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APPLICATION ON FUZZY METRICS SPACE AND COMMON FIXED POINT THEOREM

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ABSTRACT

In recent year, many author have focused on Common Fixed Point Theorem for Integer Type Mapping in Fuzzy Metric Space

Key words: *L fixed point, common fixed point, fuzzy metric space, compatability*

1. INTRODUCTION

In real world, the complexity generally arises from uncertainly in the form of ambiguity. The probability theory has been age old and effective tool to handle uncertainly, but it can be applied only to the situations whose characteristics are based on random processes, i.e., process in which the occurrence of events is strictly determined by chance. Uncertainly may arise due to partial information about the problem, or due to information which is not fully reliable, or due to inherent imprecision in the language with which the problem is defined or due to receipt of information from more than one source. Fuzzy set theory is an excellent mathematical tool to handle the uncertainly arising due to vagueness. In 1965, Lotfi A- Zadeh [30] propounded the fuzzy set theory in his paper.

The concept of Fuzzy sets was initially investigated by Zadeh [30] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [58] and modified by George and Veeramani [9]. Recently, Grebiec [10] has proved fixed point results for Fuzzy metric space. In the sequel, Singh and Chauhan [4] introduced the concept of compatible mappings of Fuzzy metric space and proved the common fixed point theorem. Jungck *et al.* [48] introduced the concept of compatible maps of type (A) in metric space and proved fixed point theorems. Cho [3] introduced the concept of compatible maps of type (α) and compatible maps of type (β) in fuzzy metric space. Using the concept of compatible maps of type (A), Jain *et al.* proved a fixed point theorem for six self maps in a fuzzy metric space. Using the concept of compatible maps of type (β), Jain *et al.* proved a fixed point theorem in fuzzy metric space. In this paper, a fixed point theorem for six self maps has been established using the concept of compatible maps of type (β) and weak compatible maps, which generalizes the result of Cho [3].

For the sake of completeness, we recall some definition and known results in Fuzzy metric space, which are used in this chapter.

Definition 1.1: Let X be any set. A fuzzy set in X is a function with domain X and values in $[0,1]$.

Definition 1.2: A binary operation $\star : [0,1] \times [0,1] \to [0,1]$ is continuous $t-$norm if $\star$ is satisfying the following conditions:
1.1 (a) $\star$ is commutative and associative,
1.2 (b) $\star$ is continuous,
1.2 (c) $a \star 1 = a$ for all $a \in [0,1]$
1.2 (d) $a \star b \le c \star d$ whenever $a \le c$ and $b \le d$,
for all $a, b, c, d \in [0,1]$
Examples of $t-$ norm are $a \star b = \min \{a, b\}$ and $a \star b = ab$.

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Definition 1.3: A triplet $(X, M, \star)$ is a fuzzy metric space whenever X is an arbitrary set, $\star$ is continuous t −norm and M is fuzzy set on $X \times X \times [0, \infty^{+})$ satisfying, for every $x, y, z \in X$ and $s, t > 0$, the following condition:
1.3 (a) $M(x, y, t) > 0$
1.3 (b) $M(x, y, 0) = 0$
1.3 (c) $M(x, y, t) = 1$ iff $x = y$
1.3 (d) $M(x, y, t) = M(y, x, t)$
1.3 (e) $M(x, y, t) \star M(y, z, s) \leq M(x, z, t + s)$
1.3 (f) $M(x, y, \cdot) : (0, \infty^{+}) \to [0,1]$ is continuous.

We note that, $M(x, y, t)$ can be realized as the measure of nearness between x and y with respect to t .It is known that $M(x, y, \cdot)$ is non decreasing for all $x, y \in X$.

Let $M(x, y, \star)$ be a fuzzy metric space for $t > 0$, the open ball

$$B(x, r, t) = \{y \in X: M(x, y, t) > 1 - r\}.$$

Now, the collection $\{B(x, r, t): x \in X, 0 < r < 1, t > 0\}$ is a neighborhood system for a topology τ on X induced by the fuzzy metric M. This topology is Housdroff and first countable.

