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SOME GOURAVA INDICES AND INVERSE SUM INDEG INDEX OF CERTAIN NETWORKS

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ABSTRACT

Chemical graph theory is a branch of graph theory whose focus of interest is to finidng topological indices of chemical graphs which correlate well with chemical properties of the chemical molecules. In this paper, we compute the first and second Gourava indices, inverse sum in deg index, sum and product connectivity Gourava indices for silicate, chain silicate, rhombus silicate, hexagonal, oxide, rhombus oxide and honeycomb networks.

Keywords: *silicate, hexagonal, oxide, honeycomb networks; first and second Gourava indices, inverse sum indeg index, connectivity Gourava indices.*

Mathematics Subject Classification: *05C05, 05C07, 05C90.*

1. INTRODUCTION

A molecular graph or a chemical graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical Chemistry which has an important effect on the development of the chemical Sciences. A single number that can be computed from the molecular graph and used to characterize some property of the underlying molecule is said to be a topological index or molecular structure descriptor. Numerous such descriptors have been considered in Theoretical chemistry and have found some applications, especially in *QSPR/QSAR* research see [1, 2].

We consider only finite, simple and connected graph G with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. We refer to [3] for undefined term and notation.

The first and second Gourava indices [4] of a molecular graph G are respectively defined as

$$GO_1(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)\right]$$

$$GO_2(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right).$$

The inverse sum indeg index of a graph G is defined as

$$ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}.$$

This index was introduced by Vukičević and Gaśperov in [5].

Motivated by the definition of the product connectivity or Randić index [6], Kulli introduced the sum connectivity Gourava index and product connectivity Gourava index of a graph as follows:

The sum connectivity Gourava index of a graph G is defined as

$$SGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)}}.$$

This index was introduced in [7] and was studied in [8].

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The product connectivity Gourava index of a graph G is defined as

$$PGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_G(u)+d_G(v))+d_G(u)d_G(v)}}.$$

This index was introduced in [9] and was studied in [8].

In this paper, the first and second Gourava indices, inverse sum indeg index, sum and product connectivity Gourava indices for silicate, chain silicate, rhombus silicate, hexagonal, oxide, rhombus oxide and honeycomb networks are computed. For more information about networks see [10, 11, 12].

2. RESULTS FOR SILICATE NETWORKS

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network is symbolized by SL_n where n is the number of hexagons between the center and boundary of SL_n. A silicatre network of dimension two is shown in Figure 1.

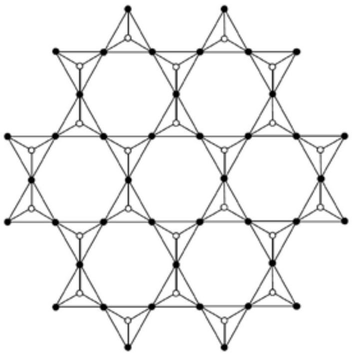

Figure-1: Silicate network of dimention two

In the following theorem, we compute the exact values of $GO_1(SL_n)$, $GO_2(SL_n)$, $ISI(SL_n)$, $SGO(SL_n)$ and $PGO(SL_n)$ for silicate networks.

Theorem 1: Let SL_n be the silicate networks. Then

(1) $GO_1(SL_n)=1350n^2-324n$.

(2) $GO_2(SL_n)=10692n^2-3888n$.

(3) $ISI(SL_n)=90n^2-15n$.

(4) $SGO(SL_n)=\left(\frac{6}{\sqrt{3}}+\frac{9}{2\sqrt{3}}\right)n^2+\left(\frac{6}{\sqrt{15}}-\frac{1}{\sqrt{3}}\right)n.$

(5) $PGO(SL_n)=\left(\frac{2}{\sqrt{2}}+\frac{3}{2\sqrt{3}}\right)n^2+\left(\frac{2}{\sqrt{6}}+\frac{2}{3\sqrt{2}}-\frac{1}{\sqrt{3}}\right)n.$

Proof: Let G be the graph of silicate network SL_n with $|V(SL_n)|=15n^2+3n$ and $|E(SL_n)|=36n^2$. By algebraic method, in SL_n, there are three types of edges based on the degree of end vertices of each edge, as given in Table 1.

$d_G(u), d_G(v)\backslash uv\in E(G)$	(3,3)	(3,6)	(6,6)
Number of edges	$6n$	$18n^2+6n$	$18n^2-12n$

Table-1: Edge partition of SL_n

(1) By using the partition given Table 1, we can apply the formula of GO_1 index for G.

Since $GO_1(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)\right]$,

this implies that

$$GO_1(SL_n)=6n\left[(3+3)+(3\times3)\right]+\left(18n^2+6n\right)\left[(3+6)+(3\times6)\right]+\left(18n^2-12n\right)\left[(6+6)+(6\times6)\right]$$

$=1350n^2-324n.$

(2) By using the partition given in Table 1, we can apply the formula of GO_2 index of G.

