted by *rcl(A).*

Definition 2.3 [7]: The union of all regular open subsets of X contained in A is called regular-interior of A and it is denoted by $rInt(A)$.

Definition 2.4 [8]: A subset A of (X,τ) is called a generalized regular closed set (briefly gr closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

Definition 2.5 [8]: The generalized regular-closure of a subset A of a space X is the intersection of all generalized-regular closed sets containing A and is denoted by grcl(A)

The generalized regular-interior of a subset A of a space X is the union of all generalized-regular open sets contained in A and is denoted by grInt(A).

Corresponding Author: K. Bhuvaneswari[1]
[1]Associate Professor, Department of Mathematics,
Mother Teresa Women's University, Kodaikanal, Tamil Nadu, India.

Definition 2.6 [6]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X) U_R(X) B_R(X)\}$ where $X \subseteq U$. Then by Property 2.10, $\tau_R(X)$ satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. $(U, \tau_R(X))$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^C$ are called nano closed sets with $[\tau_R(X)]^C$ being called nano topology of $\tau_R(X)$.

Definition 2.7 [6]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.
- The nano closure of the set A is denoted by Ncl(A). Ncl(A) is the smallest nano closed set containing A.

Definition 2.8 [6]: Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

- Nano regular open if $A \subseteq NInt[Ncl(A)]$
- Nano regular closed if $Ncl[NInt(A)] \subseteq A$

NRO(U,X), NRC(U,X) respectively denote the families of all nano regular open, nano regular closed subsets of U.

Definition 2.9 [6]: If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, Then

(i) The nano regular-closure of A is defined as the intersection of all nano regular closed sets containing A and it is denoted by Nrcl(A). Nrcl(A) is the smallest nano regular closed set containing A.
(ii) The nano regular-interior of A is defined as the union of all nano regular open subsets of A contained in A and it is denoted by NrInt(A). NrInt(A) is the largest nano regular open subset of A.

Definition 2.10 [6]: A subset A of $(U, \tau_R(X))$ is called nano generalized-regular closed set (briefly Ngr closed) if $Nrcl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano open in $(U, \tau_R(X))$.

Definition 2.11 [3]: Let $(X, \tau_{1,2})$ be a bitopological space and $A \subseteq U$. Then A is said to be

- (1,2)* Regular open if $A \subseteq \tau_{1,2}Int[\tau_{1,2}cl(A)]$
- (1,2)* Regular closed if $\tau_{1,2}cl[\tau_{1,2}Int(A)] \subseteq A$

(1,2)*RO(X), (1,2)*RC(X) respectively denote the families of all (1,2)* regular open, (1,2)* regular closed subsets of X.

Definition 2.12 [3]: If $(X, \tau_{1,2})$ is a bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The (1,2)* regular-closure of A is defined as the intersection of all (1,2)* regular closed sets containing A and it is denoted by $\tau_{1,2}$ rcl(A). $\tau_{1,2}$ rcl(A) is the smallest (1,2)* regular closed set containing A.
(ii) The (1,2)* regular-interior of A is defined as the union of all (1,2)* regular open subsets of A contained in A and it is denoted by $\tau_{1,2}$ rInt(A). $\tau_{1,2}$ rInt(A) is the largest (1,2)*regular open subset of A.

Definition 2.13 [3]: A subset A of $(X, \tau_{1,2})$ is called (1,2)* generalized-regular closed set (briefly (1,2)* gr closed) if $\tau_{1,2}rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)* open in $(X, \tau_{1,2})$.

Definition 2.14 [2]: Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $X \subseteq U$ Then $\tau_R(X)$ satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $(U, \tau_{R_{1,2}}(X))$ is called the nano bitopological space. Elements of the nano bitopology are known as nano (1, 2)* open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^c$ are called nano (1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.