Example 1.4: Let (X, d) be a metric space. Define $a \star b = \min\{a, b\}$ and $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ and all $t > 0$. Then $(X, M, \star)$ is a fuzzy metric space. It is called the fuzzy metric space induced by d.

Definition 1.5: A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, \star)$ is said to be
1. a converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.
2. Cauchy sequence converges to x iff for each $\varepsilon > 0$ and each $t > 0$, $n_0 \in N$ such that $M(x_m, x_n, t) > 1 - \varepsilon$ for all $m, n \geq n_0$.
3. Complete if every Cauchy sequence in it converges to a point in it.

Definition 1.6: Self mapping A and S of a fuzzy metric space $(X, M, \star)$ are said to be as follows:
1. compatible if and only if $M(ASx_n, SAx_n, t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some $p \in X$ as $n \to \infty$.
2. compatible of type (β) if and only if $M(AAx_n, SSx_n, t) \to 1$ for all $t > 0$, where $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p$ for some $p \in X$ as $n \to \infty$.

Note:
1. Two maps A and B from a fuzzy metric space $(X, M, \star)$ into itself are said to be weakly compatible if they commute at their coincidence points i.e., $Ax = Bx$ implies $ABx = BAx$ for some $x \in X$.
2. The concept of compatible map of type (β) is more general then the concept of compatible map in fuzzy metric space.

Definition 1.7: Let A and S be two self maps of a fuzzy metric space $(X, M, \star)$ then A and S is said to be a weakly commuting if $M(ASx_n, SAx_n, t) \leq M(Sx_n, Ax_n, t)$ for all x \in X.

It can be seen that commuting maps $(ASx = SAx \ \forall\ x \in X)$ are weakly compatible but converse is not true.

Lemma 1.8: In a fuzzy metric space $(X, M, \star)$ limit of a sequence is unique.

Lemma 1.9: Let $(X, M, \star)$ be a fuzzy metric space. Then for all $x, y \in X$ $M(x, y, .)$ is a non decreasing function.

Lemma 1.10: Let $(X, M, \star)$ be a fuzzy metric space. If there exists $k \in (0,1)$ such that for all $x, y \in X$,
1. $M(x, y, kt) \geq M(x, y, t) \ \forall t > 0, if\ x = y$.
2. Let $\{x_n\}$ be a sequence in a fuzzy metric space $(X, M, \star)$. If there exists a number $k \in (0,1)$ such that
$$M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t) \ \forall\ t > 0 \ and\ n \in N$$

Then $\{x_n\}$ is a Cauchy sequence in X.

Definition 1.11: Let A and S be two self maps of a fuzzy metric space $(X, M, \star)$ then A and S is said to be a weakly commuting if $M(ASx_n, SAx_n, t) \leq M(Sx_n, Ax_n, t)$ for all x \in X .

It can be seen that commuting maps $(ASx = SAx \ \forall\ x \in X)$ are weakly compatible but converse is not true.

Lemma 1.12: The only $t -$ norm $\star$ satisfying $r \star r = r$ for all $r \in [0,1]$ is the minimum $t -$ norm that is

$$a \star b = \min\{a, b\} \text{ for all } a, b \in [0,1].$$

Lemma 1.13: Let $(X, M, \star)$ be a fuzzy metric space. Then for all $x, y \in X$ $M(x, y, .)$ is a non decreasing function.

Lemma 1.14: Let $(X, M, \star)$ be a fuzzy metric space. If there exists $k \in (0,1)$ such that for all $x, y \in X$,

$$M(x, y, kt) \geq M(x, y, t) \; \forall t > 0, \; then\; x = y.$$

Lemma 1.15: Let $\{x_n\}$ be a sequence in a fuzzy metric space $(X, M, \star)$. If there exists a number $k \in (0,1)$ such that

$$M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t) \;\; \forall \;\; t > 0 \; and \; n \in N$$

Then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 1.16: The only $t-norm$ $\star$ satisfying $r \star r = r$ for all $r \in [0,1]$ is the minimum $t-norm$ that is

$$a \star b = \min\{a, b\} \;\text{ for all }\; a, b \in [0,1].$$

2. MAIN THEOREM

2.1 COMMON FIXED POINT THEOREM FOR INTEGER TYPE MAPPING IN FUZZY METRIC SPACE

On the way of generalization of Banach contraction principle one of the most famous generalization is introduced by Branciari [11] in general setting of lebgesgue integrable function and proved following fixed point theorems in metric spaces.