Since $GO_2(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)$,

this implies that

$$GO_2\left(SL_n\right)=6n\left[\left(3+3\right)\times\left(3\times3\right)\right]+\left(18n^2+6n\right)\left[\left(3+6\right)\times\left(3\times6\right)\right]+\left(18n^2-12n\right)\left[\left(6+6\right)\times\left(6\times6\right)\right]$$

$$=\ 10692n^2-\ 3888n$$

(3) By using the partition given in Table 1, we can apply the formula of *ISI* index of *G*. Since

$$ISI\left(G\right)=\sum_{uv\in E(G)}\frac{d_G\left(u\right)d_G\left(v\right)}{d_G\left(u\right)+d_G\left(v\right)},$$

this implies that

$$ISI\left(SL_n\right)=\left(6n\right)\left(\frac{3\times3}{3+3}\right)+\left(18n^2+6n\right)\left(\frac{3\times6}{3+6}\right)+\left(18n^2-12n\right)\left(\frac{6\times6}{6+6}\right)=90n^2-15n.$$

(4) By using the partition given in Table 1, we can apply the formula of *SGO* index of *G*.

Since $SGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_G(u)+d_G(u))+d_G(u)d_G(v)}}$

this implies that

$$SGO\left(SL_n\right)=\left(6n\right)\frac{1}{\sqrt{(3+3)(3\times3)}}+\left(18n^2+6n\right)\frac{1}{\sqrt{(3+6)+(3\times6)}}+\left(18n^2-12n\right)\frac{1}{\sqrt{(6+6)+(6\times6)}}$$

$$=\left(\frac{6}{\sqrt{3}}+\frac{9}{2\sqrt{3}}\right)n^2+\left(\frac{6}{\sqrt{15}}-\frac{1}{\sqrt{3}}\right)n.$$

(5) By using the partition given in Table 1, we can apply the formula of *PGO* index of *G*.

Since $PGO\left(G\right)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G\left(u\right)+d_G\left(v\right)\right)+d_G\left(u\right)d_G\left(v\right)}}.$

this implies that

$$PGO\left(SL_n\right)=\left(6n\right)\frac{1}{\sqrt{\left(3+3\right)\left(3\times3\right)}}+\left(18n^2+6n\right)\frac{1}{\sqrt{\left(3+6\right)\left(3\times6\right)}}+\left(18n^2-12n\right)\frac{1}{\sqrt{\left(6+6\right)\left(6\times6\right)}}$$

$$=\left(\frac{2}{\sqrt{2}}+\frac{3}{2\sqrt{3}}\right)n^2+\left(\frac{2}{\sqrt{6}}+\frac{2}{3\sqrt{2}}-\frac{1}{\sqrt{3}}\right)n.$$

3. RESULTS FOR CHAIN SILICATE NETWORKS

We now consider a family of chain silicate networks. A chain silicate network is symbolized by CS_n and is obtained by arranging n tetrahedral linearly see Figure 2.

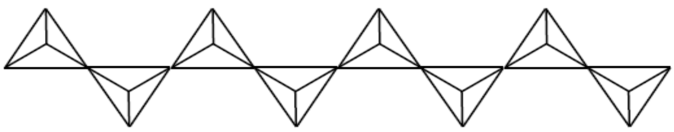

Figure-2: Chain silicate network

In the following theorem, we compute the exact values of $GO_1(CS_n)$ $GO_2(CS_n)$, $ISI(CS_n)$, $SGO(CS_n)$ and $PGO(CS_n)$ for chain silicate networks.

Theorem 2: Let CS_n, $n \geq 2$, be the chain silicate networks. Then

(1) $GO_1(CS_n)=171n-90$.

(2) $GO_2(CS_n)=1134n-972$.

(3) $ISI(CS_n)=\frac{25}{2}n-4$.

(4) $SGO\left(CS_n\right)=\left(\frac{1}{\sqrt{15}}+\frac{4}{3\sqrt{3}}+\frac{1}{4\sqrt{3}}\right)n+\left(\frac{4}{\sqrt{15}}-\frac{2}{3\sqrt{3}}-\frac{1}{2\sqrt{3}}\right).$

(5) $PGO\left(CS_n\right)=\left(\frac{1}{3\sqrt{6}}+\frac{4}{9\sqrt{2}}+\frac{1}{12\sqrt{3}}\right)n+\left(\frac{4}{3\sqrt{6}}-\frac{2}{9\sqrt{2}}-\frac{1}{6\sqrt{3}}\right)$

Proof: Let *G* be the graph of chain silicate network CS_n with $|V(CS_n)| = 3n+1$ and $|E(CS_n)| = 6n$. By algebraic method, in CS_n, $n\geq2$, there are three types of edges based on the degrees of end vertices of each edge, as given in Table 2.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(3,3)	(3,6)	(6,6)
Number of edges	$n+4$	$4n-2$	$n-2$

Table-2: Edge partition of CS_n

(1) By using the partition given in Table 2, we can apply the formula of GO_1 index of G. Since $GO_1(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)\right]$,

this implies that

$$GO_1(CS_n)=(n+4)\left[(3+3)+(3\times 3)\right]+(4n-2)\left[(3+6)+(3\times 6)\right]+(n-2)\left[(6+6)+(6\times 6)\right]$$
$$= 171n-90.$$

(2) By using the partition given in Table 2, we can apply the formula of GO_2 index of G.