Definition 2.15 [2]: If $(U, \tau_{R_{1,2}}(X))$ is a nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The nano (1, 2)* closure of A is defined as the intersection of all nano (1, 2)* closed sets containing A and it is denoted by $N\tau_{1,2}cl(A)$. $N\tau_{1,2}cl(A)$ is the smallest nano (1, 2)* closed set containing A.
- The nano (1, 2)* interior of A is defined as the union of all nano (1, 2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}Int(A)$. $N\tau_{1,2}Int(A)$ is the largest nano (1, 2)* open subset of A.

3. NANO (1, 2)* GENERALIZED REGULAR CLOSED SETS

In this section, we define and study the nano (1, 2)* generalized-regular closed sets in nano bitopological space $(U, \tau_{R_{1,2}}(X))$.

Definition 3.1: A subset A of $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)* generalized-regular closed set (briefly N(1, 2)*gr-closed) if $N\tau_{1,2}rcl(A)) \subseteq V$ whenever $A \subseteq V$ and V is nano (1, 2)* open in $(U, \tau_{R_{1,2}}(X))$.

Example 3.2: Let $U = \{a, b, c, d\}$ with $U / R = \{\{c\}, \{d\}, \{a, b\}\}$

$X_1 = \{a, c\}$ and $\tau_{R_1}(X) = \{U, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}$

$X_2 = \{a, d\}$ and $\tau_{R_2}(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$

Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{c\}, \{d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ which are (1,2)* open sets.

The nano (1, 2)* closed sets $= \{U, \phi, \{c\}, \{d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$.

The nano (1, 2)* regular closed sets = $\{U, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}\}$

The nano (1, 2)* regular open sets = $\{U, \phi, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, b\}, \{d\}, \{c\}\}$

The nano (1, 2)* generalized-regular open sets are

$$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}.$$

The nano (1, 2)* generalized-regular closed sets are

$$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}.$$

The nano (1, 2)* regular-generalized open sets are

$$\{U, \phi, \{a\}\{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}.$$

The nano (1, 2)* regular-generalized closed sets are

$$\{U,\phi,\{a\}\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}.$$

Theorem 3.3: Let $(U,\tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space $(U,\tau_{R_{1,2}}(X))$ is nano (1,2)* regular closed set in $(U,\tau_{R_{1,2}}(X))$, then A is a nano (1,2)* generalized-regular closed set in $(U,\tau_{R_{1,2}}(X))$.

Proof: Let A be a nano (1,2)* regular closed set in X and $A\subseteq V$, V is nano (1,2)* open in U. That is $N\tau_{1,2}cl[N\tau_{1,2}Int(A)]=A$. Since A is nano (1,2)* open. $N\tau_{1,2}Int(A)=A$. Every nano (1,2)* open set is nano (1,2)* regular open. Therefore $N\tau_{1,2}cl(A)=A\subseteq V$ implies $N\tau_{1,2}cl(A)\subseteq V$. Since $A\subseteq V$ then $N\tau_{1,2}cl(A)\subseteq V$ whenever V is nano (1,2)* open in U. Hence A is a nano (1,2)* generalized-regular closed set.

The converse of the above Theorem 3.3 is not true from the following example.

Example 3.4: Let $U=\{a,b,c,d\}$ with $U/R=\{\{c\},\{d\},\{a,b\}\}$

$X_1=\{a,c\}$ and $\tau_{R_1}(X)=\{U,\phi,\{c\},\{a,b,c\},\{a,b\}\}$

$X_2=\{a,d\}$ and $\tau_{R_2}(X)=\{U,\phi,\{d\},\{a,b,d\},\{a,b\}\}$

Then $\tau_{R_{1,2}}(X)=\{U,\phi,\{c\},\{d\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ which are (1, 2)* open sets.

Here is {{a},{b},{a, c},{a, d},{b, c},{b, d},{a, c, d},{b, c, d}} nano (1, 2)* generalized regular closed sets but it is not nano (1,2)* regular closed.