Theorem 2.1: Let (X, d) be a complete metric space, $\alpha \in (0,1)$ and let $T: X \to X$, be a mapping such that for each $x, y \in X$,

$$\int_0^{d(Tx,Ty)} \xi(v)\, dv \leq \int_0^{d(x,y)} \xi(v)\, dv$$

Where $\xi : [0, +\infty] \to [0, +\infty]$ is a lebgesgue integrable mapping which is summable on each compact subset of $[0, +\infty]$, non negative, and such that, $\forall\, \varepsilon > 0$, $\int_0^{\varepsilon} \xi(v)\, dv > 0$ Then, T has unique fixed point $z \in X$ such that for each $x \in X$, $T^n x \to z$ as $n \to \infty$.

It should be noted that if $\xi(v) = 1$ then Banach contraction principle is obtained.

Inspired from the result of Branciari [11] we prove following common fixed point theorems in fuzzy metric spaces.

Theorem 2.2: Let $(X, M, \star)$ be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

2.2(a) $P(X) \subset ST(X)$ and $Q(X) \subset AB(X)$,

2.2 (b) $AB = BA, ST = TS, PB = BP, QT = TQ$,

2.2 (c) either P or AB is continuous,

2.2 (d) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible,

2.2 (e) there exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\, dv \geq \int_0^{W(x,y,t)} \xi(v)\, dv$$

$$W(x, y, t) = \min\begin{Bmatrix} M^2(ABx, STy, t), \\ M^2(Qy, STy, t), \end{Bmatrix}$$

Where $\xi : [0, +\infty] \to [0, +\infty]$ is a lebgesgue integrable mapping which is summable on each compact subset of $[0, +\infty]$, non negative, and such that, $\forall\, \varepsilon > 0$, $\int_0^{\varepsilon} \xi(v)\, dv > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Let $x_0 \in X$, then from 3.3.2(a) we have $x_1, x_2 \in X$ such that

$$Px_0 = STx_1 \;\text{ and }\; Qx_1 = ABx_2$$

Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that for $n \in N$

$$Px_{2n-2} = STx_{2n-1} = y_{2n-1} \;\text{ and }\; Qx_{2n-1} = ABx_{2n} = y_{2n}$$

Step-1: Put $x = x_{2n}$ and $y = x_{2n+1}$ in 3.3.2 (e) then we have

$$\int_0^{M^2(Px_{2n},Qx_{2n+1},kt)} \xi(v)\, dv \geq \int_0^{W(x_{2n},x_{2n+1},t)} \xi(v)\, dv$$

$$W(x_{2n}, x_{2n+1}, t) = \min\begin{Bmatrix} M^2(ABx_{2n}, STx_{2n+1}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \end{Bmatrix}$$

$$\int_0^{M^2(y_{2n+1},y_{2n+2},kt)} \xi(v)\, dv \geq \int_0^{W(y_{2n+1},y_{2n+2},t)} \xi(v)\, dv$$

$$W(y_{2n+1}, y_{2n+2}, t) = \min\begin{Bmatrix} M^2(y_{2n}, y_{2n+1}, t), \\ M^2(y_{2n+2}, y_{2n+1}, t), \end{Bmatrix}$$

$$\int_0^{M^2(y_{2n+1},y_{2n+2},kt)} \xi(v)\, dv \geq \int_0^{\min\begin{Bmatrix} M^2(y_{2n},y_{2n+1},t), \\ M^2(y_{2n+2},y_{2n+1},t) \end{Bmatrix}} \xi(v)\, dv$$

From Lemma.1.13 and 1.14 we have

$$\int_0^{M^2(y_{2n+1},y_{2n+2},kt)} \xi(v)\ dv \geq \int_0^{M^2(y_{2n},y_{2n+1},t)} \xi(v)\ dv$$