Since $GO_2(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)$,

this implies that

$$GO_2(CS_n)=(n+4)\left[(3+3)(3\times 3)\right]+(4n-2)\left[(3+6)(3\times 6)\right]+(n-2)\left[(6+6)(6\times 6)\right]$$
$$=1134n-972.$$

(3) By using the partition given in Table 2, we can apply the formula of *ISI* index of *G*.

Since $ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)}$,

this implies that

$$ISI(CS_n)=(n+4)\left(\frac{3\times 3}{3+3}\right)+(4n-2)\left(\frac{3\times 6}{3+6}\right)+(n-2)\left(\frac{6\times 6}{6+6}\right)=\frac{25}{2}n-4.$$

(4) By using the partition given in Table 2, we can apply the formula S*GO* index of *G*.

Since $SGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)}}$,

this implies that

$$SGO(CS_n)=(n+4)\frac{1}{\sqrt{(3+3)+(3\times 3)}}+(4n-2)\frac{1}{\sqrt{(3+6)+(3\times 6)}}+(n-2)\frac{1}{\sqrt{(6+6)+(6\times 6)}}$$
$$=\left(\frac{1}{\sqrt{15}}+\frac{4}{3\sqrt{3}}+\frac{1}{4\sqrt{3}}\right)n+\left(\frac{4}{\sqrt{15}}-\frac{2}{3\sqrt{3}}-\frac{1}{2\sqrt{3}}\right).$$

(5) By using the partition in given table 2, we can apply the formula of P*GO* index of *G*.

Since $PGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)}}$,

this implies that

$$PGO(CS_n)=(n+4)\frac{1}{\sqrt{(3+3)(3\times 3)}}+(4n-2)\frac{1}{\sqrt{(3+6)(3\times 6)}}+(n-2)\frac{1}{\sqrt{(6+6)(6\times 6)}}$$
$$=\left(\frac{1}{3\sqrt{6}}+\frac{4}{9\sqrt{2}}+\frac{1}{12\sqrt{3}}\right)n+\left(\frac{4}{3\sqrt{6}}-\frac{2}{9\sqrt{2}}-\frac{1}{6\sqrt{3}}\right).$$

4. RESULTS FOR RHOMBUS SILICATE NETWORKS

We now consider a family of rhombus silicate networks. A rhombus silicate network is symbolized by $RHSL_n$. A rhombus silicate network of dimension 3 is shown in Figure 3.

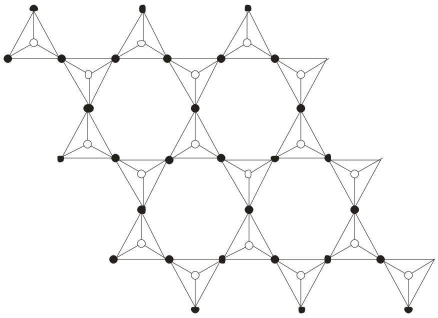

Figure-3: Rhombus silicate network of dimension 3

In the following theorem, we compute the exact values of $GO_1(RHSL_n)$, $GO_2(RHSL_n)$, $ISI(RHSL_n)$, $SGO(RHSL_n)$ and $PGO(RHSL_n)$ for rhombus silicate networks.

Theorem 3: Let $RHSL_n$ be the rhombus silicate networks. Then

(1) $GO_1(RHSL_n) = 450n^2 - 216n + 18$.

(2) $GO_2(RHSL_n) = 3564n^2 - 2592n + 324$.

(3) $ISI(RHSL_n) = 30n^2 - 10n + 1$.

(4) $SGO(RHSL_n) = \left(\frac{2}{\sqrt{3}} + \frac{3}{2\sqrt{3}}\right)n^2 + \left(\frac{4}{\sqrt{15}} + \frac{4}{3\sqrt{3}} - \frac{2}{\sqrt{3}}\right)n . + \left(\frac{2}{\sqrt{15}} - \frac{4}{3\sqrt{3}} + \frac{1}{2\sqrt{3}}\right)$

(5) $PGO(RHSL_n) = \left(\frac{2}{3\sqrt{2}} + \frac{1}{2\sqrt{3}}\right)n^2 . + \left(\frac{4}{3\sqrt{6}} + \frac{4}{9\sqrt{2}} - \frac{2}{3\sqrt{3}}\right)n + \left(\frac{2}{3\sqrt{6}} - \frac{4}{9\sqrt{2}} + \frac{1}{6\sqrt{3}}\right)$.

Proof: Let G be the graph of rhombus silicate network $RHSL_n$ with $|V(RHSL_n)| = 5n^2+2n$ and $|E(RHSL_n)| = 12n^2$. By algebraic method, in $RHSL_n$, there are three types of edges based on the degrees of endvertices of each edge as given in Table 3.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(3,3)	(3,6)	(6,6)
Number of edges	$4n+2$	$6n^2+4n-4$	$6n^2-8n+2$

Table-3: Edge partition of $RHSL_n$

(1) By using the partition given in Table 3, we can apply the formula of GO_1 index of G.

Since $GO_1(G) = \sum_{uv \in E(G)} \left[\left(d_G(u) + d_G(v)\right) + d_G(u)d_G(v)\right]$,

this implies that

$$GO_1(RHSL_n) = (4n+2)\left[(3+3)+(3\times 3)\right] + \left(6n^2+4n-4\right)\left[(3+6)+(3\times 6)\right] + \left(6n^2-8n+2\right)\left[(6+6)+(6\times 6)\right]$$
$$= 450n^2 - 216n + 18.$$

(2) By using the partition given in Table 3, we can apply the formula of GO_2 index of G.