Remark 3.5: Every nano (1, 2)* regular-generalized closed set is a nano (1,2)* generalized-regular closed set. In the Example 3.2, all nano (1,2)* regular-generalized closed sets are nano (1,2)* generalized-regular closed sets. The converse of the Remark 3.5 is true.

Remark 3.6: In the Example 3.2, let $A=\{a\}\subseteq V$, $V=\{a,b,c,d\}$, V is nano (1, 2)*open. $N\tau_{1,2}cl(A)=\{a,b,c\}\subseteq V$

Now $N\tau_{1,2}rcl(A)=\{a,b\}\subseteq N\tau_{1,2}cl(A)$. If $N\tau_{1,2}cl(A)\subseteq V$, then $N\tau_{1,2}rcl(A)\subseteq N\tau_{1,2}cl(A)$.

Theorem 3.7: Let $(U,\tau_{R_{1,2}}(X))$ be a nano bitopological space. If a subset A of a nano bitopological space $(U,\tau_{R_{1,2}}(X))$ is nano (1,2)* generalized closed set in $(U,\tau_{R_{1,2}}(X))$, then A is a nano (1,2)* generalized regular closed set in $(U,\tau_{R_{1,2}}(X))$.

Proof: Let V be any nano (1,2)* generalized closed set. Then $N\tau_{1,2}cl(A)\subseteq V$ whenever $A\subseteq V$ and V is nano (1,2)* open in U. But $N\tau_{1,2}rcl(A)\subseteq N\tau_{1,2}cl(A)$ whenever $A\subseteq V$, V is nano (1,2)* open in U. Now we have $N\tau_{1,2}rcl(A)\subseteq V$, $A\subseteq V$, V is nano (1,2)* open in U. Hence A is nano (1,2)* generalized regular closed set.

Remark 3.8: The converse of the Theorem 3.7 need not be true. In the Example 3.2, let A={a}, V={a, b, d} whenever $A\subseteq V$, V is nano (1,2)* open. Now $N\tau_{1,2}rcl(A)=\{a,b\}\subseteq V$. Hence A={a, b} is nano (1,2)* generalized regular closed set. But $N\tau_{1,2}cl(A)=\{a,b,c\}\not\subseteq V$. Hence the subset A = {a} is not nano (1,2)* generalized closed set. Hence every nano (1,2)* generalized regular closed set need not be a nano (1,2)* generalized closed set.

Theorem 3.9: ϕ and U are nano (1,2)* generalized regular closed subset of U.

REFERENCES

1. K.Bhuvaneswari and P.Sulochana Devi, On Nano Regular Generalized and Nano Generalized Regular Closed Sets in Nano Topological Spaces. International Journal of Engineering Trends and Technology (IJETT) Vol.13, No.8, Jul 2014.
2. K.Bhuvaneswari and J.Sheeba Priyadharshini, On Nano (1,2)* Regular Generalized Closed Sets in Nano bitopological Spaces. International Journal of Applied Mathematics & Statistical Sciences (IJMASS) Vol.5, Issue-3, Apr-May 2016, 19-30.
3. T.Fukutake, On generalized closed sets in bitopological spaces, Bull. Fukuoka Univ. Ed. III 35 (1986) 19-28.
4. J.C.Kelly Bitopological Spaces, Proc. London Math. Soc., 13 (1963), 71-89.
5. N.Levine, (1970), Generalized closed sets in topological spaces, Rend, Circ.Mat,Palermo. (2), 19, Pp.89-96.
6. M.Lellis Thivagar, and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, Volume I Issue I, August 2013, Pp.31-37
7. N.Palaniappan and K.Chandarasekhara Rao, Regular generalized closed sets, Kyungpook math.J, 33(1993), 211-219.
8. Sharmistha Bhattacharya (Halder), on generalized regular closed sets, Int. J. Contemp. Math.Sci., 6(2011), 145-152.
9. Stone M.H, Applications of the theory of Boolean rings to general topology, Trans.Amer.Math.Soc., 41 (1937) 375-381.

Source of Support: Nil, Conflict of interest: None Declared