Since $\xi(v)$ is Lesbesgue integrable function so that

$$M^2(y_{2n+1},y_{2n+2},kt) \geq M^2(y_{2n},y_{2n+1},t)$$

That is

$$M(y_{2n+1},y_{2n+2},kt) \geq M(y_{2n},y_{2n+1},t)$$

Similarly we have

$$M(y_{2n+2},y_{2n+3},kt) \geq M(y_{2n+1},y_{2n+2},t)$$

Thus we have

$$M(y_{n+1},y_{n+2},kt) \geq M(y_n,y_{n+1},t)$$
$$M(y_{n+1},y_{n+2},t) \geq M\left(y_n,y_{n+1},\frac{t}{k}\right)$$
$$M(y_n,y_{n+1},t) \geq M\left(y_0,y_1,\frac{t}{k^n}\right) \to 1 \text{ as } n \to \infty,$$

and hence $M(y_n,y_{n+1},t) \to 1$ as $n \to \infty$ for all $t > 0$.

For each $\epsilon > 0$ *and* $t > 0$, we can choose $n_0 \in N$ such that

$$M(y_n,y_{n+1},t) > 1-\epsilon \text{ for all } n > n_0.$$

For any $m,n \in N$ we suppose that $\geq n$. Then we have

$$M(y_n,y_m,t) \geq M\left(y_n,y_{n+1},\frac{t}{m-n}\right) \star M\left(y_{n+1},y_{n+2},\frac{t}{m-n}\right) \star \ldots. \star M\left(y_{m-1},y_m,\frac{t}{m-n}\right)$$
$$M(y_n,y_m,t) \geq (1-\epsilon) \star (1-\epsilon) \star \ldots\ldots \star (1-\epsilon)(m-n)\text{times}$$
$$M(y_n,y_m,t) \geq (1-\epsilon)$$

And hance $\{y_n\}$ is a Cauchy sequence in X.

Since $(X,M,\star)$ is complete, $\{y_n\}$ converges to some point $z \in X$. Also its subsequences converges to the same point $z \in X$.
That is

$$\{Px_{2n+2}\} \to z \text{ and } \{STx_{2n+1}\} \to z \quad 2.1(i)$$
$$\{Qx_{2n+1}\} \to z \text{ and } \{ABx_{2n}\} \to y_{2n} \quad 2.2(ii)$$

Case-1: Suppose AB is continuous

Since AB is continuous, we have

$$(AB)^2x_{2n} \to ABz \text{ and } ABPx_{2n} \to ABz$$

As (P,AB) is compatible pair of type (β), we have

$$M(PPx_{2n},(AB)(AB)x_{2n},t) = 1, \text{ for all } t > 0$$

Or $\quad M(PPx_{2n},ABz,t) = 1$

Therefore, $\quad PPx_{2n} \to ABz.$

Step-2: Put $x = (AB)x_{2n}$ and $y = x_{2n+1}$ in 2.2(e) we have

$$\int_0^{M^2(P(AB)x_{2n},Qy,kt)} \xi(v)\ dv \geq \int_0^{W(P(AB)x_{2n},Qy,kt)} \xi(v)\ dv$$
$$W(P(AB)x_{2n},Qy,t) = \min\begin{Bmatrix} M^2(AB(AB)x_{2n},STx_{2n+1},t), \\ M^2(Qx_{2n+1},STx_{2n+1},t), \end{Bmatrix}$$

Taking $n \to \infty$ we get

$$\int_0^{M^2(P(AB)x_{2n},Qy,kt)} \xi(v)\ dv \geq \int_0^{W(P(AB)x_{2n},Qy,t)} \xi(v)\ dv$$
$$M^2\big((AB)z,z,kt\big) \geq \min\begin{Bmatrix} M^2\big((AB)z,z,t\big) \\ M^2\big((AB)z,z,t\big) \end{Bmatrix}$$
$$\int_0^{M^2((AB)z,z,kt)} \xi(v)\ dv \geq \int_0^{\min\begin{Bmatrix} M^2((AB)z,z,t), \\ M^2((AB)z,z,t) \end{Bmatrix}} \xi(v)\ dv$$