Since $GO_2(G) = \sum_{uv \in E(G)} \left(d_G(u) + d_G(v)\right)\left(d_G(u)d_G(v)\right)$,

this implies that

$$GO_2(RHSL_n) = (4n+2)\left[(3+3)(3\times 3)\right] + \left(6n^2+4n-4\right)\left[(3+6)(3\times 6)\right] + \left(6n^2-8n+2\right)\left[(6+6)(6\times 6)\right]$$
$$= 3564n^2 - 2592n + 324.$$

(3) By using the partition given in Table 3, we can apply the formula of ISI index of G.

Since $ISI(G) = \sum_{uv \in E(G)} \frac{d_G(u)d_G(v)}{d_G(u) + d_G(v)}$,

this implies that

$$ISI(RHSL_n) = (4n+2)\left(\frac{3\times 3}{3+3}\right) + \left(6n^2+4n-4\right)\left(\frac{3\times 6}{3+6}\right) + \left(6n^2-8n+2\right)\left(\frac{6\times 6}{6+6}\right)$$
$$= 30n^2 - 10n + 1.$$

(4) By using the partition given in Table 3, we can apply the formula SGO index of G.

Since $SGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\left(d_G(u) + d_G(v)\right) + d_G(u)d_G(v)}}$,

this implies that

$$SGO\left(RHSL_n\right) = \left(4n+2\right)\frac{1}{\sqrt{(3+3)+(3\times 3)}} + \left(6n^2+4n-4\right)\frac{1}{\sqrt{(3+3)+(3\times 3)}}$$

$$+\left(6n^2-8n+2\right)\frac{1}{\sqrt{(6+6)+(6\times 6)}}$$

$$=\left(\frac{2}{\sqrt{3}}+\frac{3}{2\sqrt{3}}\right)n^2+\left(\frac{4}{\sqrt{15}}+\frac{4}{3\sqrt{3}}-\frac{2}{\sqrt{3}}\right)n+\left(\frac{2}{\sqrt{15}}-\frac{4}{3\sqrt{3}}-\frac{1}{2\sqrt{3}}\right).$$

(5) By using the partition given in Table 3, we can apply the formula *PGO* index of *G*. Since

$$PGO(G) = \sum_{uv\in w(G)} \frac{1}{\sqrt{(d_G(u)+d_G(v))+d_G(u)d_G(v)}}$$

this implies that

$$PGO\left(RHSL_n\right) = \left(4n+2\right)\frac{1}{\sqrt{(3+3)(3\times 3)}} + \left(6n^2+4n-4\right)\frac{1}{\sqrt{(3+6)(3\times 6)}}$$

$$+\left(6n^2-8n+2\right)\frac{1}{\sqrt{(6+6)(6\times 6)}}$$

$$=\left(\frac{2}{3\sqrt{2}}+\frac{1}{2\sqrt{3}}\right)n^2+\left(\frac{4}{3\sqrt{6}}+\frac{4}{9\sqrt{2}}-\frac{2}{3\sqrt{3}}\right)n+\left(\frac{2}{3\sqrt{6}}-\frac{4}{9\sqrt{2}}+\frac{1}{6\sqrt{3}}\right).$$

5. RESULTS FOR HEXAGONAL NETWORKS

It is known that there exist three regular plane tilings with composition of same kind of regular polygons such as triangular, hexagonal and square. Triangulor tiling is used in the construction of hexagonal networks. This network is symbolized by HX_n where n is sthe number of vertices in each side of hexagon. A hexagonal network of dimension six is shown in Figure 4.

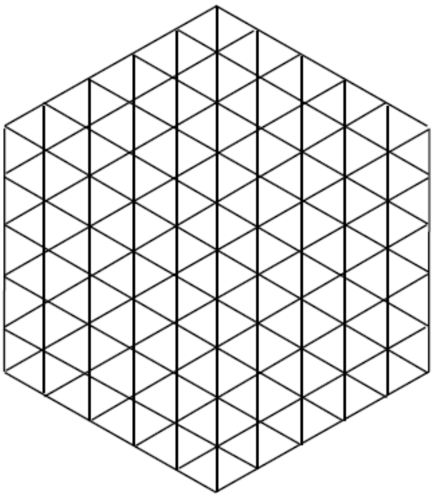

Figure-4: Hexagonal network of dimension six

In the following theorem, we compute the exact values of $GO_1(HX_n)$, $GO_2(HX_n)$, $ISI(HX_n)$, $SGO(HX_n)$ and $PGO(HX_n)$ for hexagonal networks.

Theorem 4: Let HX_n be the hexagonal networks. Then

(1) $GO_1(HX_n) = 432n^2 - 1032n + 582$.

(2) $GO_2(HX_n) = 3888n^2 - 10608n + 6876$.

(3) $ISI(HX_n) = 27n^2 - \frac{291}{5}n + \frac{42}{5}$.

(4) $SGO\left(HX_n\right)=\frac{9}{\sqrt{48}}n^2+\left(\frac{6}{\sqrt{24}}+\frac{12}{\sqrt{34}}-\frac{33}{\sqrt{48}}\right)n+\left(\frac{12}{\sqrt{19}}+\frac{6}{\sqrt{27}}-\frac{18}{\sqrt{24}}-\frac{24}{\sqrt{34}}+\frac{30}{\sqrt{48}}\right).$

(5) $PGO\left(HX_n\right)=\frac{3}{4\sqrt{3}}n^2+\left(\frac{3}{4\sqrt{2}}+\frac{3}{\sqrt{15}}-\frac{11}{4\sqrt{3}}\right)n+\left(\frac{6}{\sqrt{21}}+\frac{2}{3\sqrt{2}}-\frac{9}{4\sqrt{2}}-\frac{6}{\sqrt{15}}+\frac{15}{6\sqrt{3}}\right).$

Proof: Let G be the graph of hexagonal network HX_n with $|V(HX_n)| = 3n^2-3n+1$ and $|E(HX_n)| = 9n^2 - 15n + 6$. By algebraic method, in HX_n, there are five types of edges based on the degrees of end vertices of each edge, as given in Table 4.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(3,4)	(3,6)	(4,4)	(4, 6)	(6,6)
Number of edges	12	6	$6n-18$	$12n-24$	$9n^2-33n+30$

Table-4: Edge partition of HX_n

(1) By using the partition given in Table 4, we can apply the formula of GO_1 index of G.

Since $$GO_1(G) = \sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)\right],$$

this implies that

$$GO_1(HX_n) = 12\left[(3+4)+(3\times4)\right]+6\left[(3+6)+(3\times6)\right]+(6n-18)\left[(4+4)+(4\times4)\right]$$
$$+(12n-24)\left[(4+6)+(4\times6)\right]+\left(9n^2-33n+30\right)\left[(6+6)+(6\times6)\right]$$
$$= 432n^2 - 1032n + 582.$$

(2) By using the partition given in Table 4, we can apply the formula of GO_2 index of G. Since $$GO_2(G) = \sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right),$$

this implies that

$$GO_2(HX_n) = 12\left[(3+4)(3\times4)\right]+6\left[(3+6)(3\times6)\right]+(6n-18)\left[(4+4)(4\times4)\right]$$
$$+(12n-24)\left[(4+6)(4\times6)\right]+\left(9n^2-33n+30\right)\left[(6+6)(6\times6)\right]$$
$$= 3888n^2 - 10608n + 6876.$$

(3) By using the partition given in Table 4, we can apply the formula of ISI index of G.

Since $$ISI(G) = \sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)},$$

this implies that

$$ISI(HX_n) = 12\left(\frac{3\times4}{3+4}\right)+6\left(\frac{3\times6}{3+6}\right)+(6n-18)\left(\frac{4\times4}{4+4}\right)+(12n-24)\left(\frac{4\times6}{4+6}\right)+\left(9n^2-33n+30\right)\left(\frac{6\times6}{6+6}\right)$$
$$= 27n^2 - \frac{291}{5}n + \frac{42}{5}.$$

(4) By using the partition given in Table 4, we can apply the formula of SGO index of G.

Since $$SGO(G) = \sum_{uv\in E(G)}\frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)}},$$

this implies that

$$SGO(HX_n) = (12)\frac{1}{\sqrt{(3+4)+(3\times4)}}+(6)\frac{1}{\sqrt{(3+6)+(3\times6)}}+(6n-18)\frac{1}{\sqrt{(4+4)+(4\times4)}}$$
$$+(12n-24)\frac{1}{\sqrt{(4+6)+(4\times6)}}+\left(9n^2-33n+30\right)\frac{1}{\sqrt{(6+6)+(6\times6)}}$$
$$= \frac{9}{\sqrt{48}}n^2+\left(\frac{6}{\sqrt{24}}+\frac{12}{\sqrt{34}}-\frac{33}{\sqrt{48}}\right)n+\left(\frac{12}{\sqrt{19}}+\frac{6}{\sqrt{27}}-\frac{18}{\sqrt{24}}-\frac{24}{\sqrt{34}}+\frac{30}{\sqrt{48}}\right).$$

(5) By using the partition given in Table 4, we can apply the formula of PGO index of G.

Since $$PGO(G) = \sum_{uv\in W(G)}\frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)}},$$

this implies that

$$PGO(HX_n) = (12)\frac{1}{\sqrt{(3+4)(3\times4)}}+(6)\frac{1}{\sqrt{(3+6)(3\times6)}}+(6n-18)\frac{1}{\sqrt{(4+4)(4\times4)}}$$
$$+(12n-24)\frac{1}{\sqrt{(4+6)(4\times6)}}+\left(9n^2-33n+30\right)\frac{1}{\sqrt{(6+6)(6\times6)}}$$
$$= \frac{3}{4\sqrt{3}}n^2+\left(\frac{3}{4\sqrt{2}}+\frac{3}{\sqrt{15}}-\frac{11}{4\sqrt{3}}\right)n+\left(\frac{6}{\sqrt{21}}+\frac{2}{3\sqrt{2}}-\frac{9}{4\sqrt{2}}-\frac{6}{\sqrt{15}}+\frac{15}{6\sqrt{3}}\right).$$

6. RESULTS FOR OXIDE NETWORKS

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension five is shown in Figure 5.