That is form the property of $\xi(v)$ we have

$$M\big((AB)z, z, kt\big) \geq M\big((AB)z, z, t\big)$$

Therefore by lemma 3.1.14 we have

$$ABz = z. \qquad 2.2(iii)$$

Step -3 Put $x = z$ and $y = x_{2n+1}$ in 3.3.2(e) we have

$$\int_0^{M^2(Pz,Q\,x_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(Pz,Q\,x_{2n+1},t)} \xi(v)\ dv$$

$$W(Pz, Q\,x_{2n+1}, t) = \min\begin{Bmatrix} M^2(ABz, ST\,x_{2n+1}, t) \\ M^2(Q\,x_{2n+1}, ST\,x_{2n+1}, t) \end{Bmatrix}$$

Taking $n \to \infty$ and using equation 3.3.2 (i) we have

$$\int_0^{M^2(Pz,z,kt)} \xi(v)\ dv \geq \int_0^{W(Pz,z,t)} \xi(v)\ dv$$

$$W(Pz, z, t) \geq \min\begin{Bmatrix} M^2(ABz, z, t) \\ M^2(z, z, t) \end{Bmatrix}$$

So that $M^2(Pz, z, kt) \geq M^2(Pz, z, t)$

And hance $M(Pz, z, kt) \geq M(Pz, z, t)$

Therefore by using lemma 1.14, we get $Pz = z$

So we have $ABz = Pz = z$.

Step-4: Putting $x = Bz$ and $y = x_{2n+1}$ in 2.2(e), we get

$$\int_0^{M^2(PBz,Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(PBz,Qx_{2n+1},t)} \xi(v)\ dv$$

$$W(PBz, Qx_{2n+1}, t) = \min\begin{Bmatrix} M^2(ABBz, STx_{2n+1}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \end{Bmatrix}$$

As $BP = PB$ and $AB = BA$, so we have
$P(Bz) = B(Pz) = Bz$ and $(AB)(Bz) = (BA)(Bz) = B(ABz) = Bz.$

Taking $n \to \infty$ and using 2.2(i) we get

$$\int_0^{M^2(PBz,Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(PBz,Qx_{2n+1},t)} \xi(v)\ dv$$

$$\int_0^{M^2(Bz,z,kt)} \xi(v)\ dv \geq \int_0^{W(Bz,z,t)} \xi(v)\ dv$$

$$W(Bz, z, t) = \min\begin{Bmatrix} M^2(Bz, z, t), \\ M^2(z, z, t), \end{Bmatrix}$$

So we have $M^2(Bz, z, kt) \geq M^2(Bz, z, t)$

That is $M(Bz, z, kt) \geq M(Bz, z, t)$

Therefore by Lemma 1.14 we have $Bz = z$

And also we have $ABz = z$ implies $Az = z$

Therefore

$$Az = Bz = Pz = z. \qquad 2.2(iv)$$

Step–5: As $P(X) \subset ST(X)$ there exists $u \in X$ such that

$$z = Pz = STu$$

Putting $x = x_{2n}$ and $y = u$ in 2.2(e) we get

$$\int_0^{M^2(Px_{2n},Qu,kt)} \xi(v)\ dv \geq \int_0^{W(Px_{2n},Qu,t)} \xi(v)\ dv$$

$$W(Px_{2n}, Qu, t) = \min\begin{Bmatrix} M^2(ABx_{2n}, STu, t), \\ M^2(Qu, STu, t), \end{Bmatrix}$$

Taking $n \to \infty$ and using 2.2(i) and 2.2(ii) we get

$$\int_0^{M^2(z,Qu,kt)} \xi(v)\ dv \geq \int_0^{W(z,Qu,t)} \xi(v)\ dv$$

$$W(z, Qu, t) = \min\begin{Bmatrix} M^2(z, STu, t) \\ M^2(Qu, STu, t), \end{Bmatrix}$$

So we have $M^2(z, Qu, kt) \geq M^2(z, Qu, t)$

That is $M(z, Qu, kt) \geq M(z, Qu, t)$

Therefore by using Lemma 1.13 we have $Qu = z$

Hence $STu = z = Qu$.

Hence (Q, ST) is weak compatible, therefore, we have

$$QSTu = STQu$$

Thus $Qz = STz$.