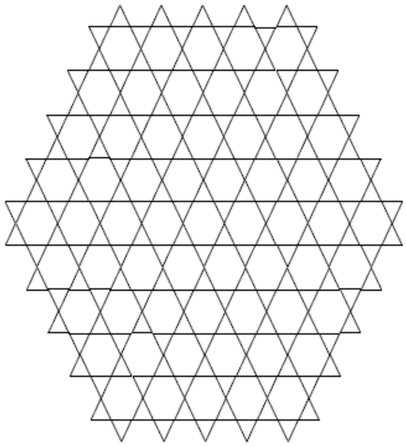

Figure-5: Oxide network of dimension 5

In the following theorem, we compute the extact values of $GO_1(OX_n)$, $GO_2(OX_n)$, $ISI(OX_n)$, $SGO(OX_n)$ and $PGO(OX_n)$ for oxide networks.

Theorem 5: Let OX_n be the oxide networks. Then

(1) $GO_1(OX_n) = 432n^2 - 120n$.

(2) $GO_2(OX_n) = 2304n^2 - 960n$.

(3) $ISI(OX_n) = 36n^2 - 8n$.

(4) $SGO\left(OX_n\right)=\frac{9}{\sqrt{6}}n^2+\left(\frac{12}{\sqrt{14}}-\sqrt{6}\right)n.$

(5) $PGO\left(OX_n\right)=\frac{9}{\sqrt{32}}n^2+\left(\sqrt{3}-\frac{3}{\sqrt{8}}\right)n$

Proof: Let G be the graph of oxide network OX_n with $|V(OX_n)| = 9n^2+3n$ and $|E(OX_n)|=18n^2$. By algebraic method, in OX_n, there are two types of edges based on the degrees of end vertices of each edge, as given in Table 5.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2, 4)	(4, 4)
Number of edges	$12n$	$18n^2 - 12n$

Table-5: Edge partition of OX_n

(1) By using the partition given in Table 5, we can apply the formula of GO_1 index of G.

Since $GO_1(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+d_G(u)d_G(v)\right],$

this implies that

$GO_1(OX_n)=12n\left[(2+4)+(2\times4)\right]+\left(18n^2-12n\right)\left[(4+4)+(4\times4)\right] = 432n^2-120n.$

(2) By using the partition given in Table 5, we can apply the formula of GO_2 index of G.

Since $GO_2(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right),$

this implies that

$GO_2(OX_n)=12n\left[(2+4)(2\times4)\right]+\left(18n^2-12n\right)\left[(4+4)(4\times4)\right]= 2304n^2 - 960n.$

(3) By using the partition given in Table 5, we can apply the formula of ISI index of G.

Since $ISI(G)=\sum_{uv\in E(G)}\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)},$

this implies that

$ISI(OX_n)=(12n)\left(\frac{2\times4}{2+4}\right)+\left(18n^2-12n\right)\left(\frac{4\times4}{4+4}\right)= 36n^2 - 8n.$

(4) By using the partition given in Table 5, we can apply the formula of *SGO* index of *G*.

Since $SGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_G(u)+d_G(v))+d_G(u)d_G(v)}}$

this implies that

$$PGO\left(OX_n\right)=\left(12n\right)\frac{1}{\sqrt{(2+4)+(2\times 4)}}+\left(18n^2-12n\right)\frac{1}{\sqrt{(4+4)+(4\times 4)}}=\frac{9}{\sqrt{6}}n^2+\left(\frac{12}{\sqrt{14}}-\sqrt{6}\right)n.$$

(5) By using the partition given in Table 5, we can apply the formula of *PGO* index of *G*.

Since $PGO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{(d_G(u)+d_G(v))d_G(u)d_G(v)}}$

this implies that

$$PGO\left(OX_n\right)=\left(12n\right)\frac{1}{\sqrt{(2+4)(2\times 4)}}+\left(18n^2-12n\right)\frac{1}{\sqrt{(4+4)(4\times 4)}}=\frac{9}{\sqrt{32}}n^2+\left(\sqrt{3}-\frac{3}{\sqrt{8}}\right)n.$$

7. RESULTS FOR RHOMBUS OXIDE NETWORKS

A rhombus oxide network of dimension n is denoted by $RHOX_n$. A rhombus oxide network of dimension 3 is shown in Figure 6.

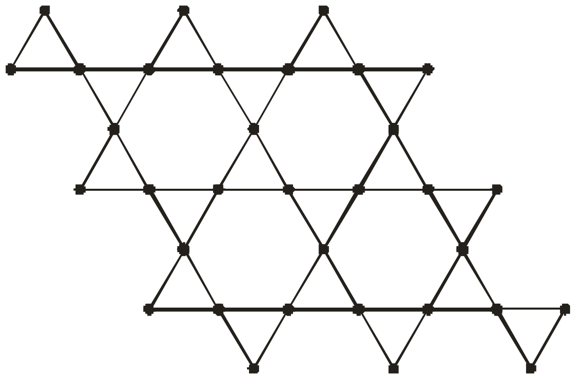

Figure-6: Rhombus oxide network of dimension 3

In the following theorem, we compute the exact values of $GO_1(RHOX_n)$, $GO_2(RHOX_n)$, $ISI(RHOX_n)$ $SGO(RHOX_n)$ and $PGO(RHOX_n)$ for rhombus oxide networks.

Theorem 6: Let $RHOX_n$ be the rhombus oxide networks. Then

(1) $GO_1(RHOX_n) = 144n^2 – 80n + 8$.

(2) $GO_2(RHOX_n) = 768n^2 – 640n + 96$.

(3) $ISI(RHOX_n) = 12n^2-\frac{16}{3}n+\frac{2}{3}$.

(4) $SGO\left(RHOX_n\right)=.\frac{3}{\sqrt{6}}n^2+\left(\frac{8}{\sqrt{14}}-\frac{4}{\sqrt{6}}\right)n+\left(\frac{1}{\sqrt{2}}-\frac{4}{\sqrt{14}}+\frac{1}{\sqrt{6}}\right)$

(5) $SGO\left(RHOX_n\right)=\frac{3}{\sqrt{32}}n^2+\left(\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{2}}\right)n+\left(\frac{1}{2}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{32}}\right)$

Proof: Let *G* be the graph of rhombus oxide network $RHOX_n$ with $|V(RHOX_n)| = 3n^2 + 2n$ and $|E(RHOX_n)| = 6n^2$. By algebraic method, in $RHOX_n$, there are three types of edges based on the degrees of end vertices of each edge, as given in Table 6.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2,2)	(2,4)	(4,4)
Number of edges	2	$8n – 4$	$6n^2 – 8n + 2$

Table-6: Edge partition of $RHOX_n$

(1) By using the partition given in Table 6, we can apply the formula of GO_1 index of *G*. Since $GO_1\left(G\right)=\sum_{uv\in E(G)}\left[\left(d_G\left(u\right)+d_G\left(v\right)\right)+d_G\left(u\right)d_G\left(v\right)\right]$,

this implies that

$GO_1(RHOX_n) = 2[(2+2) + (2\times2)] + (8n - 4)[(2+4) + (2\times4)] + (6n^2 - 8n + 2)[(4+4) + (4\times4)]$
$= 144n^2 - 80n + 8.$

(2) By using the partition given in Table 6, we can apply the formula of GO_2 index of G.

Since $GO_2(G) = \sum_{uv\in E(G)} \left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right),$

this implies that

$GO_2(RHOX_n) = 2[(2+2)(2\times2)] + (8n - 4)[(2+4)(2\times4)] + (6n^2 - 8n + 2)[(4+4)(4\times4)]$
$= 768n^2 - 640n + 96.$

(3) By using thre partition given in Table 6, we can apply the formula of *ISI* index of *G*.

Since $ISI(G) = \sum_{uv\in E(G)} \frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)},$

this implies that

$$ISI(RHOX_n) = 2\left(\frac{2\times2}{2+2}\right) + (8n-4)\left(\frac{2\times4}{2+4}\right) + \left(6n^2-8n+2\right)\left(\frac{4\times4}{4+4}\right) = 12n^2 - \frac{16}{3}n + \frac{2}{3}.$$

(4) By using the partition given in Table 6, we can apply the formula of *SGO* index of *G*.

Since $SGO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_G(u)+d_G(v)) + d_G(u)d_G(v)}}$

this implies that

$$SGO(RHOX_n) = 2\frac{1}{\sqrt{(2+2)+(2\times2)}} + (8n-4)\frac{1}{\sqrt{(2+4)+(2\times4)}} + \left(6n^2-8n+2\right)\frac{1}{\sqrt{(4+4)+(4\times4)}}$$
$$= \frac{3}{\sqrt{6}}n^2 + \left(\frac{8}{\sqrt{14}} - \frac{4}{\sqrt{6}}\right)n + \left(\frac{1}{\sqrt{2}} - \frac{4}{\sqrt{14}} - \frac{1}{\sqrt{6}}\right).$$

(5) By using the partition given in Table 6, we can apply the formula *PGO* index of *G*. Since $PGO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)}},$

this implies that

$$PGO(RHOX_n) = 2\frac{1}{\sqrt{(2+2)(2\times2)}} + (8n-4)\frac{1}{\sqrt{(2+4)(2\times4)}} + \left(6n^2-8n+4\right)\frac{1}{\sqrt{(4+4)(4\times4)}}$$
$$= \frac{3}{\sqrt{32}}n^2 + \left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{2}}\right)n + \left(\frac{1}{2} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{32}}\right).$$

8. RESULTS FOR HONEYCOMB NETWORKS

If we recursively use hexagonal tiling in a particular parttern, honeycomb networks are formed. These networks are very useful in chemistry and also in computer graphics. A honeycomb network of dimension n is denoted by HC_n where n is the number of hexagons between central and boundary hexagon. A honeycomb network of dimension four is shown in Figure 7.