Step–6: Putting $x = x_{2n}$ and $y = z$ in 2.2(e) we get

$$\int_0^{M^2(Px_{2n},Qz,kt)} \xi(v)\ dv \geq \int_0^{W(Px_{2n},Qz,t)} \xi(v)\ dv$$

$$W(Px_{2n}, Qz, t) = \min\begin{Bmatrix} M^2(ABx_{2n}, STz, t), \\ M^2(Qz, STz, t), \end{Bmatrix}$$

Taking $n \to \infty$ and using 2.2(ii) and step 5 we get

$$\int_0^{M^2(z,Qz,kt)} \xi(v)\ dv \geq \int_0^{W(z,Qz,t)} \xi(v)\ dv$$

$$W(z, Qz, t) = \min\begin{Bmatrix} M^2(z, STz, t) \\ M^2(Qz, STz, t), \end{Bmatrix}$$

That is $M^2(z, Qz, kt) \geq M^2(z, Qz, t)$

And hence $M(z, Qz, kt) \geq M(z, Qz, t)$

Therefore by using Lemma 1.13 we get $Qz = z$.

Step–7: Putting $x = x_{2n}$ and $y = Tz$ in 2.2(e) we get

$$\int_0^{M^2(Px_{2n},QTz,kt)} \xi(v)\ dv \geq \int_0^{W(Px_{2n},QTz,t)} \xi(v)\ dv$$

$$W(Px_{2n}, QTz, t) = \min\begin{Bmatrix} M^2(ABx_{2n}, STTz, t) \\ M^2(QTz, STTz, t) \end{Bmatrix}$$

As $QT = TQ$ and $ST = TS$ we have $QTz = TQz = Tz$

And $ST(Tz) = T(STz) = TQz = Tz$.

Taking $n \to \infty$ we get

$$\int_0^{M^2(z,Tz,kt)} \xi(v)\ dv \geq \int_0^{W(z,Tz,t)} \xi(v)\ dv$$

$$W(z, Tz, t) = \min\begin{Bmatrix} M^2(z, Tz, t), \\ M^2(Tz, Tz, t), \end{Bmatrix}$$

And hence $M^2(z, Tz, kt) \geq M^2(z, Tz, t)$

Therefore $M(z, Tz, kt) \geq M(z, Tz, t)$

Therefore by Lemma 1.13 we have $Tz = z$

Now $STz = Tz = z$ implies $Sz = z$.

Hence

$$Sz = Tz = Qz = z \qquad 2.2(v)$$

Combining 2.2(iv) and 2.2(v) we have

$$Az = Bz = Pz = Sz = Tz = Qz = z$$

Hence z is the common fixed point of A, B, S, T, P and Q.

Case-II: suppose P is continuous

As P is continuous

$$P^2x_{2n} \to Pz \text{ and } P(AB)x_{2n} \to Pz$$

As (P, AB) is compatible pair of type (β),

$$M(PPx_{2n}, (AB)(AB)x_{2n}, t) = 1 \text{ forall } t > 0$$

Or $$M(Pz, (AB)(AB)x_{2n}, t) = 1$$

Therefore $(AB)^2x_{2n} \to Pz$.

Step-8: Putting $x = Px_{2n}$ and $y = x_{2n+1}$ in 2.2(e) then we get

$$\int_0^{M^2(PPx_{2n},Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(PPx_{2n},Qx_{2n+1},t)} \xi(v)\ dv$$

$$W(PPx_{2n}, Qx_{2n+1}, t) = \min\begin{Bmatrix} M^2(ABPx_{2n}, STx_{2n+1}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \end{Bmatrix}$$

Taking n → ∞, we get

$$\int_0^{M^2(Pz,z,kt)} \xi(v)\ dv \geq \int_0^{W(Pz,z,t)} \xi(v)\ dv$$

$$W(Pz, z, t) = \min\{M^2(z, z, t), M^2(Pz, z, t)\}$$

$$\int_0^{M^2(Pz,z,kt)} \xi(v)\ dv \geq \int_0^{M^2(Pz,z,t)} \xi(v)\ dv$$

Therefore we have

$$M^2(Pz, z, kt) \geq M^2(Pz, z, t)$$

Hence $$M(Pz, z, kt) \geq M(Pz, z, t)$$

Therefore by Lemma 1.13 we get $Pz = z$

Step-9: Put $x = ABx_{2n}$ and $y = x_{2n+1}$ in 3.3.2(e) then we get

$$\int_0^{M^2(P\ ABx_{2n},Qx_{2n+1},kt)} \xi(v)\ dv \geq \int_0^{W(P\ ABx_{2n},Qx_{2n+1},t)} \xi(v)\ dv$$