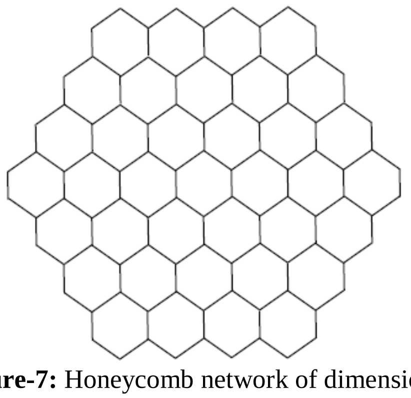

Figure-7: Honeycomb network of dimension 4

In the following theorem, we compute the exact values of $GO_1(HC_n)$, $GO_2(HC_n)$, $ISI(HC_n)$, $SGO(HC_n)$ and $PGO(HC_n)$ for honeycomb networks.

Theorem 7: Let HC_n be the honeycomb networks. Then

(1) $GO_1(HC_n) = 135n^2 - 93n + 6$.

(2) $GO_2(HC_n) = 486n^2 - 450n + 60$.

(3) $ISI(HC_n) = \frac{27}{2}n^2 - \frac{81n}{10} + \frac{3}{5}$.

(4) $SGO(HC_n) = \frac{9}{\sqrt{15}}n^2 + \left(\frac{12}{\sqrt{11}} - \frac{15}{\sqrt{15}}\right)n + \left(\frac{3}{\sqrt{2}} - \frac{12}{\sqrt{11}} + \frac{6}{\sqrt{15}}\right)$

(5) $PGO(HC_n) = \frac{9}{\sqrt{54}}n^2 + \left(\frac{12}{\sqrt{30}} - \frac{15}{\sqrt{54}}\right)n + \left(\frac{3}{2} - \frac{12}{\sqrt{30}} + \frac{6}{\sqrt{54}}\right)$

Proof: Let G be the graph of honeycomb network HC_n with $|V(HC_n)| = 6n^2$ and $|E(HC_n)| = 9n^2 - 3n$. In HC_n, by algebraic method, there are three types of edges based on the degrees of end vertices of each edge, as given in Table 7.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	6	$12n - 12$	$9n^2 - 15n + 6$

Table-7: Edge partition of HC_n

(1) By using the partition given in Table 7, we can apply the formula of GO_1 index of G.

Since $GO_1(G) = \sum_{uv \in E(G)} \left[\left(d_G(u) + d_G(v)\right) + d_G(u)d_G(v)\right]$,

this implies that

$$GO_1(HC_n) = 6[(2+2) + (2\times2)] + (12n - 12)[(2+3) + (2\times3)] + (9n^2 - 15n + 6)[(3+3) + (3\times3)]$$
$$= 135n^2 - 93n + 6.$$

(2) By using the partition given in Table 7, we can apply the formula of GO_2 index of G.

Since $GO_2(G) = \sum_{uv \in E(G)} \left(d_G(u) + d_G(v)\right)\left(d_G(u)d_G(v)\right)$,

this implies that

$$GO_2(HC_n) = 6[(2+2)(2\times2)] + (12n - 12)[(2+3)(2\times3)] + (9n^2 - 15n + 6)[(3+3)(3\times3)]$$
$$= 486n^2 - 450n + 60.$$

(3) By using the partition given in Table 7, we can apply the formula of ISI index of G.

Since $ISI(G) = \sum_{uv \in E(G)} \frac{d_G(u)d_G(v)}{d_G(u) + d_G(v)}$,

this implies that

$$ISI(HC_n) = 6\left(\frac{2\times2}{2+2}\right) + (12n-12)\left(\frac{2\times3}{2+3}\right) + \left(9n^2 - 15n + 6\right)\left(\frac{3\times3}{3+3}\right) = \frac{27}{2}n^2 - \frac{81}{10}n + \frac{3}{5}.$$

(4) By using the partition given in Table 7, we can apply the formula of SGO index of G.

Since $SGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\left(d_G(u) + d_G(v)\right) + d_G(u)d_G(v)}}$,

this implies that

$$SGO(HC_n) = (6)\frac{1}{\sqrt{(2+2) + (2\times2)}} + (12n-12)\frac{1}{\sqrt{(2+3) + (2\times3)}} + \left(9n^2 - 15n + 6\right)\frac{1}{\sqrt{(3+3) + (3\times3)}}$$
$$= \frac{9}{\sqrt{15}}n^2 + \left(\frac{12}{\sqrt{11}} - \frac{15}{\sqrt{15}}\right)n + \left(\frac{3}{\sqrt{2}} - \frac{12}{\sqrt{11}} + \frac{6}{\sqrt{15}}\right).$$

(5) By using the partition given in Table 7, we can apply the formula PGO index of G. Since

$$PGO(HC_n) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\left(d_G(u) + d_G(v)\right)\left(d_G(u)d_G(v)\right)}}$$

this implies that

$$PGO(HC_n) = 6\frac{1}{\sqrt{(2+2)(2\times2)}} + (12n-12)\frac{1}{\sqrt{(2+3)(2\times3)}} + \left(9n^2 - 15n + 6\right)\frac{1}{\sqrt{(3+3)(3\times3)}}$$
$$= \frac{9}{\sqrt{54}}n^2 + \left(\frac{12}{\sqrt{30}} - \frac{15}{\sqrt{54}}\right)n + \left(\frac{3}{2} - \frac{12}{\sqrt{30}} + \frac{6}{\sqrt{54}}\right).$$

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