$$W(P\ ABx_{2n}, Qx_{2n+1}, t) = \min\begin{Bmatrix} M^2(AB\ ABx_{2n}, STx_{2n+1}, t), \\ M^2(Qx_{2n+1}, STx_{2n+1}, t), \end{Bmatrix}$$

Taking n → ∞ we get

$$\int_0^{M^2(ABz,z,kt)} \xi(v)\ dv \geq \int_0^{W(ABz,z,t)} \xi(v)\ dv$$

$$W(ABz, z, t) = \min\begin{Bmatrix} M^2(AB\ z, z, t), \\ M^2(z, z, t), M^2 \end{Bmatrix}$$

$$\int_0^{M^2(ABz,z,kt)} \xi(v)\ dv \geq \int_0^{M^2(ABz,z,t)} \xi(v)$$

Therefore $M^2(ABz, z, kt) \geq M^2(ABz, z, t)$

And hence $M(ABz, z, kt) \geq M(ABz, z, t)$

By lemma 1.13 we get $ABz = z$

By applying step 4, 5, 6, 7, 8 we get

$$Az = Bz = Sz = Tz = Pz = Qz = z.$$

That is z is a common fixed point of A,B,S,T,P,Q in X.

Uniqueness: Let u be another common fixed point of A, B, S, T, P and Q. Then

$$Au = Bu = Su = Tu = Pu = Qu = u$$

Putting $x = u$ and $y = z$ in 2.1(e) then we get

$$\int_0^{M^2(Pu,Qz,kt)} \xi(v)\ dv \geq \int_0^{W(Pu,Qz,t)} \xi(v)\ dv$$

$$W(Pu, Qz, t) = \min\begin{Bmatrix} M^2(ABu, STz, t), \\ M^2(Qz, STz, t), \end{Bmatrix}$$

Taking limit both side then we get

$$\int_0^{M^2(u,z,kt)} \xi(v)\ dv \geq \int_0^{W(u,z,t)} \xi(v)\ dv$$
$$W(u,z,t) = \min\begin{Bmatrix} M^2(u,z,t), \\ M^2(z,z,t)\ , \end{Bmatrix}$$

That is $M^2(u,z,kt) \geq M^2(u,z,t)$

And hence $M(u,z,kt) \geq M(u,z,t)$

By lemma 1.13 we get $z = u$.

That is z is a unique common fixed point of A, B, S, T, P and Q in X.

Remark 2.3: Theorem 3.3.1 is a special case of the Theorem 3.3.2. It is sufficient if we take $\xi(v) = 1$ in Theorem 3.3.2.

Remark 2.4: If we take $B = T = I$ identity map on X in Theorem 2.2.3.2 then condition 2.3.2(b) is satisfy trivially and we get following Corollary

Corollary 2.5: Let $(X, M, \star)$ be a complete fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:
2.5(a) $P(X) \subset S(X)$ and $Q(X) \subset A(X)$,
2.5 (b) either P or AB is continuous,
2.5 (c) (P, AB) is compatible of type (β) and (Q, ST) is weak compatible,
2.5 (d) there exists $k \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$\int_0^{M^2(Px,Qy,kt)} \xi(v)\ dv \geq \int_0^{W(x,y,t)} \xi(v)\ dv$$
$$W(x,y,t) = \min\begin{Bmatrix} M^2(Ax,Sy,t), \\ M^2(Qy,Sy,t)\ , \end{Bmatrix}$$

Where $\xi : [0,+\infty] \to [0,+\infty]$ is a lebgesgue integrable mapping which is summable on each compact subset of $[0,+\infty]$, non negative, and such that, $\forall\ \varepsilon > 0$, $\int_0^{\varepsilon} \xi(v)\ dv > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

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